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Non-Extremal Black Holes of $N = 2, d = 5$ Supergravity

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Abstract

We study the generalization of the Ansatz of Galli *et al.* [1] for non-extremal black holes of $N = 2, d = 4$ supergravities for a simple model of $N = 2, d = 5$ supergravity with a vector multiplet whose moduli space has two branches. We use the formalism of Ferrara, Gibbons and Kallosh [2], which we generalize to any dimension d . We find that the equations of motion of the model studied can be completely integrated without the use of our Ansatz (which is, nevertheless, recovered in the integration). The family of solutions found (common to both branches) is characterized by five independent parameters: the mass M , the electric charges q_0, q_1 , the asymptotic value of the scalar at infinity ϕ_∞ and the scalar charge Σ . The solutions have a singular horizon whenever Σ differs from a specific expression $\Sigma_0(M, q_0, q_1, \phi_\infty)$ (*i.e.* when there is primary scalar hair $\Sigma - \Sigma_0 \neq 0$). The family of regular black holes interpolates between its two extremal limits. The supersymmetry properties of the extremal solutions depend on the choice of branch: one is always supersymmetric and the other non-supersymmetric in one branch and the reverse in the other one.

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Introduction

In a recent paper [1] Galli *et al.* proposed a general Ansatz to find non-extremal black-hole solutions of $N = 2, d = 4$ supergravity theories coupled to vector multiplets, that makes crucial use of the formalism developed by Ferrara, Gibbons and Kallosh (FGK) in ref. [2].³ The Ansatz consists of a systematic deformation of the corresponding supersymmetric (extremal) solutions to the same model which has to be plugged into the equations of motion derived by FGK to determine the values of the integration constants, something that needs to be done for each particular model.

The Ansatz can be generalized to higher dimensions by using the corresponding generalization of the FGK formalism, but it may only work for $N = 2$ -type theories for which the metric functions of supersymmetric solutions are homogenous of a certain degree in harmonic seed functions. In this paper we want to study a generalization of ref. [1]'s Ansatz for the $N = 2, d = 5$ supergravity case, and we will generalize the FGK formalism and the results obtained in refs. [12, 2] to arbitrary dimensions. We will then construct the non-extremal black-hole solutions of a simple model of $N = 2, d = 5$ supergravity with just one vector multiplet (and, therefore, one scalar field).

This paper is organized as follows: Section 1 is devoted to the generalization of the results of [2] to $d \geq 4$ dimensions. In Section 1.1 we adapt the results of the previous section to the particular case of $N = 2, d = 5$ theories with vector multiplets. In Section 2 we construct the general black-hole solutions of a simple model of $N = 2, d = 5$ supergravity, studying first the supersymmetric ones, which can be constructed using well-known recipes. Section 3 contains our conclusions.

1 The FGK formalism in $d \geq 4$

In order to generalize the results of ref. [2] to $d \geq 4$ we first need to find a suitable generalization of their radial coordinate τ , a goal that can be achieved relatively easily:⁴ consider the d -dimensional non-extremal Reissner-Nordström (RN) family of solutions. If we normalize the d -dimensional Einstein-Maxwell action as (see *e.g.* ref. [13])

$$\mathcal{I}[g_{\mu\nu}, A_\mu] = \frac{1}{16\pi G_N^{(d)}} \int d^d x \sqrt{|g|} \left[R - \frac{1}{4} F^2 \right], \quad (1.1)$$

where $G_N^{(d)}$ is the d -dimensional Newton constant. Then, the metric can be put in the form

$$ds^2 = H^{-2} W dt^2 - H^{\frac{2}{d-3}} \left[W^{-1} dr^2 + r^2 d\Omega_{(d-2)}^2 \right], \quad (1.2)$$

$$H = 1 + \frac{h}{r^{d-3}}, \quad W = 1 - \frac{2\mathcal{B}}{r^{d-3}}, \quad (1.3)$$

where $d\Omega_{(d-2)}^2$ is the metric of the unit $(d-2)$ -sphere and the constant h and the non-extremality parameter \mathcal{B} are given by⁵

$$\mathcal{B} = \frac{4\pi G_N^{(d)}}{(d-2)\omega_{(d-2)}} \sqrt{M^2 - 2 \frac{(d-2)}{(d-3)} q^2}, \quad h = \frac{4\pi G_N^{(d)}}{(d-2)\omega_{(d-2)}} M - \mathcal{B}. \quad (1.4)$$

³ For previous work on near-extremal and non-extremal solutions see *e.g.* refs. [3, 4, 5, 6, 7, 8, 9, 10, 11].

⁴ Observe that the case $d = 5$ was treated in ref. [11].

⁵ In $d = 4$, \mathcal{B} is usually called r_0 or c .

In the above expressions $\omega_{(d-2)}$ is the volume of the unit $(d-2)$ -sphere, M is the ADM mass and q the canonically-normalized electric charge.

The metric (1.2) describes the exterior of a RN black hole with the (outer) event horizon being located at $r^{d-3} = 2\mathcal{B} \geq 0$. The (inner) Cauchy horizon would, in principle, be located at $r^{d-3} = -h \leq 0$: this corresponds to a real value of r only for even d ; for odd d , the Cauchy horizon is not covered by these coordinates.

When $\mathcal{B} = 0$ the function W effectively disappears from the metric and we recover the extremal RN black hole in isotropic coordinates. As is well-known, in this limit there are many other solutions of the same form with H replaced by an arbitrary function harmonic on Euclidean \mathbb{R}^{d-1} . In this sense, the above non-extremal metric can be understood as a deformation of the extremal one by an additional *harmonic* function W (called *Schwarzschild* or *non-extremality factor*) containing the (non-)BPS parameter \mathcal{B} . This kind of deformations have been used to find non-extremal solutions in *e.g.* refs. [5, 11].⁶

If we perform the coordinate transformation

$$r^{d-3} = \frac{2\mathcal{B}}{1 - e^{-2\mathcal{B}\rho}}, \quad (1.5)$$

in the above metric we find that it takes the conformastatic form

$$ds^2 = e^{2U} dt^2 - e^{-\frac{2}{d-3}U} \gamma_{\underline{m}\underline{n}} dx^m dx^n, \quad (1.6)$$

where the function e^{2U} is given by

$$e^{2U} = \hat{H}^{-2} e^{-2\mathcal{B}\rho} \quad \text{with} \quad \hat{H} = \frac{h + 2\mathcal{B}}{2\mathcal{B}} - \frac{h}{2\mathcal{B}} e^{-2\mathcal{B}\rho}, \quad (1.7)$$

and the spatial background metric, γ , is given by

$$\gamma_{\underline{m}\underline{n}} dx^m dx^n = \left[\frac{\mathcal{B}}{\sinh(\mathcal{B}\rho)} \right]^{\frac{2}{d-3}} \left[\left(\frac{\mathcal{B}}{\sinh(\mathcal{B}\rho)} \right)^2 \frac{d\rho^2}{(d-3)^2} + d\Omega_{(d-2)}^2 \right]. \quad (1.8)$$

The coordinate ρ is the higher-dimensional generalization of the τ of ref. [2] we were looking for. In fact, in $d = 4$ their relation is $\rho = -\tau$. The main difference with τ is that the event horizon is at $\rho \rightarrow +\infty$ instead of $-\infty$; furthermore, the Cauchy horizon, which in $d = 4$ could be reached at $\tau \rightarrow +\infty$, is not covered by ρ because, in general, it cannot take negative values due to the fractional power in γ . In the extremal limit, *i.e.* when $\mathcal{B} \rightarrow 0$, the background metric takes the form

$$\gamma_{\underline{m}\underline{n}} dx^m dx^n = \frac{1}{\rho^{\frac{2}{d-3}}} \left[\left(\frac{d\rho}{(d-3)\rho} \right)^2 + d\Omega_{(d-2)}^2 \right], \quad (1.9)$$

which is nothing but the Euclidean metric on \mathbb{R}^{d-1} as can be seen by the coordinate change $\rho = r^{3-d}$; needless to say, in the limit $\mathcal{B} = 0$ the function \hat{H} becomes a harmonic function on \mathbb{R}^{d-1} .

It is reasonable to expect that all static black-hole metrics in $d \geq 4$ dimensions can be brought to the conformastatic form eq. (1.6) with the background metric (1.8). In the next section we will also assume that the metric function e^{-2U} of the non-extremal black holes of $N = 2$, $d = 5$ supergravity

⁶ As one can see from ref. [11] the solution for non-extreme black holes that we are going to construct, can, due to the special properties of supersymmetric couplings, be coordinate-transformed to a solution with a Schwarzschild factor.

can be obtained from that of the extremal ones by replacing the harmonic functions H_I by hatted harmonic functions of the form $\hat{H}_I = a_I + b_I e^{-\mathcal{B}\rho}$ and adding an overall factor of $e^{\mathcal{B}\rho}$ as in eq. (1.7).

Let us consider now the d -dimensional action

$$\mathcal{I}[g_{\mu\nu}, A^\Lambda{}_\mu, \phi^i] = \int d^d x \left\{ R + \mathcal{G}_{ij} \partial_\mu \phi^i \partial^\mu \phi^j + 2I_{\Lambda\Sigma} F^\Lambda{}_{\mu\nu} F^{\Sigma\mu\nu} \right\}, \quad (1.10)$$

where the $I_{\Lambda\Sigma}$ are given functions of the scalars ϕ^i and are supposed to form an invertible, negative definite matrix.

In $d > 4$ dimensions there could be higher-rank potentials in the action, but they should not couple to black holes. Of course, their consistent truncation from the action could place additional constraints on the remaining fields, but this analysis has to be made on a case by case basis and one could always impose those constraints on the solutions to the above unconstrained action. In odd dimensions there could also be Chern-Simons terms for the 1-forms $A^\Lambda{}_\mu$. However, those terms will only contribute to the equations of motion when we consider objects magnetically charged with respect to the 1-forms, *i.e.* electrically charged with respect to their dual $(d-3)$ -forms, but these would not be black holes in $d > 4$. Therefore, we can conclude that the above action is general enough to cover all or most of the possible (necessarily electrically) charged black-hole solutions in $d > 4$. In $d = 4$ there is an additional term involving only scalars and 1-forms related to the fact that only in $d = 4$ dimensions black holes can have magnetic charges on top of the electric ones.

Plugging the assumptions of time-independence of all fields and a metric of the form eqs. (1.6,1.8) into the equations of motion resulting from the action, and using the conservation of the electric charges q_Λ , we are left with a reduced system of differential equations in ρ that can be derived from the so-called *geodesic action*

$$\mathcal{I}[U, \phi^i] = \int d\rho \left\{ (\dot{U})^2 + \frac{(d-3)}{(d-2)} \mathcal{G}_{ij} \dot{\phi}^i \dot{\phi}^j - e^{2U} V_{\text{bh}} \right\}, \quad (1.11)$$

where the black-hole potential is given by

$$V_{\text{bh}} = \alpha^2 \frac{2(d-3)}{(d-2)} I^{\Lambda\Sigma} q_\Lambda q_\Sigma, \quad (1.12)$$

α being a constant related to the normalization of the charge to be determined later; one also finds a relation between the Hamiltonian corresponding to the action (1.11) and the non-extremality parameter \mathcal{B} , namely

$$(\dot{U})^2 + \frac{(d-3)}{(d-2)} \mathcal{G}_{ij} \dot{\phi}^i \dot{\phi}^j + e^{2U} V_{\text{bh}} = \mathcal{B}^2. \quad (1.13)$$

Assuming regularity of the fields at the horizon, it is possible to derive generalizations of the theorems proven in ref. [2]: for extremal black holes, in the $\rho \rightarrow +\infty$ limit

$$e^U \sim \frac{1}{\rho} \left[\frac{A_{\text{h}}}{\omega_{(d-2)}} \right]^{-\frac{(d-3)}{(d-2)}}, \quad (1.14)$$

where A_{h} is the area of the event horizon. Furthermore, this area is given by

$$A_{\text{h}} = \omega_{(d-2)} \left[-V_{\text{bh}}(\phi_{\text{h}}^i) \right]^{\frac{(d-2)}{2(d-3)}}, \quad (1.15)$$

where the values of the scalars at the horizon, ϕ_{h}^i , extremize the black-hole potential

$$\partial_i V_{\text{bh}}|_{\phi_{\text{h}}^i} = 0. \quad (1.16)$$

For general (extremal or non-extremal) black holes, defining the mass M and the scalar charges Σ^i by the asymptotic (*i.e.* $\rho \rightarrow 0$) behavior of the metric function and scalars as

$$U \sim -M\rho \quad , \quad \phi^x \sim \phi_\infty^x - \Sigma^x \rho \quad , \quad (1.17)$$

we find from eq. (1.13)

$$M^2 + \frac{(d-3)}{(d-2)} \mathcal{G}_{ij}(\phi_\infty) \Sigma^i \Sigma^j + V_{\text{bh}}(\phi_\infty, q) = \mathcal{B}^2 \quad . \quad (1.18)$$

Finally, the entropy $S = A_{\text{h}}/(4G_N^{(d)})$ and temperature, T , of the black-hole event horizon are related to the non-extremality parameter by generalization of the Smarr formula [12]

$$\mathcal{B} = \frac{16\pi G_N^{(d)}}{(d-3)\omega_{(d-2)}} ST \quad . \quad (1.19)$$

Observe that the mass M defined above is identically to the ADM mass if we set

$$\frac{8\pi G_N^{(d)}}{(d-2)\omega_{(d-2)}} = 1 \quad , \quad (1.20)$$

as we will do from now on,⁷ we have

$$S = \frac{2\pi}{(d-2)\omega_{(d-2)}} A_{\text{h}} \quad \text{whence} \quad ST = \frac{(d-3)}{2(d-2)} \mathcal{B} \quad . \quad (1.23)$$

1.1 The FGK formalism for $N = 2, d = 5$ theories

The relevant part of the bosonic action of $N = 2, d = 5$ supergravity theories coupled to n vector multiplets is, using the conventions of refs. [14, 15],

$$\mathcal{I}[g_{\mu\nu}, A^I_\mu, \phi^x] = \int d^5x \left\{ R + \frac{1}{2} g_{xy} \partial_\mu \phi^x \partial^\mu \phi^y - \frac{1}{8} a_{IJ} F^I_{\mu\nu} F^{J\mu\nu} \right\} \quad , \quad (1.24)$$

where $I, J = 0, 1, \dots, n$ and $x, y = 1, \dots, n$. The scalar target spaces are determined by the existence of $n + 1$ functions $h^I(\phi)$ of the n physical scalar subject to the constraint

$$C_{IJK} h^I h^J h^K = 1 \quad , \quad (1.25)$$

where C_{IJK} is a completely symmetric constant tensor that determines the model. Defining

$$h_I \equiv C_{IJK} h^J h^K \quad \text{(whence } h_I h^I = 1) \quad (1.26)$$

⁷ With this choice, to have canonically-normalized charges in the black-hole potential α must take the value

$$\alpha = \frac{(d-2)}{\sqrt{2}(d-3)} \quad . \quad (1.21)$$

This is not the most convenient normalization, though, because, with it, the relation between mass and charge of an extremal RN black hole is

$$M^2 = \frac{2(d-2)}{(d-3)} q^2 \quad , \quad (1.22)$$

and we will choose a different one in the next section.

the matrix a_{IJ} can be expressed as

$$a_{IJ} = -2C_{IJK}h^K + 3h_Ih_J, \quad (1.27)$$

and can be used to raise and lower the index of the functions h^I . We also define

$$h^I{}_x \equiv -\sqrt{3}\partial_x h^I, \quad h_{Ix} \equiv a_{IJ}h^J = +\sqrt{3}\partial_x h_I, \quad (1.28)$$

which are orthogonal to the h^I with respect to the metric a_{IJ} . Finally, the target-space metric is given by

$$g_{xy} \equiv a_{IJ}h^I{}_x h^J{}_y \xrightarrow{\text{which implies}} a^{IJ} = h^I h^J + g^{xy}h^I{}_x h^J{}_y. \quad (1.29)$$

Adapting the results of the previous section to these conventions and definitions we get the effective action

$$\mathcal{I}[U, \phi^x] = \int d\rho \left\{ (\dot{U})^2 + \frac{1}{3}g_{xy}\dot{\phi}^x\dot{\phi}^y - e^{2U}V_{\text{bh}} \right\}, \quad (1.30)$$

and Hamiltonian constraint (1.13) becomes

$$(\dot{U})^2 + \frac{1}{3}g_{xy}\dot{\phi}^x\dot{\phi}^y + e^{2U}V_{\text{bh}} = \mathcal{B}^2, \quad (1.31)$$

where the black-hole potential with the choice of normalization $\alpha^2 = 3/32$, is given by

$$-V_{\text{bh}} = a^{IJ}q_I q_J = \mathcal{Z}^2 + 3g^{xy}\partial_x \mathcal{Z} \partial_y \mathcal{Z}, \quad (1.32)$$

where we defined the *central charge* $\mathcal{Z}(\phi, q) \equiv h^I q_I$ and used eq. (1.29) in order to obtain the last expression. The supersymmetric black holes of these theories satisfy

$$\partial_x \mathcal{Z}|_{\phi_{\text{h}}} = 0 \xrightarrow{\text{whence}} \partial_x V_{\text{bh}}|_{\phi_{\text{h}}} = 0, \quad (1.33)$$

i.e. the values the physical scalar fields take at the horizon extremize the central charge and the black hole potential; in fact, all extremal black-hole solutions of the theory satisfy the latter equation but only the supersymmetric ones satisfy also the former. Furthermore, the supersymmetric ones saturate the BPS bound

$$M = \mathcal{Z}(\phi_{\infty}, q). \quad (1.34)$$

The supersymmetric, and therefore extremal, black-hole solutions [16, 17, 18] are completely determined by $n + 1$ real harmonic functions on Euclidean \mathbb{R}^4

$$I_I = I_{I\infty} + q_I \rho. \quad (1.35)$$

The fields of the supersymmetric solutions are related to these function by

$$e^{-U} h_I(\phi) = I_I. \quad (1.36)$$

These equations must be solved for $U = U_{\text{susy}}(I)$ and the physical scalars $\phi^x = \phi^x_{\text{susy}}(I)$ using the constraints of real special geometry.

Galli *et al.*'s Ansatz [1] for the non-extremal black-holes solutions is a deformation of the supersymmetric extremal solutions $U_{\text{susy}}(I), \phi^x_{\text{susy}}(I)$, namely

$$U = U_{\text{susy}}(\hat{I}) - 2\mathcal{B} \rho, \quad \phi^x = \phi^x_{\text{susy}}(\hat{I}), \quad (1.37)$$

where the functions \hat{I}_I have the form

$$\hat{I}_I = a_I + b_I e^{-2\mathcal{B}\rho}. \quad (1.38)$$

2 A simple model of $N = 2$, $d = 5$ supergravity and its black holes

Let us consider a simple model with one vector multiplet determined by $C_{011} = 1/3$;⁸ in terms of the physical, unconstrained, scalar ϕ we find that the model has two branches, labeled by $\sigma = \pm 1$:

$$\begin{aligned} h_{(\sigma)}^0 &= e^{\sqrt{\frac{2}{3}}\phi}, & h_{(\sigma)}^1 &= \sigma e^{-\frac{1}{\sqrt{6}}\phi}, \\ h_{(\sigma)0} &= \frac{1}{3}e^{-\sqrt{\frac{2}{3}}\phi}, & h_{(\sigma)1} &= \frac{2}{3}\sigma e^{\frac{1}{\sqrt{6}}\phi}. \end{aligned} \quad (2.1)$$

The scalar metric $g_{\phi\phi}$ and the vector field strengths metric a_{IJ} take exactly the same values in both branches:

$$g_{\phi\phi} = 1 \quad , \quad a_{IJ} = \frac{1}{3} \begin{pmatrix} e^{-2\sqrt{\frac{2}{3}}\phi} & 0 \\ 0 & e^{\sqrt{\frac{2}{3}}\phi} \end{pmatrix}, \quad (2.2)$$

and, therefore, the bosonic parts of both models and their classical solutions are identical. Since the functions $h_{(\sigma)}^I(\phi)$ differ, the fermionic structure will be different. In particular, the central charge in the σ -branch is

$$\mathcal{Z}_{(\sigma)} = q_0 e^{\sqrt{\frac{2}{3}}\phi} + \sigma q_1 e^{-\frac{1}{\sqrt{6}}\phi}. \quad (2.3)$$

The black-hole potential, being a property of the bosonic part of the theory, is identical in both branches:

$$-V_{\text{bh}} = \frac{3}{2} \left[2q_0^2 e^{2\sqrt{\frac{2}{3}}\phi} + q_1^2 e^{-\sqrt{\frac{2}{3}}\phi} \right]. \quad (2.4)$$

The black-hole potential is extremized for

$$\phi_{\text{h}} = -\sqrt{\frac{2}{3}} \log \left(\pm \sigma \frac{2q_0}{q_1} \right). \quad (2.5)$$

Since $\pm \sigma 2q_0/q_1 > 0$, the upper sign (which corresponds to the supersymmetric case in the σ -branch, as it extremizes the central charge) requires the following relation between the signs $s_I (\equiv q_I/|q_I|)$ of the charges q_I

$$s_0 = \sigma s_1, \quad (2.6)$$

while the lower one (non-supersymmetric in the σ -branch) requires

$$s_0 = -\sigma s_1. \quad (2.7)$$

The same bosonic solution will be supersymmetric in the σ -branch and non-supersymmetric in the $(-\sigma)$ -branch. We are going to construct the supersymmetric solutions of the σ -branch next; the non-supersymmetric solutions of the $(-\sigma)$ -branch will be constructed at the same time.

2.1 Supersymmetric and non-supersymmetric extremal solutions

According to the general prescription, the extremal solutions are given by two real harmonic functions of the form eq. (1.35), and are related to U and ϕ by eqs. (1.36), which in this case take the form

$$I_0 = \frac{1}{3} e^{-U_{\text{susy}}} e^{-\sqrt{\frac{2}{3}}\phi_{\text{susy}}}, \quad I_1 = \frac{2}{3} \sigma e^{-U_{\text{susy}}} e^{\frac{1}{\sqrt{6}}\phi_{\text{susy}}}. \quad (2.8)$$

⁸ This model can be obtained by dimensional reduction of minimal $d = 6$ $N = (1, 0)$ supergravity.

Solving for U_{susy} and ϕ_{susy} we get

$$e^{-U_{\text{susy}}} = \left(\frac{3^3}{2^2} I_0 I_1^2 \right)^{1/3}, \quad \phi_{\text{susy}} = -\sqrt{\frac{2}{3}} \log \left(\sigma \frac{2I_0}{I_1} \right). \quad (2.9)$$

The regularity and well-definedness of these fields impose some restrictions on the harmonic functions, to wit

- i) They should not vanish at any finite value of ρ : this requirement relates the signs of q_I and I_I .
- ii) $\text{sign}(I_0) = \sigma \text{sign}(I_1)$ everywhere for ϕ_{susy} to be well-defined in the σ -branch. This implies, in particular, that $s_0 = \sigma s_1$ which is the relation we found for the supersymmetric critical points. There are therefore for each branch two supersymmetric cases which are disjoint in charge space: $s_0 = +1, s_1 = \sigma$ and $s_0 = -1, s_1 = -\sigma$.
- iii) For U_{susy} to be well-defined ($e^{-U} > 0$) only $I_0 > 0$ seems to be allowed. However, if we take into account that the spatial metric eq. (1.9) is odd in ρ , we can compensate the wrong sign in e^{-U} with a change of sign in ρ .

In principle we have to consider the two aforementioned cases separately, but in the end both can be written in a unified way, with the harmonic functions given by

$$I_0 = \frac{1}{3} e^{-\sqrt{\frac{2}{3}} \phi_\infty} + |q_0| \rho, \quad I_1 = \sigma \left\{ \frac{2}{3} e^{\frac{1}{\sqrt{6}} \phi_\infty} + |q_1| \rho \right\}, \quad (2.10)$$

and the mass and scalar charge are given by

$$M = |\mathcal{Z}_{(\sigma)}(\phi_\infty, q)|, \quad \Sigma = 3 \partial_\phi \mathcal{Z}_{(\sigma)}(\phi_\infty, q). \quad (2.11)$$

Studying the near-horizon, *i.e.* $\rho \rightarrow \infty$, behavior we find that

$$\phi_{\text{susy}}|_{\text{h}} = -\sqrt{\frac{2}{3}} \log \left(\sigma \frac{2q_0}{q_1} \right), \quad (2.12)$$

$$\frac{A_{\text{h}}}{2\pi^2} = \sqrt{\frac{3^3}{2^2} |q_0| q_1^2} = [-V_{\text{bh}}(\phi_{\text{h}}, q)]^{\frac{3}{4}} = |\mathcal{Z}_{(\sigma)}(\phi_{\text{h}}, q)|^{\frac{3}{2}}. \quad (2.13)$$

These field configurations solve the same equations of motion all values of σ , but they are only supersymmetric in the σ -branch of the theory.

2.2 Non-extremal solutions

The most general solution can be obtained by observing that the geodesic Lagrangian is separable: by defining

$$x \equiv U + \sqrt{\frac{2}{3}} \phi, \quad y \equiv U - \frac{1}{\sqrt{6}} \phi, \quad (2.14)$$

the effective action eq. (1.30) takes the form

$$\mathcal{I}[x, y] = \int d\rho \left[\frac{1}{3} (\dot{x})^2 + \frac{2}{3} (\dot{y})^2 + 3q_0^2 e^{2x} + \frac{3}{2} q_1^2 e^{2y} \right], \quad (2.15)$$

and its equations of motion can be integrated immediately. We do not need to make any particular Ansatz, but should rather be able to recover it from the general solution, which is⁹

$$e^{-3U} = \frac{3^3}{2^2} |q_0 q_1^2| \left(\frac{\sinh(B\rho + D)}{B} \right)^2 \left(\frac{\sinh(A\rho + C)}{A} \right), \quad (2.16)$$

$$\phi = -\sqrt{\frac{2}{3}} \log \left\{ \left| \frac{2q_0}{q_1} \right| \left(\frac{B}{\sinh(B\rho + D)} \right) \left(\frac{\sinh(A\rho + C)}{A} \right) \right\}, \quad (2.17)$$

where A, B, C and D are (positive) integration constants. Their values are constrained by the requirement of asymptotic flatness and related to the non-extremality parameter \mathcal{B} by the Hamiltonian constraint eq. (1.31)

$$2B^2 + A^2 = 3\mathcal{B}^2. \quad (2.18)$$

There are, then, three independent integration constants that must correspond to the three independent physical parameters that are not the electric charges: the mass M , the asymptotic value of the scalar ϕ_∞ and the scalar charge Σ (according to eq. (1.18) \mathcal{B} is a function of these three). As the scalar charge is not an attribute of point-like objects, we do not expect the existence of regular black holes with $\Sigma \neq 0$ (scalar hair). However, we know that regular black holes with $\Sigma \neq 0$ exist when Σ is a function of the other physical parameters $\Sigma_0(M, q, \phi_\infty)$ (see *e.g.* the supersymmetric case studied in the previous section). This kind of hair is known as *secondary hair* [19], while $\Delta\Sigma \equiv \Sigma - \Sigma_0$ is called *primary hair* and its presence is generically associated to singularities.

In order to make contact with Galli *et al.*'s Ansatz, we rewrite eqs. (2.16) as

$$e^{-U} = e^{-U_{\text{susy}}(\hat{I})} e^{(A+2B)\rho/3}, \quad (2.19)$$

$$\phi = \phi_{\text{susy}}(\hat{I}) - \sqrt{\frac{2}{3}}(B - A)\rho, \quad (2.20)$$

where $e^{-U_{\text{susy}}(I)}$ is given in eqs. (2.9) and

$$\phi_{\text{susy}}(I) = -\sqrt{\frac{2}{3}} \log \left(\frac{2I_0}{I_1} \right), \quad (2.21)$$

so there is no distinction between the branches. The hatted ‘‘harmonic’’ functions are given by

$$\hat{I}_0 = \frac{1}{3} e^{-\sqrt{\frac{2}{3}}\phi_\infty} (2A)^{-1} \left\{ \left(A + M + \sqrt{\frac{2}{3}}\Sigma \right) + \left(A - M - \sqrt{\frac{2}{3}}\Sigma \right) e^{-2A\rho} \right\}, \quad (2.22)$$

$$\hat{I}_1 = \frac{2}{3} e^{\frac{1}{\sqrt{6}}\phi_\infty} (2B)^{-1} \left\{ \left(B + M - \frac{1}{\sqrt{6}}\Sigma \right) + \left(B - M + \frac{1}{\sqrt{6}}\Sigma \right) e^{-2B\rho} \right\}, \quad (2.23)$$

and the constants A and B are given by, taking the positive roots,

$$A = \sqrt{\left(M + \sqrt{\frac{2}{3}}\Sigma \right)^2 - 3^2 q_0^2 e^{2\sqrt{\frac{2}{3}}\phi_\infty}}, \quad (2.24)$$

$$B = \sqrt{\left(M - \frac{1}{\sqrt{6}}\Sigma \right)^2 - \frac{3^2}{2^2} q_1^2 e^{-\sqrt{\frac{2}{3}}\phi_\infty}}. \quad (2.25)$$

⁹ Please observe that this solution could also have been obtained by using the results obtained by Mohaupt & Vaughan in ref. [11].

A necessary condition for the solutions to become a product spacetime in the $\rho \rightarrow +\infty$ limit, thus signaling the occurrence of a horizon, can be read off from eq. (2.19): $A + 2B = \mathcal{B}$. This constraint together with the Hamiltonian constraint (2.18) implies not only $A = B = \mathcal{B} \equiv \mathcal{B}_0$, but also $\Sigma = \Sigma_0$ with¹⁰

$$\Sigma_0 = -\sqrt{6} \left\{ M - \sqrt{M^2 + 3q_0^2 e^{2\sqrt{\frac{2}{3}}\phi} - \frac{3}{4}q_1^2 e^{-\sqrt{\frac{2}{3}}\phi}} \right\}. \quad (2.26)$$

In that case, the form of the non-extremal solution is the one proposed by Galli *et al.* as a deformation of the supersymmetric one. In what follows we will only consider the regular solutions with no primary scalar hair $\Sigma = \Sigma_0$, $\mathcal{B} = \mathcal{B}_0$. It is useful to express these constants in terms of the asymptotic values of the central charges of the two branches of the supersymmetric theory $\mathcal{Z}_{(+)}$ and $\mathcal{Z}_{(-)}$:

$$\Sigma_0 = -\sqrt{6} \left\{ M - \sqrt{C} \right\} \quad \text{and} \quad \mathcal{B}_0^2 = 5M^2 - 3\mathcal{Z}_{(+)\infty} \tilde{\mathcal{Z}}_{(-)\infty} - 4M\sqrt{C}. \quad (2.27)$$

where

$$C \equiv M^2 + \frac{3}{16} \left(3\mathcal{Z}_{(+)\infty}^2 + 3\mathcal{Z}_{(-)\infty}^2 + 10\mathcal{Z}_{(+)\infty}\mathcal{Z}_{(-)\infty} \right). \quad (2.28)$$

Further conditions for regularity of the bh's are the reality and positivity of \mathcal{B}_0^2 , which is the case if

$$M^2 \geq \mathcal{Z}_{(+)\infty}^2 \quad \text{and} \quad M^2 \geq \mathcal{Z}_{(-)\infty}^2. \quad (2.29)$$

\mathcal{B}_0 vanishes only when one of the bounds is saturated, so there are in a given σ -branch two extremal limits: one is supersymmetric and the other non-supersymmetric.

At the horizon, the scalar goes to the finite, yet ϕ_∞ -dependent value

$$\phi_h = \phi_\infty - \sqrt{\frac{2}{3}} \log \left(\frac{\mathcal{B}_0 - M + 2\sqrt{C}}{\mathcal{B}_0 + 2M - \sqrt{C}} \right). \quad (2.30)$$

The area of the horizon is easily found to be

$$\frac{A_h}{2\pi^2} = \sqrt{\left(\mathcal{B}_0 - M + 2\sqrt{C} \right) \left(\mathcal{B}_0 + 2M - \sqrt{C} \right)^2}, \quad (2.31)$$

and the entropy can be computed from eq. (1.23) $S = A_h/3\pi$. Also, using eq. (1.23) the temperature is just $T = \mathcal{B}_0/(3S)$ and vanishes in the extremal limits.

Let us end this section with a quick word on the extremal solutions: as we found in the previous section the general family of non-extremal solutions has two extremal limits, namely one given by $M = |\mathcal{Z}_{(+)\infty}|$ and the other one by $M = |\mathcal{Z}_{(-)\infty}|$; the supersymmetry properties of the limiting solution will depend on the choice of branch. In order to study them we have to take into account that when one of the extremality bounds eq. (2.29) is saturated, the other one still holds. In other words: if (the absolute values of) the two supercharges are different, the first bound that becomes saturated when we vary the mass, will correspond to that of the largest supercharge. Which supercharge is largest depends on the signs of the charges:

$$\begin{aligned} s_0 = s_1, & \quad \Rightarrow \quad |\mathcal{Z}_{(+)\infty}| \geq |\mathcal{Z}_{(-)\infty}|, \\ s_0 = -s_1, & \quad \Rightarrow \quad |\mathcal{Z}_{(-)\infty}| \geq |\mathcal{Z}_{(+)\infty}|. \end{aligned} \quad (2.32)$$

As in the 4-dimensional examples studied in ref. [1], the values of the charges determine completely the extremal limit. Taking this into account is easy to see that we recover the extremal solutions found before, whose supersymmetry properties depend on our choice of branch.

¹⁰ Only one of the solutions of the second degree equation for Σ_0 is valid, *i.e.* gives rise to regular black holes.

3 Conclusions

In this paper we have studied the generalization of the formalism of Ferrara, Gibbons and Kallosh [2] to higher dimensions and we have applied it to the construction of the non-extremal solutions of a simple model of $N = 2, d = 5$ supergravity with just one modulus to check a proposal for a generalization of the Ansatz of [1] to higher dimensions.

Instead of using this Ansatz directly, we have been able to integrate directly the effective equations of motion of the model and we have found a general solution with an independent scalar charge parameter Σ . Only when Σ is related to the mass, electric charges and asymptotic value of the scalar by a given formula $\Sigma = \Sigma_0(M, q_0, q_1, \phi_\infty)$ the solutions are regular, *i.e.* black hole solutions and not naked singularities. We can interpret these regular solutions as not having *primary scalar hair* in the sense of ref. [19] and their form fits perfectly in [1]’s Ansatz.

Only a few examples of general families of solutions including singular solutions with and regular solutions without primary scalar hair are known [20]. Most of the solutions known have only secondary hair: their scalar charges are related to the masses, charges, and asymptotic values of the moduli by certain expressions. In the supersymmetric cases these expressions are related to the asymptotic values of the derivatives of the central charges (or to the matter central charges¹¹ but in the general case it is not known how to determine them before finding the explicit solutions. This is an important problem for which no solution has been proposed.

Here we have dealt with an extremely simple model. It is clear that to confirm (or refute) the validity of [1]’s Ansatz more examples need to be studied. Work in this direction is in progress.

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¹¹ This follows from the general relation eq. (1.18) for $\mathcal{B} = 0$ plus the BPS bound $M = |\mathcal{Z}|$ and the expression of the black-hole potential in terms of the central charges.

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