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Ultracold spherical horizons in gauged $N = 1$, $d = 4$ supergravity

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ABSTRACT

We show that the near-horizon limit of ultracold magnetic Reissner–Nordström–De Sitter black holes, whose geometry is the direct product of 2-dimensional Minkowski spacetime and a 2-sphere, preserves half of the supersymmetries of minimal R-gauged $N = 1$, $d = 4$ supergravity.

The supersymmetric black-hole solutions of 4-dimensional supergravity theories can often be understood as solitons interpolating between two maximally supersymmetric vacua of the theory to which they approach in the far-field and near-horizon (NH) regions. The NH geometry (the product space $aDS_2 \times S^2$, known as Bertotti–Robinson solution, in the typical asymptotically-flat cases) contains a great deal of information about the constituents of the original solution and is amenable to a dual description by a gauge theory living in the boundary of the aDS_2 space. In particular, the radius of the S^2 factor of the NH geometry, which corresponds to a horizon with the same topology, is directly related to the entropy. The sufficiency of the NH description to describe the black-hole entropy, independently of the asymptotic behavior of the solution, is a consequence of the attractor mechanism [1].

The topology of the spatial factor of the NH solution (S^2 in the above example) coincides with that of the spatial sections of the black-hole horizon. In the 4-dimensional, asymptotically flat (vanishing cosmological constant) case, a classical theorem by Hawking [2][1] and the "topological censorship theorem" of Ref. [4] constrain that topology to be that of S^2. However, black holes with event horizons topologically inequivalent to S^2 have been discovered in dimensions higher than four [5,6][2]; in four dimensions and in presence of a negative cosmological constant *topological black holes* with horizons which are Riemann surfaces or arbitrary genus have also been constructed [8].

The requirement of unbroken supersymmetry of the NH solution strongly constrains the possible NH geometries and horizon topologies. In Ref. [9] Reall showed that, in 5-dimensional supergravity, supersymmetry only allows three possible horizon topologies: T^3, $S^1 \times S^2$ (the topology of the supersymmetric black ring of Ref. [6]) and (possibly a quotient of) a homogeneously squashed S^3. On the other hand, in Ref. [10] it was shown that only the genus bigger than one horizons may have unbroken supersymmetry in minimal gauged $N = 2$, $d = 4$ supergravity.

In a recent paper [11] Gutowski and Papadopoulos have studied possible topologies of supersymmetric horizons of black hole solutions of $N = 1$, $d = 4$ supergravity finding that, regardless of the supersymmetry properties of the complete black-hole solution,[3] if the horizon is compact and supersymmetric (i.e. if the NH geometry is), then its constant time sections have to be, topologically, tori. Our purpose in this note is to investigate possible simple realizations of these NH geometries in simple $N = 1$, $d = 4$ supergravity theories.

Since, in order to have topological black hole solutions, we need a negative cosmological constant, we should consider a $N = 1$, $d = 4$ theory providing a minimal supersymmetric embedding of the cosmological Einstein–Maxwell (EM-Λ) theory

$$S = \int dx^4 \sqrt{|g|} \left\{ R - F^2 - \Lambda \right\}, \tag{1}$$

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[1] A recent paper with references on the generalization of this theorem to higher dimensions and non-vanishing cosmological constant is [3].

[2] For a recent review with references, see [7].

[3] There are no supersymmetric asymptotically flat black-hole solutions in $N = 1$, $d = 4$ supergravities [12,13]. No supersymmetric black holes with other asymptotic behaviors are known, either.

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for negative (*aDS*) cosmological constant Λ. However, just to be general (which will prove fortunate in he end), we are also going to consider another $N=1$, $d=4$ supergravity theory providing a supersymmetric embedding of EM-Λ for positive (*DS*) cosmological constant.

The bosonic equations of motion of these two theories take the common form

$$R_{\mu\nu} = \frac{\Lambda}{2} g_{\mu\nu} + 2\left[F_\mu{}^\rho F_{\nu\rho} - \frac{1}{4} g_{\mu\nu} F^2\right], \tag{2}$$

$$d \star F = 0. \tag{3}$$

The two theories that we are going to consider have the same matter content: an Abelian vector supermultiplet $\{A_\mu, \lambda\}$ coupled to the $N=1$, $d=4$ supergravity multiplet $\{e^a_\mu, \psi_\mu\}$. The first theory, constructed by Townsend in Ref. [16], has, in more modern parlance, a constant superpotential $W=g/2$ which gives a negative cosmological constant $\Lambda = -8g^2$ and possesses a maximally supersymmetric aDS_4 solution. In the second theory, constructed by Freedman in Ref. [17], the Abelian vector field is used to gauge the global $U(1)$ R-symmetry via a Fayet–Iliopoulos term which gives a positive cosmological constant $\Lambda = +g^2/2$ where g is the gauge coupling constant. These two possibilities cannot be combined because the constant superpotential breaks R-symmetry.

Although the bosonic sectors of these two theories are identical, up to the sign of the cosmological constant, the couplings of the fermionic sectors and the supersymmetry transformations are substantially different, which results in very different supersymmetric configurations even though all the Killing spinors of the supersymmetric configurations of any $N=1$, $d=4$ supergravity must satisfy the condition [12,13]

$$\gamma^u \epsilon = 0, \tag{4}$$

where u is a null coordinate, or, equivalently

$$\gamma^{01}\epsilon = \pm\epsilon. \tag{5}$$

1. Supersymmetry of Plebański–Hacyan geometries

We are going to consider configurations whose metric is the direct product of two 2-dimensional subspaces of constant curvature, the first one parametrized by the first two (timelike and spacelike) coordinates and the second one parametrized by the last two (spacelike) coordinates. This generic class of solutions to EM-Λ was first obtained by Plebański and Hacyan in Ref. [14], and includes as special cases the Bertotti–Robinson solution ($aDS_2 \times S^2$) and the Nariai universe ($DS_2 \times S^2$) [15], whose discovery predates the work [14].

The geometry of the purely spacelike 2-dimensional subspace is expected to correspond to that of the constant-time sections of a black-hole horizon. The Maxwell field will have non-vanishing components $F_{01} = \alpha$ and $F_{23} = \beta$, where α and β are real constants (that is: the components of the Maxwell field are proportional to the volume 2-forms of the two subspaces). We will make this more precise Ansatz later on.

1.1. $N=1$, $d=4$ *supergravity with constant superpotential*

As was mentioned before, the minimal version of this theory was constructed by Townsend in Ref. [16] and when coupled to a vector multiplet corresponds to a supersymmetric version of the EM-Λ theory with the cosmological constant $\Lambda = -8g^2$ being of the anti-De Sitter kind. The supersymmetry transformations of the fermions for vanishing fermions are[4]

$$0 = \delta_\epsilon \psi_\mu = \nabla_\mu \epsilon + \frac{i}{2} g \gamma_\mu \epsilon^*, \tag{6}$$

$$0 = 2\delta_\epsilon \lambda = \not{F}^+ \epsilon, \tag{7}$$

where ∇ is the general and Lorentz-covariant derivative.

That this theory does not admit supersymmetric solution of the type we are after is easily deduced by calculating the integrability condition for Eq. (6):

$$[\not{R}_{\mu\nu} + g^2 \gamma_{\mu\nu}]\epsilon = 0. \tag{8}$$

The split into 2-dimensional spaces of constant curvature, implies that e.g. $\not{R}_{02} = 0$, which immediately implies that $\epsilon = 0$, whence no supersymmetric PH solutions exist.

1.2. *Minimal gauged* $N=1$, $d=4$ *supergravity*

This $N=1$, $d=4$ theory was constructed by Freedman in Ref. [17] and has the curiosity that it corresponds to supergravity theory with a De Sitter-like cosmological constant ($\Lambda = g^2/2$). The relevant supersymmetry transformations for vanishing fermions are

$$0 = \delta_\epsilon \psi_\mu = \left[\nabla_\mu + \frac{i}{2} g A_\mu\right]\epsilon, \tag{9}$$

$$0 = \delta_\epsilon \lambda = \left[\not{F}^+ - \frac{i}{2} g\right]\epsilon. \tag{10}$$

De Sitter spacetime is a solution of the theory but breaks all supersymmetries.

The Killing spinor equation (10) only admits solutions for our Ansatz if $\alpha = 0$ and

$$\beta = \pm g/2 \quad \text{and} \quad [1 \pm i\gamma^{23}]\epsilon = 0, \tag{11}$$

so that we are dealing with a purely magnetic configuration.

The integrability condition of the Killing spinor equation (9) reads

$$[\not{R}_{\mu\nu} - igF_{\mu\nu}]\epsilon = 0. \tag{12}$$

The product structure of the metric that we have assumed indicates that the first factor must be flat 2-dimensional Minkowski spacetime and the second a 2-sphere whose curvature is related to β and, therefore, to g.

At this point a more precise form for the Ansatz becomes necessary: using standard spherical coordinates for the 2-sphere we write

$$ds^2 = dt^2 - dx^2 - R^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right),$$
$$A_\phi = -\beta R^2 \cos\theta. \tag{13}$$

The non-vanishing components of the Ricci and Maxwell field strength tensors are, in the obvious tetrad basis

$$R_{22} = R_{33} = -\frac{1}{R^2}, \qquad F_{23} = \beta. \tag{14}$$

The Maxwell equations are automatically solved as the field strength is an invariant 2-form on a symmetric space; the Einstein equations are solved if

[4] For clarity's sake we mention that we are using a normalized version of the slash, i.e. for the 2-form F we have $2\not{F} \equiv F_{ab}\gamma^{ab}$.

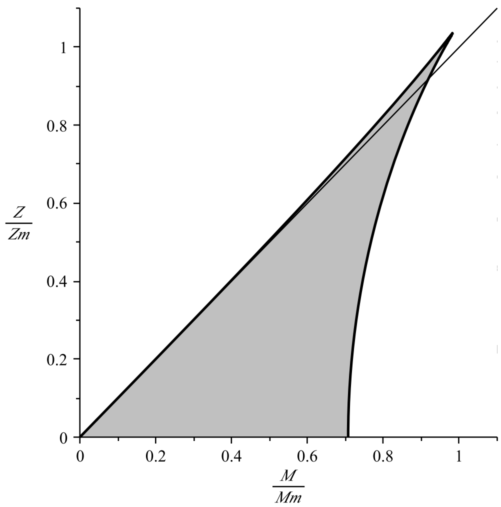

Fig. 1. A plot of the values of M and Z for which the RNDS black holes exist. The straight line are the extreme bh's, i.e. the ones for which $M^2 = Z^2$.

$$R^2 = \frac{g^2}{4} + \beta^2, \tag{15}$$

which due to Eq. (11) implies:

$$R = \sqrt{2}/g. \tag{16}$$

In order to finish the analysis we need to solve the Killing spinor equations (9); the 0, 1, 2 components are trivial and are solved for any t, x and θ independent spinor. The last component is also trivially satisfied once we take into account the following relation between the spin and the gauge connections $A_3 = \pm g^{-1}\omega_{323}$ and use the projection in Eq. (11).

In conclusion we found a half-BPS solution to Freedman's gauged $N = 1$, $d = 4$ supergravity that is purely magnetic and whose geometry is $\mathbb{R}^{1,1} \times S^2$. The obvious question then is: can this geometry be the NH limit of a black hole? A first naive worrisome point is about the occurrence of the $\mathbb{R}^{1,1}$ factor in the NH geometry, as the usual one of supersymmetric black holes would not give rise to $\mathbb{R}^{1,1}$ but rather to aDS_2. But, as said, this is a naive preoccupation as, following Gutowski and Papadopoulos, we are asking for the NH-geometry to be supersymmetric and not the complete solution. If we then couple this to the fact that the NH geometry of black holes with non-vanishing temperature, such a Schwarzschild's, leads to a 2-dimensional Rindler space which is locally isometric to $\mathbb{R}^{1,1}$, the preoccupation should cease to exist. So in order to find the candidate black hole whose NH-limit gives rise to the supersymmetric solution, we should analyze the NH-limits of magnetically-charged black holes with spherical topology in De Sitter spaces.

2. Reissner–Nordström–De Sitter black holes

The Reissner–Nordström–De Sitter (RNDS) black holes can be written in standard coordinates as

$$ds^2 = f\,dt^2 - f^{-1}\,dr^2 - r^2\,dS^2_{[\theta,\varphi]}, \tag{17}$$

$$A = \frac{Q}{r}\,dt - P\cos\theta\,d\varphi, \tag{18}$$

where $dS^2_{[\theta,\varphi]}$ stands for the round metric on S^2 with coordinates θ and φ, and the function $f = f(r)$ is given by

$$f = -\frac{\Lambda}{6}r^2 + 1 - \frac{2M}{r} + \frac{Z^2}{r^2}, \quad \text{with } Z^2 \equiv Q^2 + P^2. \tag{19}$$

As is well known, De Sitter black holes need not exist for all values of the mass, M, and the electro-magnetic charge, Z; a plot of the pairs (M, Z) that can give rise to black holes are indicated in Fig. 1 by the grey area and its boundary. As is paramount from the figure M and $|Z|$ are bounded by maximal values that in our normalization of Λ are given by

$$M_{crit} = \frac{2}{3\sqrt{\Lambda}} = \frac{2\sqrt{2}}{3g} \quad \text{and} \quad Z^2_{crit} = \frac{1}{2\Lambda} = \frac{1}{g^2}. \tag{20}$$

A point in the grey area corresponds to a black hole with three horizons, namely an inner one at $r = r_i$, an outer one at $r = r_o$ and a cosmological horizon at $r = r_c$, the nomenclature deriving from the fact that $0 < r_i < r_o < r_c$. Furthermore, all these horizons are *warm* in the sense that they correspond to single zeroes of f, whence one can associate a temperature to at least the outer and the cosmological horizon.[5]

The left boundary corresponds to those black holes for which the inner and the outer horizon coincide $0 < r_i = r_o < r_c$, implying that this coincident horizon, but not the cosmological horizon, has zero temperature: these black holes are called *cold black holes*. The right boundary corresponds to the situation where the outer and the cosmological horizons coincide $0 < r_i < r_o = r_c$ and are also cold black holes; they receive the name *Nariai* black holes. The intersection of these two boundaries, corresponding to the pair (M_{crit}, Z_{crit}), for which all three horizons coincide, goes by the name *ultracold black hole* [18].

This small discussion then brings us to the question: How are we to identify the RNDS black-hole solution whose NH limit gives us the supersymmetric Plebański–Hacyan solution? The answer is simple: by looking at the NH limit of the gauge field! First of all, a non-zero Q would lead to a non-zero F_{01} so we will take $Q = 0$. The NH limit of the vector field strength for a horizon located at $r = r_H$ is

$$F = d(-P\cos\theta\,d\varphi) = P\,d\theta \wedge \sin\theta\,d\varphi \to \frac{P}{r_H^2} e^2 \wedge e^3, \tag{21}$$

and leads to the identification that $P = \beta r_H^2$. Seeing that the value of β for the supersymmetric solution is given in Eq. (11) and that r_H is effectively the radius of the 2-sphere in the NH limit, Eq. (16), we can deduce that our candidate black hole must have

$$P = \beta r_H^2 = \pm\frac{g}{2}\left(\frac{\sqrt{2}}{g}\right)^2 = \pm 1/g, \tag{22}$$

implying that our candidate black hole is none other than the ultracold black hole.

This poses, however, an immediate problem, one already pointed out by Romans [18]: as the horizon of the ultracold black hole corresponds to a triple zero of the function f in Eq. (19), the naive NH limit does not give as NH geometry Rindler spacetimes S^2 but a different one, one that is not even a solution to the equations of motion: the reason for this is that in this case the usual procedure of zooming in does not conform to Geroch's criteria of limiting spaces [19].

There is an alternative limiting procedure that does give rise to the desired result [20,21] which basically consists in going first to the cold limit in which $f(r)$ has a double zero and then taking the NH limit simultaneously with the ultracold limit in a particular

[5] As is well known, by expanding f in Eq. (19) around the horizon location $r = r_H$ as $f = (r - r_H)\,h(r)$ with h being regular at r_H, one finds that the NH geometry is that of a Rindler space of temperature $T = h(r_H)/(4\pi)$ times a 2-sphere of radius r_H.

way. The result is the supersymmetric Plebański–Hacyan solution[6] which can, therefore, be identified as the NH limit of the ultracold, purely magnetic, RNDS black hole.

3. Conclusions

In this Letter we have tried to find simple examples of supersymmetric horizons in $N=1$, $d=4$ supergravity theories motivated by the prediction made in Ref. [11] that, if any, their spatial sections would always be topologically equivalent to tori. We have focused on two $N=1$, $d=4$ theories (Freedman's and Townsend's) whose bosonic sector is the cosmological Einstein–Maxwell theory with positive and negative cosmological constant, respectively, and on candidate near-horizon geometries which are the direct product of two 2-dimensional spaces of constant curvature. We have shown that none of our candidates is supersymmetric in Townsend's theory ($\Lambda < 0$) but we have also shown that one of them, with the geometry Minkowski$_2 \times S^2$ is actually supersymmetric in Freedman's ($\Lambda > 0$). Then we have shown that this supersymmetric solution is the NH limit of the ultracold RNDS black-hole solution when the NH limit is correctly computed, which means that, even though no RNDS black-hole solution is supersymmetric, the horizon of the ultracold one, which has the topology of S^2, is. We can also say that the non-supersymmetric ultracold RNDS black hole solution interpolates between non-supersymmetric DS spacetime at infinity and a half-supersymmetric Plebański–Hacyan solution at the horizon.

This result is a clear counterexample for the generic prediction of Ref. [11]. The reason why our spherically-symmetric NH geometry was missed is, as far as we can see, that the analysis made in that reference is based on a gravitino Killing spinor equation that is not general enough, and in particular does not include Freedman's theory.

Of course, our results do not imply that these are the only possible supersymmetric NH geometries nor that Freedman's theory and its generalizations are the only possible $N=1$, $d=4$ supergravities in which supersymmetric NH geometries can be found.

At this moment we do not have a clear physical interpretation of this result. We can only stress the fact that the supersymmetric solution has mass and magnetic charge which are extremized for a given value of the cosmological/coupling constant. Furthermore, we would like to point out that, while Townsend's theory is sometimes called $N=1$, $d=4$, *aDS* supergravity, Freedman's (studied, for instance, in Refs. [22,23]) is very different from a naive (and inconsistent) $N=1$, $d=4$, *DS* supergravity and can be embedded in string theory [24].

As a final comment let us point out that a fake version of Freedman's gauged supergravity can be constructed and the existence of fake-supersymmetric NH-geometries can be studied, which shows that indeed there is a fake-supersymmetric $aDS_2 \times \Sigma^2_{g>1}$ solution. One can then also show that there is no *aDS*-black hole which has this NH-geometry.

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[6] Notice that we can arrive at the same result in a more pedestrian way by taking the NH limit of a warm or a cold horizon in a first step and then taking the ultracold limit in a second step. In the first case, we arrive at the NH geometry Rindler$_2 \times S^2$ in the first step and then adjust the physical parameters to those of the supersymmetric PH solution in the second. In the second case, we arrive to the NH geometry $aDS_2 \times S^2$ in the first step while the second step flattens out the aDS_2 factor because the ultracold limit is the limit of infinite *aDS* radius. We get the same result in all cases.