



Universidad de  
Oviedo



ESCUELA POLITÉCNICA DE INGENIERÍA DE GIJÓN  
GRADO EN INGENIERÍA ELECTRÓNICA INDUSTRIAL  
Y AUTOMÁTICA

TRABAJO DE FIN DE GRADO

**MÉTODOS DE ANÁLISIS  
Y EVALUACIÓN DE LOS EFECTOS  
DE LA CALIDAD DE LA ENERGÍA  
EN SISTEMAS DESEQUILIBRADOS  
Y NO SINUSOIDALES**

**Candidata:**  
Andrea Martínez Álvarez

**Tutores:**  
Prof. Francesco Grasso  
Dott. Libero Paolucci

**Coordinador Erasmus:**  
Víctor M. González Suárez

Año Académico 2018-2019



# Introducción

En términos de un circuito eléctrico, la potencia eléctrica es la tasa, por unidad de tiempo, a la cual la energía eléctrica es transferida por un circuito eléctrico.

La clasificación de la energía eléctrica depende de la naturaleza de la corriente, pudiendo esta ser corriente alterna (CA) o corriente continua (CC).

En la década de 1880, surgió una discusión sobre qué tipo de energía, CA o CC, sería la mejor para la generación y transmisión de energía que se resolvió con la victoria de la corriente alterna, con numerosas ventajas sobre la corriente continua.

Basado en este último tipo, en los circuitos monofásicos, solo hay una fase, es decir, la corriente alterna fluye a través de un solo cable y existe una ruta de retorno llamada línea neutra para completar el circuito.

En 1882, se descubrió que se puede usar más de una fase para la activación, transmisión y sistemas de energía y que estos sistemas tienen muchas ventajas sobre los monofásicos. De esta forma el circuito trifásico, es el sistema polifásico en el que se envían tres fases juntas del generador a la carga.

En lo que concierne a la generación, para la misma cantidad de energía eléctrica, el tamaño del alternador trifásico es pequeño en comparación con el monofásico debido a la relación entre potencia y peso. Debido a esta reducción de peso, el transporte y la instalación del alternador se vuelven convenientes y se requiere menos espacio para acomodar el alternador en las centrales eléctricas.

Con respecto a la transmisión y distribución de la misma cantidad de potencia, el requisito de material conductor es menor.

Finalmente, lo mismo sucede con los transformadores trifásicos y los motores de inducción, ya que para la misma cantidad de energía eléctrica y mecánica, respectivamente, el tamaño de los trifásicos es menor. Por lo tanto, se reduce el costo global de la generación, transmisión y distribución.

Otra ventaja es desde el punto de vista funcional. En los sistemas trifásicos, la potencia instantánea es casi constante durante el ciclo, lo que da como resultado un funcionamiento suave y sin vibraciones de la máquina, mientras que en los sistemas monofásicos la potencia instantánea es pulsante, lo que conduce a vibraciones en las máquinas. Por ello, los motores de inducción trifásicos se encienden automáticamente, ya que el flujo magnético producido por el suministro trifásico es de magnitud constante, mientras que el producido por el suministro monofásico es de naturaleza pulsátil. Además, si ocurre una falla en cualquier devanado de un transformador trifásico, el resto de los dos devanados se pueden usar para servir a la carga.

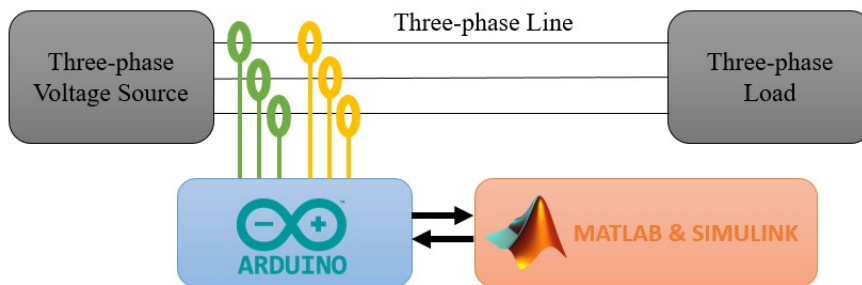
Por último, el punto fuerte es que un sistema trifásico se puede utilizar para alimentar una carga monofásica, mientras que viceversa no es posible.

Por todo ello, está claro que los sistemas trifásicos son más económicos, eficientes, confiables y es por eso que son el caso de estudio de esta tesis.

## Descripción del proyecto

Específicamente, el objetivo del proyecto consiste en la medición de la potencia de un sistema trifásico en un régimen distorsionado.

El propósito es diseñar un sistema en el que se midan las corrientes y tensiones de línea de un sistema trifásico bajo condiciones no sinusoidales y desequilibradas y se calcule la potencia del sistema como en el que se muestra en la figura a continuación.



Tres sensores de efecto Hall de corriente y un sensor de tensión trifásico medirán las corrientes de línea y las tensiones que serán adquiridas y procesadas por un microcontrolador, y a partir de ellas, este será responsable de calcular los valores de potencia. Los resultados se enviarán a través de la comunicación del puerto serie a un ordenador para su visualización. Para llevar esto a cabo, se utilizará la tarjeta del microcontrolador Arduino Mega 2560 y será programado por Simulink, el entorno de programación gráfica de MATLAB, desarrollado por MathWorks. La visualización de resultados se realizará también a través de este último programa.

Las fases para la realización del proyecto se detallarán a continuación:

- Estudiar la formulación para el cálculo de potencia.
- Modelar y simular en Matlab & Simulink el sistema de medición.
- Adapta el software para programar el Arduino y adquirir y procesar las señales.
- Diseñar y construir el hardware del sistema.
- Resultados y conclusiones.

# Sistemas trifásicos

Como se indicó en la introducción, la energía eléctrica trifásica es el método más común para la generación, transmisión y distribución de energía eléctrica. Es comúnmente utilizado por las redes eléctricas de todo el mundo para transferir energía y para alimentar motores grandes y otras cargas pesadas.

Se basa en los sistemas trifásicos, en los cuales tres conductores de circuito transportan tres corrientes alternas producidas por una fuente de tensión que suministra una carga del generador a la carga. Éstos pueden o no tener un cable neutro que permite al sistema trifásico para usar una tensión más alta y al mismo tiempo es compatible con aparatos monofásicos de voltaje más bajo.

Tanto la fuente como la carga pueden conectarse de dos maneras, es decir, la conexión en estrella (Y) y la conexión delta ( $\Delta$ ).

Sin embargo, nuestro sistema estará diseñado para medir voltajes y corrientes de línea, independientemente del tipo de conexión de la fuente y la carga, lo que hará que nuestro sistema sea más versátil y con la capacidad de medir la potencia de cualquier sistema trifásico.

## Condiciones de los sistemas trifásicos

En los sistemas trifásicos, las tres corrientes alternas son de la misma frecuencia y alcanzan sus valores máximos instantáneos en diferentes momentos. Tomando como referencia un conductor, las otras dos corrientes son de la misma frecuencia y se retrasan en el tiempo en un tercio y dos tercios de un ciclo de la corriente eléctrica.

Suponiendo un sistema de secuencia positiva giratoria en sentido antihorario, a, b, c, los voltajes de línea a neutro son los siguientes:

$$v_a = \sqrt{2}V \sin(\omega t) \tag{1}$$

$$v_b = \sqrt{2}V \sin(\omega t - 120^\circ) \tag{2}$$

$$v_c = \sqrt{2}V \sin(\omega t + 120^\circ) \tag{3}$$

donde V es el valor rms de la tensión e I es el valor rms de la corriente.

Las corrientes de línea, supuestamente retrasadas respecto a la tensión, tienen expresiones similares y son las siguientes:

$$i_a = \sqrt{2}I \sin(\omega t - \theta) \quad (4)$$

$$i_b = \sqrt{2}I \sin(\omega t - \theta - 120^\circ) \quad (5)$$

$$i_c = \sqrt{2}I \sin(\omega t - \theta + 120^\circ) \quad (6)$$

Cuando alguna de las condiciones anteriores no se cumple (diferentes corrientes o diferentes cambios de fase entre ellas), el sistema se denomina sistema desequilibrado.

### Sistemas trifásicos sinusoidales desequilibrados

El desequilibrio de carga conduce a corrientes asimétricas que a su vez causan asimetría de voltaje. Hay situaciones en las que los tres fasores de tensión no son simétricos, lo que conduce a corrientes asimétricas incluso cuando la carga está perfectamente equilibrada. En este caso, los tres fasores de corriente  $I_a$ ,  $I_b$  e  $I_c$ , no tienen magnitudes iguales, y no se desplazan exactamente  $120^\circ$  entre sí.

Los voltajes de línea a neutro son los siguientes:

$$v_a = \sqrt{2}V_a \sin(\omega t + \alpha_a) \quad (7)$$

$$v_b = \sqrt{2}V_a \sin(\omega t + \alpha_b - 120^\circ) \quad (8)$$

$$v_c = \sqrt{2}V_b \sin(\omega t + \alpha_c + 120^\circ) \quad (9)$$

Las corrientes de línea tienen expresiones similares:

$$i_a = \sqrt{2}V_a \sin(\omega t + \beta_a) \quad (10)$$

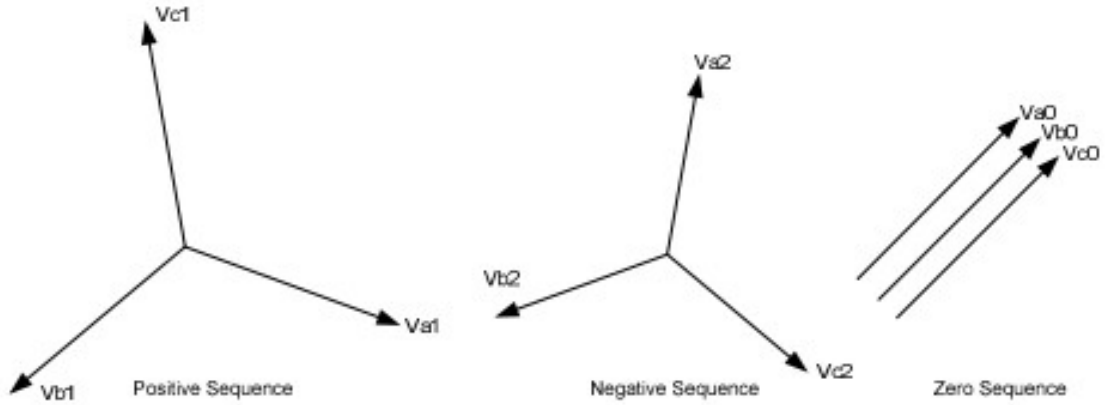
$$i_b = \sqrt{2}V_a \sin(\omega t + \beta_b - 120^\circ) \quad (11)$$

$$i_c = \sqrt{2}V_b \sin(\omega t + \beta_c + 120^\circ) \quad (12)$$

En redes no balanceadas, puede ser importante conocer la secuencia de las fases, ya que un cambio en la secuencia puede resultar en una distribución de las corrientes de línea completamente diferente, incluso cuando las tensiones de suministro y las cargas siguen siendo las mismas.

En 1918, el Dr. C. L. Fortescue escribió un artículo titulado "Método de coordenadas simétricas aplicadas a la solución de redes polifásicas", donde describía cómo las tensiones (o corrientes) trifásicas desequilibradas arbitrarias se podían transformar en tres conjuntos de componentes trifásicos equilibrados. Llamó a estos componentes "componentes simétricos" que se conocen como componentes de secuencia positiva, negativa y cero como se muestran en la figura a continuación.

En los tres sistemas de los componentes simétricos, los subíndices denotan los com-



ponentes en las diferentes fases. Podemos simplificar aún más la notación cambiando el subíndice (1) por (+) para la secuencia positiva, el subíndice (2) por (-) para la negativa mientras conservamos el subíndice (0) para la secuencia cero.

Ahora es posible escribir los componentes simétricos en términos de las tensiones de línea  $V_a$ ,  $V_b$  y  $V_c$ .

$$V_0 = \frac{1}{3}(V_a + V_b + V_c) \quad (13)$$

$$V_+ = \frac{1}{3}(V_a + aV_b + a^2V_c) \quad (14)$$

$$V_- = \frac{1}{3}(V_a + a^2V_b + aV_c) \quad (15)$$

Y de la misma forma resulta para la corriente.

### Sistemas trifásicos no sinusoidales

Se trata de sistemas bajo condiciones periódicas y no sinusoidales caracterizadas por formas de onda de corriente y tensión distorsionadas.

Los armónicos del sistema de potencia se definen como tensiones y corrientes sinusoidales en frecuencias que son múltiplos enteros de la frecuencia fundamental. Constituyen los principales componentes distorsionadores de la tensión de red y de las formas de onda de la corriente de carga.

Dado que las formas de onda son periódicas, la variación de tiempo de cada onda se puede expresar por medio de una serie de Fourier. Se consideran entonces la tensión de alimentación periódica distorsionada y la corriente de carga periódica distorsionada de la siguiente manera:

$$v(t) = V_0 + \sum_{h=1}^n \sqrt{2}V_h \sin(h\omega t + \alpha_h) \quad (16)$$

$$i(t) = I_0 + \sum_{h=1}^n \sqrt{2}I_h \sin(h\omega t + \beta_h) \quad (17)$$

Para condiciones de estado estable, una tensión o corriente instantánea no sinusoidal tiene dos componentes distintos: los componentes de frecuencia fundamental del sistema  $v_1$  y  $i_1$  y los términos restantes  $v_H$  y  $i_H$ , respectivamente.

$$v = v_1 + v_H \quad (18)$$

$$i = i_1 + i_H \quad (19)$$

Los valores rms correspondientes al cuadrado son los siguientes:

$$V^2 = V_1^2 + V_H^2 \quad (20)$$

$$I^2 = I_1^2 + I_H^2 \quad (21)$$

donde

$$V_H^2 = V_0^2 + \sum_{h=1}^n V_h^2 \quad (22)$$

$$I_H^2 = I_0^2 + \sum_{h=1}^n I_h^2 \quad (23)$$

El voltaje directo y los términos de corriente directa  $V_0$  y  $I_0$ , deben incluirse en  $V_H$  y  $I_H$  aunque rara vez están presentes en los sistemas de alimentación de corriente alterna.

Desde el último tercio del siglo XX, la consideración idealizada de las cargas lineales y equilibradas dejó de ser válida debido a la evolución de la tecnología y la creciente complejidad de los sistemas eléctricos, caracterizada por el uso cada vez mayor de cargas no lineales, equipos electrónicos de potencia, convertidores de frecuencia, cargas monofásicas, etc., que han llevado al crecimiento constante de corrientes no sinusoidales con un alto contenido de armónicos. Esto se traduce en potencias distorsionadas y desequilibrios debidos a la asimetría de las tensiones y corrientes en las redes, lo que da lugar a potencias de desequilibrio que, en cualquier caso, causan problemas en las redes eléctricas y en las cargas que están conectadas a ellas.

El equilibrio perfecto es técnicamente inalcanzable. Solo en simulaciones por computadora y laboratorios bien equipados se encuentran sistemas trifásicos equilibrados absolutamente perfectos.

Como ya se ha dicho anteriormente, se pretende diseñar un sistema de medición de potencia adaptable, de modo que se estudiarán las condiciones no sinusoidales y desequilibradas.



# Medida de la potencia

Los conceptos de potencia activa, potencia reactiva, potencia aparente y factor de potencia se han utilizado en ingeniería eléctrica durante casi un siglo. Para sistemas de energía monofásicos sinusoidales y sistemas trifásicos sinusoidales equilibrados, han demostrado ser muy útiles y eficientes para caracterizar la calidad de la transmisión de energía, para diseñar los equipos, para fines de facturación y para compensaciones. Estas definiciones sirven bien a la industria, siempre que las formas de onda de corriente y voltaje permanezcan casi sinusoidales y balanceadas.

Sin embargo, como se ha citado en el capítulo anterior, importantes cambios han ocurrido en los últimos 50 años modificando el entorno de la energía. Por ello, los conceptos de potencia activa, reactiva y aparente y el concepto relacionado de factor de potencia deben adaptarse, de modo que puedan diseñarse algoritmos de medición e instrumentación que brinden orientación con respecto a las cantidades que deben medirse o monitorearse.

La medición, el análisis y la definición de los diferentes términos de la señal de potencia trifásica, donde las tensiones y las corrientes están desequilibrados y distorsionados, se han estudiado para estandarizar los índices correctos que cuantifican el nivel de armónicos y distorsión.

Es bien sabido que los medidores que miden la energía (kWh) y la potencia activa (kW) proporcionan mediciones precisas también en condiciones no senoidales o desequilibradas. Sin embargo, los medidores dedicados a las mediciones de potencia aparente (kVA) y de potencia reactiva (kvar) son propensos a errores cuando se distorsionan las formas de onda de corriente y voltaje. La razón principal de tales incertidumbres se deriva de las definiciones de potencia inadecuadas que dictan el diseño conceptual de dicha instrumentación.

En general, las cantidades utilizadas en los sistemas de energía eléctrica se definen para condiciones sinusoidales. Bajo condiciones no sinusoidales, algunas cantidades pueden conducir a interpretaciones erróneas. Solo la potencia activa tiene un significado físico claro, ya que representa el valor promedio de la potencia instantánea durante un período fijo. La potencia aparente es la medida más afectada.

Evidentemente esta situación llevó a la búsqueda de una solución práctica. La literatura abunda en estudios de tales condiciones que presentan enfoques que se completan o se contradicen entre sí.

Lamentablemente no se ha logrado ningún acuerdo universal todavía.

A pesar de todos los enfoques existentes, el desarrollo de este proyecto se basa en el estándar IEEE 1459-2010, que se ha elegido debido a su objetivo de proponer los conceptos y las definiciones que pueden ser útiles e interesantes para la evaluación de la calidad de transmisión de energía eléctrica, el desarrollo de algoritmos de medición y el diseño de instrumentos de medida. Esta norma incluye definiciones para las mediciones de las magnitudes de potencia de los sistemas eléctricos en condiciones sinusoidales o no sinusoidales y equilibradas o desequilibradas.

## Medición de potencia en condiciones no sinusoidales y desequilibradas

Tras repasar las definiciones de potencia en condiciones ideales, se estudian las definiciones para las condiciones distorsionadas anteriormente citadas.

### Potencia activa

La potencia activa en una fase no sinusoidal se define de la siguiente manera:

$$P_{1\phi} = P_1 + P_H \quad (24)$$

donde  $P_1$  es la potencia activa fundamental, referida a la frecuencia fundamental y  $P_H$  la potencia activa de armónicos.

$$P_1 = \frac{1}{kT} \int_{\tau}^{\tau+kT} v_1 i_1 dt = V_1 I_1 \cos \theta_1 \quad (25)$$

$$P_H = V_0 I_0 + \sum_{h \neq 1} V_h I_h \cos \theta_h \quad (26)$$

La potencia activa total se define como la suma de la potencia activa de cada fase  $a, b, c$ :

$$P = P_a + P_b + P_c \quad (27)$$

Además la potencia activa total se puede obtener por medio de las componentes simétricas definidas anteriormente.

$$P_+ = 3V_+ I_+ \cos \theta_+ \quad (28)$$

$$P_- = 3V_- I_- \cos \theta_- \quad (29)$$

$$P_0 = 3V_0 I_0 \cos \theta_0 \quad (30)$$

$$P = P_+ + P_- + P_0 \quad (31)$$

## Potencia reactiva

De formas similares se puede calcular la potencia reactiva como sigue a continuación:

$$Q_a = \frac{\omega}{kT} \int_{\tau}^{\tau+kT} i_a \left[ \int v_a \right] dt = V_a I_a \sin \theta_a \quad (32)$$

$$Q_b = \frac{\omega}{kT} \int_{\tau}^{\tau+kT} i_b \left[ \int v_b \right] dt = V_b I_b \sin \theta_b \quad (33)$$

$$Q_c = \frac{\omega}{kT} \int_{\tau}^{\tau+kT} i_c \left[ \int v_c \right] dt = V_c I_c \sin \theta_c \quad (34)$$

$$Q = Q_a + Q_b + Q_c \quad (35)$$

$$Q_+ = 3V_+ I_+ \sin \theta_+ \quad (36)$$

$$Q_- = 3V_- I_- \sin \theta_- \quad (37)$$

$$Q_0 = 3V_0 I_0 \sin \theta_0 \quad (38)$$

$$Q = Q_+ + Q_- + Q_0 \quad (39)$$

## Potencia aparente

Entre las diversas teorías para la potencia aparente se encuentran la definición de la potencia aparente aritmética y la potencia aparente vectorial. Sin embargo, de acuerdo con la norma IEEE 1459 se recomienda renunciar a esta definición de potencia aparente aritmética y vectorial, y reemplazarlos con la potencia aparente efectiva definida a continuación.

En el pasado, la potencia aparente efectiva se dividió en potencia activa P y potencia no activa N de la siguiente manera:

$$S_e^2 = P^2 + N^2 \quad (40)$$

Sin embargo, este enfoque no separa las potencias fundamentales de secuencia positiva, por lo que conduce a la separación de la corriente y la tensión en términos fundamentales y armónicos para resolver la potencia aparente como se describe a continuación.

La línea y la carga reales se reemplazan por un sistema hipotético, perfectamente compensado que dibuja perfectamente corrientes de secuencia positiva sinusoidal,  $I_e$  y una corriente neutral nula. Matemáticamente éstas se obtienen simplificada como sigue:

$$I_e = \sqrt{\frac{I_a^2 + I_b^2 + I_c^2 + I_n^2}{3}} \quad (41)$$

$$I_{e1} = \sqrt{\frac{I_{a1}^2 + I_{b1}^2 + I_{c1}^2 + I_{n1}^2}{3}} \quad (42)$$

$$I_{eH} = \sqrt{\frac{I_{aH}^2 + I_{bH}^2 + I_{cH}^2 + I_{nH}^2}{3}} = \sqrt{I_e^2 - I_{e1}^2} \quad (43)$$

La corriente efectiva también puede referirse a los componentes simétricos:

$$I_e = \sqrt{I_+^2 + I_-^2 + (1 + 3\rho)I_0^2} \quad (44)$$

Nota: Considere que para sistemas de tres cables,  $I_{n1} = I_{nh} = 0$  y  $I_0 = 0$ .

Las expresiones prácticas para la tensión efectiva,  $V_e$ , se obtienen de manera parecida y resultan como a continuación.

$$V_e = \sqrt{V_{e1}^2 + V_{eH}^2} \quad (45)$$

$$V_e = \sqrt{\frac{1}{18}[3(V_a^2 + V_b^2 + V_c^2) + V_{ab}^2 + V_{bc}^2 + V_{ca}^2]} \quad (46)$$

$$V_{e1} = \sqrt{\frac{1}{18}[3(V_{a1}^2 + V_{b1}^2 + V_{c1}^2) + V_{ab1}^2 + V_{bc1}^2 + V_{ca1}^2]} \quad (47)$$

$$V_{eH} = \sqrt{\frac{1}{18}[3(V_{aH}^2 + V_{bH}^2 + V_{cH}^2) + V_{abH}^2 + V_{bcH}^2 + V_{caH}^2]} = \sqrt{V_e^2 - V_{e1}^2} \quad (48)$$

y para sistemas de tres hilos:

$$V_e = \sqrt{\frac{V_{ab}^2 + V_{bc}^2 + V_{ca}^2}{9}} \quad (49)$$

$$V_{e1} = \sqrt{\frac{V_{ab1}^2 + V_{bc1}^2 + V_{ca1}^2}{9}} \quad (50)$$

$$V_{eH} = \sqrt{\frac{V_{abH}^2 + V_{bcH}^2 + V_{caH}^2}{9}} = \sqrt{V_e^2 - V_{e1}^2} \quad (51)$$

Con estos valores, la resolución de la potencia aparente efectiva,  $S_e$  se separa en dos términos principales, donde  $S_{e1}$  es la potencia aparente efectiva fundamental y el término  $S_{eN}$  es la no fundamental.

$$S_e^2 = S_{e1}^2 + S_{eN}^2 \quad (52)$$

$$S_{e1} = 3V_{e1}I_{e1} \quad (53)$$

$$S_{eN}^2 = S_e^2 - S_{e1}^2 = D_{eI}^2 + D_{eV}^2 + S_{eH}^2 \quad (54)$$

La potencia aparente efectiva no fundamental,  $S_{eN}$  tiene tres componentes:

La potencia de distorsión actual, generalmente el componente más grande;

$$D_{eI} = 3V_{e1}I_{eH} \quad (55)$$

la potencia de distorsión de tensión,

$$D_{eV} = 3V_{eH}I_{e1} \quad (56)$$

y la potencia aparente armónica efectiva que se caracteriza por la potencia armónica total activa  $P_H$  y la potencia de distorsión armónica  $D_{eH}$

$$S_{eH} = 3V_{eH}I_{eH} \quad (57)$$

$$S_{eH}^2 = P_H^2 + D_{eH}^2 \quad (58)$$

$$D_{eH} = \sqrt{S_{eH}^2 - P_H^2} \quad (59)$$

El factor de potencia, PF sigue su definición clásica:

$$PF = \frac{P}{S_e} = \frac{P_1 + P_H}{S_e} \quad (60)$$

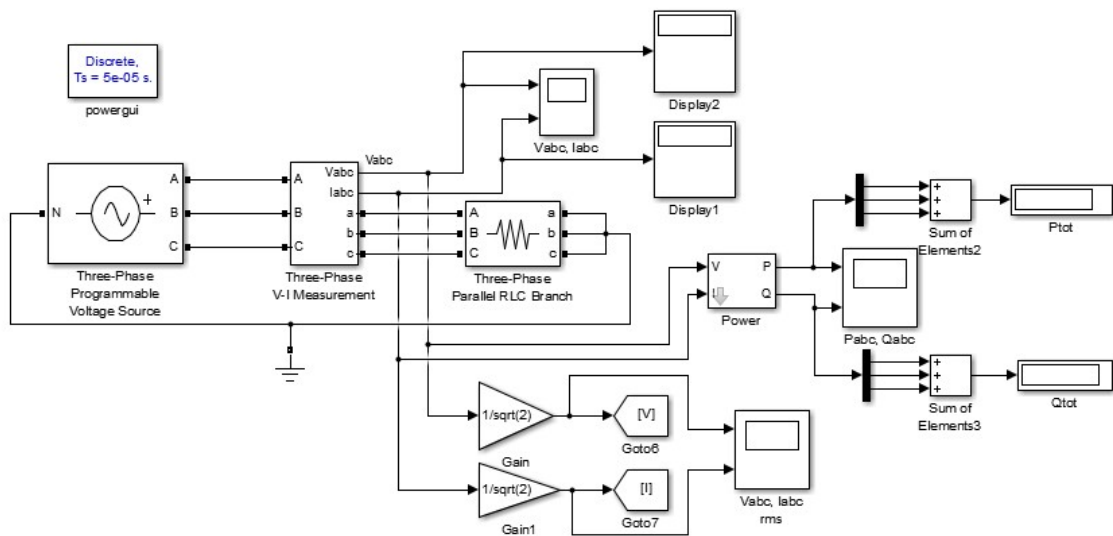


# Modelado y simulación

El modelado y la simulación de sistemas diseñados es una herramienta muy valiosa por varias razones y es por ello que antes de la construcción del circuito físico real, y de acuerdo con las técnicas de cálculo de potencia descritas en el capítulo anterior, se construirá un modelo digital, utilizando la herramienta Simulink de MATLAB.

Como contacto inicial, se llevará a cabo el modelado de un sistema trifásico sinusoidal y balanceado para aprender a medir correctamente las corrientes y tensiones correspondientes.

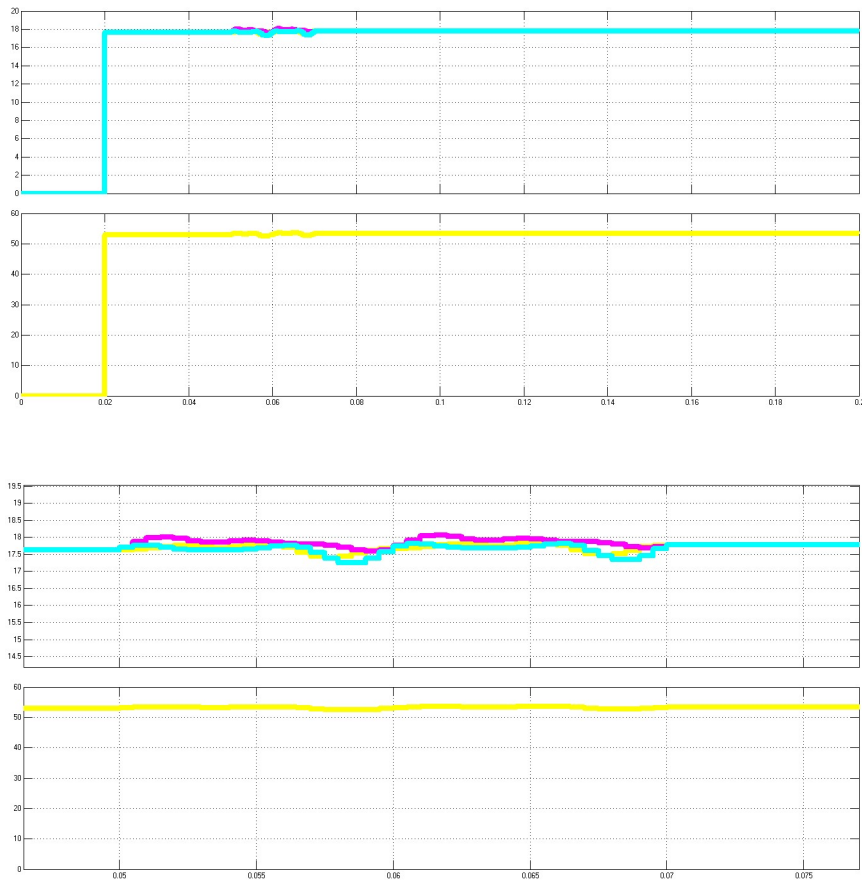
A continuación se realiza el modelado de un sistema real distorsionado con presencia de armónicos.



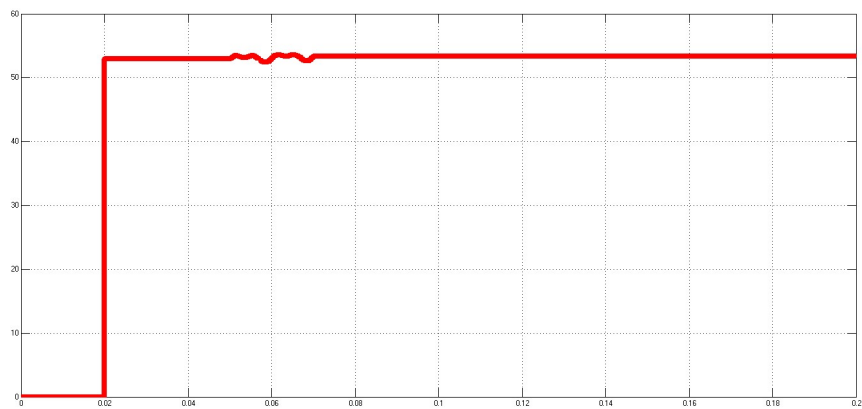
Posteriormente, persiguiendo el objetivo de este proyecto y basándose en la formulación descrita anteriormente, se realizan las operaciones necesarias para calcular la potencia.

Se realiza una simulación de 0.2 segundos en la cual, un generador de 230 Vrms y 50 Hz con contenidos de los armónicos tercero y quinto, alimentará una carga resistiva pura compuesta por una resistencia de 1000  $\Omega$ . Además, se ha elegido un tiempo de discretización Ts de  $5 \times 10^{-5}$ .

El resultado del modelo se establece en 53.17W para la potencia activa y, como se espera para un circuito resistivo, nula para la reactiva. Como puede verse en la figura siguiente y con detalle en la posterior, la potencia se altera con la introducción de los armónicos a 0.05 segundos.



Se obtiene un resultado similar para la potencia aparente, en la que la potencia se establece en 53.16 VA como se muestra:



Como conclusión de estos datos, podemos afirmar que el uso de la estimación estándar IEEE 1459 es apropiado y que se ha realizado un modelado correcto de la medición de potencia.



# Implementación física

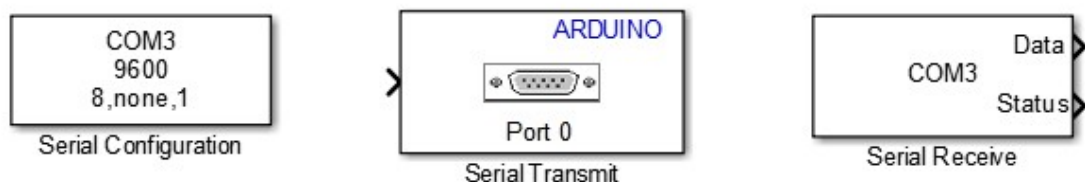
Después de haber realizado el modelado del sistema de medición, comienza la implementación física.

El primer paso es programar "el cerebro" de nuestro sistema. Esto se refiere al microcontrolador ATmega 2560, presente en la tarjeta Arduino Mega 2560. El objetivo es programar el microcontrolador para que, lea y procese las señales de tensión y corriente de línea, calcule los valores de potencia de la misma manera que en la simulación del capítulo anterior y envíe los resultados a un ordenador para su análisis y visualización.

## Comunicación Simulink-Arduino

Inicialmente se establece la conexión entre el Arduino y el Simulink con el fin de programar el microcontrolador.

Tres bloques hacen posible nuestra comunicación:



Estos bloques utilizan el puerto serie universal (USB) para enviar y recibir datos. Este puerto es del tipo Half-Duplex, lo que significa que solo puede recibir o enviar datos al mismo tiempo, es decir, que para enviar o recibir datos, el puerto debe estar libre de tráfico.

El bloque de configuración en serie configura los parámetros de un puerto en serie. Se deben dar valores a todos sus parámetros antes de colocar un envío o una recepción en serie. A través del bloque "Serial Transmit" se envían datos almacenados a la especificado puerto serie. "Serial Receive" configura y abre una interfaz a una dirección remota especificada mediante el protocolo en serie. La configuración e inicialización ocurre una vez al comienzo de la simulación. El bloque adquiere datos durante el tiempo de ejecución.

Numerosos modelos se llevan a cabo hasta conseguir la conexión por el puerto serie.

## Adaptación del software

Para adaptar el programa de cálculo de potencia modelado, se crea un nuevo modelo de un sistema monofásico con el propósito de facilitar el procesamiento de dos señales, una corriente y una tensión, en comparación con las seis presentes en el sistema trifásico. En este modelo, se introducen dos señales generadas con un generador de señales en los pines analógicos A0 y A4, aunque se podría usar cualquiera de los comprendidos entre A0 y A15. Estas señales representarán una señal de voltaje de 5 Vpp y 2,5 V de desplazamiento y una señal de corriente de 2 Vpp y 1 V de desplazamiento. Ambos se generarán en las primeras pruebas con una frecuencia de 5 Hz.

Un microcontrolador no tiene capacidad para trabajar con señales analógicas, por lo que es necesario convertir las señales analógicas en señales digitales para trabajar con ellas. En el Arduino Mega, el convertidor analógico digital interno tiene una resolución de 10 bits, lo que significa que la tensión de entrada analógica se convierte en un valor numérico entre 0 y 1023, y una tensión de referencia de 5 V, ya que es tecnología TTL.

Posteriormente, se calculan la magnitud y la fase de las señales para el cálculo de la potencia.

Finalmente, los valores de potencia calculados deben adaptarse para que puedan ser enviados por el puerto serie. Esta comunicación basada en simulink tiene la limitación de 8 bits por datos (uint8), y nuestros resultados son de 16 bits (uint16). La solución propuesta es que se enviarán como un vector compuesto por 2 datos de 8 bits precedidos y terminados por 2 datos limitantes sin información valiosa.

Todas estas adaptaciones de enviar los datos divididos, deben tenerse en cuenta en la lectura de los datos, de modo que se deben generar nuevamente como un solo dato uint16 para facilitar su comprensión y representación.

Una vez que se ha llevado a cabo toda esta adaptación teórica, el microcontrolador se graba con el código anteriormente descrito, lo que resulta en un error sin solución. Se trata de la incompatibilidad del bloque para el cálculo de la magnitud y la fase, Fourier, de la biblioteca Simscape con el lenguaje aceptado por el Arduino.

Como solución a este problema, se propone el diseño de un nuevo bloque de Fourier basado en otros bloques que no pertenecen a esta biblioteca. Sin embargo, otro problema surge debido al bloque "Mean" para el cálculo de los coeficientes a través de los cuales se pretende calcular la magnitud y la fase.

Este nuevo obstáculo no se detecta debido a un error de compilación, sino al realizar pruebas que varían la frecuencia de las señales a partir de las cuales queremos analizar la magnitud y la fase. Para intentar resolver este error, se ha llevado a cabo un nuevo modelo de simulación en el que solo se busca la magnitud y la fase de diferentes señales y en el que se han investigado las posibles causas descubriendo que el error persiste en aquellas frecuencias cuyo período es un número irracional.

Se concluyó que estas distorsiones se deben a las aproximaciones decimales internas de la herramienta Matlab y, lamentablemente, no se ha encontrado una solución aceptable para este error.

Además, este error no se puede asumir para el cálculo preciso de la potencia que intentamos calcular, por lo que no se ha continuado con el desarrollo del proyecto en esta línea.



# Conclusiones

En este punto y antes de finalizar el proyecto, es necesario evaluar el trabajo, y analizar si cumple con los objetivos que se establecieron originalmente.

El objetivo principal del proyecto, la construcción de un sistema para calcular la potencia de un sistema trifásico en condiciones desequilibradas y no sinusoidales, no se ha completado debido a las incompatibilidades de las herramientas propuestas para su desarrollo.

Sin embargo, siguiendo las líneas propuestas, se han alcanzado otros objetivos.

- Se ha realizado un estudio exhaustivo de la formulación disponible para el cálculo de la potencia real de un sistema en condiciones distorsionadas.
- Se ha ejecutado una buena adaptación de la interfaz, obteniendo comunicación entre Arduino y Simulink por lo que ha podido adquirir y procesar señales.
- Se han conocido las limitaciones de compatibilidad entre dos grandes herramientas como Arduino y Matlab.





UNIVERSITÀ  
DEGLI STUDI  
FIRENZE

**Scuola  
di Ingegneria**

Corso di Laurea in  
Ingegneria Elettrica e  
dell'Automazione

TESI DI LAUREA

**Metodi di analisi e valutazione  
degli effetti della Power Quality  
in sistemi in condizioni  
di regime distorto**

**Methods of analysis  
and evaluation of the effects  
of the Power Quality in unbalanced  
and non-sinusoidal systems**

**Candidata:**

Andrea Martínez Álvarez

**Relatori:**

Prof. Francesco Grasso

Dott. Libero Paolucci





*A tutti quelli che ancora non credono in se stessi, ma stanno per farlo*



*Il tutto è più della somma delle parti*



# Ringraziamenti

Vorrei ringraziare la mia famiglia per aver realizzato questo sogno e per essere sempre il mio sostegno. Ringrazio il professor Francesco Grasso per l'opportunità di lavorare nella sua attività e far parte della sua squadra. A Libero, per la sua disponibilità, per tutto il suo aiuto quotidiano e per rendere il lavoro più piacevole. Un ringraziamento speciale all'intero gruppo di lavoro del laboratorio di elettronica e in particolare a Fabio con cui ho condiviso più tempo. Infine, grazie a tutti con cui ho condiviso questa meravigliosa esperienza Erasmus.



# Contents

<b>Contents</b>	<b>viii</b>
<b>List of figures</b>	<b>x</b>
<b>1 Introduction</b>	<b>1</b>
1.1 Description of the project . . . . .	3
1.2 Structure . . . . .	3
<b>2 Three-phase systems</b>	<b>5</b>
2.1 Introduction . . . . .	5
2.2 Three-phase systems conditions . . . . .	6
2.2.1 Three-phase sinusoidal unbalanced systems . . . . .	7
2.2.1.1 Symmetric components method . . . . .	8
2.2.2 Three-phase non-sinusoidal systems . . . . .	10
<b>3 Power measurement</b>	<b>13</b>
3.1 The IEEE Std 1459-2010 . . . . .	14
3.2 Background: Power measurement under sinusoidal and balanced conditions	15
3.3 Power measurement under nonsinusoidal and unbalanced conditions . . . . .	16
3.3.1 Active power . . . . .	16
3.3.1.1 Positive-, negative-, and zero-sequence active powers . . . . .	17
3.3.2 Reactive power . . . . .	17
3.3.2.1 Positive-, negative-, and zero-sequence reactive powers . . . . .	18
3.3.3 Apparent power . . . . .	18
3.3.3.1 Arithmetic apparent power . . . . .	18
3.3.3.2 Vector apparent power . . . . .	19
3.3.3.3 Effective apparent power (VA) . . . . .	19
<b>4 Modeling and simulation</b>	<b>25</b>
4.1 Starting point . . . . .	26
4.2 Common model . . . . .	27
4.3 Simulation results . . . . .	34

<b>5 Physical implementation</b>	<b>37</b>
5.1 Simulink-Arduino communication . . . . .	40
5.1.1 Signal processing . . . . .	40
5.1.2 Software adaptation . . . . .	41
<b>Conclusions</b>	<b>51</b>
<b>Bibliography</b>	<b>53</b>



# List of Figures

1.1	Waveform of direct and alternating current over time. . . . .	1
1.2	System design for power measurement. . . . .	3
2.1	Three phase system . . . . .	5
2.2	Difference between Star and Delta Connections . . . . .	6
2.3	Symmetrical balanced three phase AC signal. . . . .	6
2.4	Difference between balanced and unbalanced system . . . . .	7
2.5	Difference between balanced and unbalanced system . . . . .	8
3.1	Arithmetic and vector apparent powers . . . . .	18
4.1	Balanced and sinusoidal system model. . . . .	26
4.2	Electrical system design . . . . .	27
4.3	Fourier block . . . . .	27
4.4	Distorted supplying voltage . . . . .	28
4.5	Block diagram for active and reactive powers calculation. . . . .	28
4.6	<i>Active-Reactive subsystem.</i> . . . . .	29
4.7	Block diagram for total active and reactive powers calculation. . . . .	29
4.8	Symmetrical components calculation. . . . .	29
4.9	Sequence analyzer block. . . . .	30
4.10	Total active and reactive powers based on positive, negative and zero powers. . . . .	30
4.11	Fundamental and nonfundamental frequency effective current decomposition. . . . .	31
4.12	Effective current calculated from the symmetrical components. . . . .	31
4.13	Block diagram for $V_e$ calculation. . . . .	32
4.14	Fundamental frequency effective voltage. . . . .	32
4.15	Nonfundamental frequency effective voltage. . . . .	32
4.16	Calculation of effective apparent power. . . . .	33
4.17	System distortion values. . . . .	33
4.18	Active and reactive power results from <i>Power</i> block. . . . .	34
4.19	Active power result. . . . .	35
4.20	Active power result detailed at distortion. . . . .	35
4.21	Reactive power result. . . . .	35
4.22	Effective apparent power result. . . . .	36
5.1	Arduino Mega 2560 board. . . . .	39

---

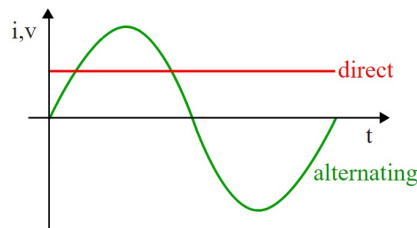
5.2	Blocks of the Simulink package for Arduino. . . . .	39
5.3	Blocks of the Simulink package for Arduino for Serial Communication. . . . .	40
5.4	Generation of a square wave. . . . .	41
5.5	Function-Call Subsystem block with a data entry and a clock for its activation by rising edge. . . . .	41
5.6	Block diagram of the single-phase system. . . . .	42
5.7	Adaptation of the analog signal. . . . .	42
5.8	Adaptation of the power results. . . . .	42
5.9	Adaptation of the power results for its transmission by the serial port. . . . .	43
5.10	Matlab Function for data vector generation. . . . .	43
5.11	Block diagram for results reading. . . . .	44
5.12	Matlab function for results reading adaptation. . . . .	45
5.13	Error due to Fourier block. . . . .	46
5.14	New "Fourier" block. . . . .	46
5.15	Block diagram of the new block for magnitude and phase measurement . . . . .	47
5.16	Block diagram of the new "Mean" block. . . . .	47
5.17	Block diagram for frequency variation tests. . . . .	48
5.18	50Hz signal magnitude and phase. . . . .	48
5.19	60Hz signal magnitude and phase. . . . .	49

# Chapter 1

## Introduction

In terms of an electric circuit, electrical power is the rate, per unit time, at which electrical energy is transferred by an electric circuit.

The classification of the electric power depends on the nature of the current. Alternating current (AC) is the movement of electrical charge through a medium that changes direction periodically. This is in contrast with direct current (DC), where the movement of charge is only in one direction and is constant.



**Figure 1.1:** Waveform of direct and alternating current over time.

In the 1880s, a discussion arose about what type of energy, AC or DC, would be best for the generation and transmission of energy that was solved with victory for alternating current, with numerous advantages over direct current.

The first advantage is in power transmission. Both AC and DC have power loss in long lines because of the resistance in the wires. For a fixed power, higher voltage results in lower current through the power line, and lower current means lower power line losses. Furthermore, the early engineers realized that very high voltage is needed for efficient power transmission. Today, long-haul power lines operate at high voltages to minimize power loss. Using transformers, it is easy to boost AC voltage to these high levels and then reverse the process at the consumer end meanwhile DC voltage does not work in a transformer.

The next point is in power generation. Mechanical generation of DC is much more complicated, and most of it today is generated by batteries, solar cells, fuel cells, or by converting AC to DC.

Alternating current also has an benefit when it comes to power consumption.

DC machines required brushes and commutators to operate, thus increasing complexity and maintenance. The simplicity of AC motors, along with the ability to use AC power readily, makes them a better choice.

In single phase circuits, there will be only one phase, i.e the AC current will flow through only one wire and there will be one return path called neutral line to complete the circuit. The generating station and load station will also be single phase.

This is an old system used from previous time.

In 1882, it has been discovered that more than one phase can be used for generating, transmitting and power systems. Three phase circuit is the polyphase system where three phases are sent together from the generator to the load.

Three-phase systems has a lot of advantages over single-phase ones.

### **Three-Phase Systems Advantages**

From the economic point of view, the first benefit of three-phase systems is presented. Starting with generation, for the same amount of electrical energy, the size of the three-phase alternator is small compared to the one-phase due to power to weight ratio. Because of this weight reduction, the transportation and installation of the alternator becomes convenient and less space is required to accommodate the alternator in the power plants. In addition, with regard to the transmission and distribution of same amount of power, the requirement of conductor material is less. Finally, the same happens with three-phase transformers and induction motors since for the same amount of electrical and mechanical power, respectively, the size of the three-phase ones is smaller. Therefore, the overall cost of the generation, transmission and distribution is reduced.

Other asset is from the functional view.

In the three-phase systems, the instantaneous power is almost constant over the cycle results in smooth and vibration-free operation of the machine whereas in one-phase systems the instantaneous power is pulsating hence change over the cycle, which leads to vibrations in machines.

A purpose of this, three-phase induction motors are self-started as the magnetic flux produced by three-phase supply is a constant magnitude while the produced by one-phase supply is pulsating in nature. Moreover, if a fault occurs in any winding of a three-phase transformer, the rest of two winding can be used in the open delta to serve the load.

At last, the strong point is that a three-phase system can be used to feed a one-phase load, whereas vice-versa is not possible.

From above, it is clear the three-phase systems are more economical, efficient, reliable and convenient and that is why they concern the case of study of this thesis.

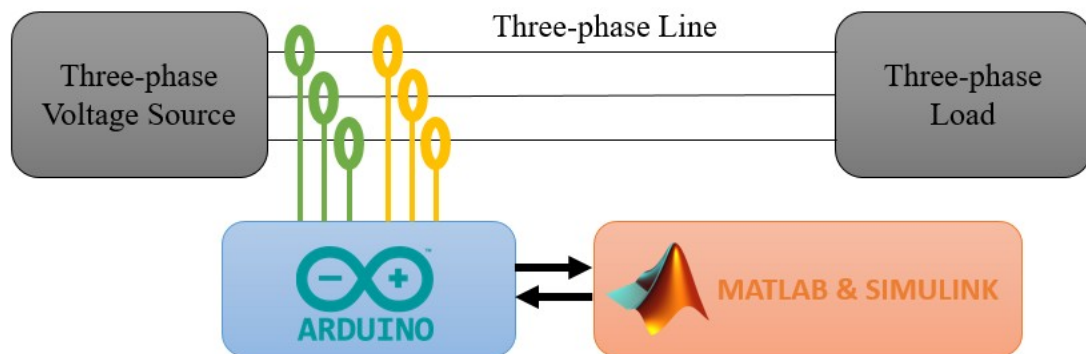
## 1.1 Description of the project

Specifically, the objective of the project consists on the power measurement of a three-phase system in a distorted regime.

The aim is to design a system (Figure 1.2) in which the line currents and voltages of a three-phase system under non-sinusoidal and unbalanced conditions are measured, and the power of the system is calculated.

Three hall effect sensors of current and a three-phase voltage sensor will measure the line currents and voltages that will be read and processed by a microcontroller, which from them, will be responsible of calculating the power values. Finally, the power results will be sent through the serial port communication to a computer for their visualization.

For this purpose, the Arduino Mega 2560 microcontroller board will be used and it will be programmed by Simulink, the graphical programming environment of MATLAB, developed by MathWorks.



**Figure 1.2:** System design for power measurement.

The measurement system designed may be applied on any three-phase system.

## 1.2 Structure

The phases for the realization of the project will be listed below:

- Study the formulation for the power calculation.
- Program and simulate in Matlab / Simulink the measurement system.
- Adapt the software to acquire and process the signals in Arduino.
- Design and build the system hardware.
- Results and conclusions.



## Chapter 2

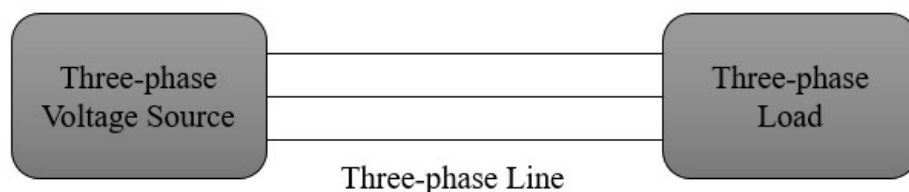
# Three-phase systems

### 2.1 Introduction

As stated in the introduction, three-phase electric power is the most common method of alternating current electric power generation, transmission, and distribution. It is commonly used by electrical grids worldwide to transfer power and to power large motors and other heavy loads.

It is based on three-phase systems, in which three circuit conductors carry three alternating currents produced by a voltage source supplying a charge from the generator to the load (Figure 2.1).

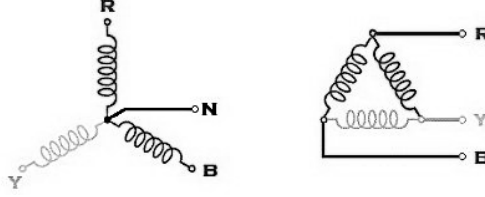
Three phase systems may or may not have a neutral wire. A neutral wire allows the three phase system to use a higher voltage while still supporting lower voltage single phase appliances. In high voltage distribution situations it is common not to have a neutral wire as the loads can simply be connected between phases.



**Figure 2.1:** Three phase system

Both the source and the load may be connected in two ways, i.e., the star connection (Y) and the delta connection ( $\Delta$ ) (Figure 2.2).

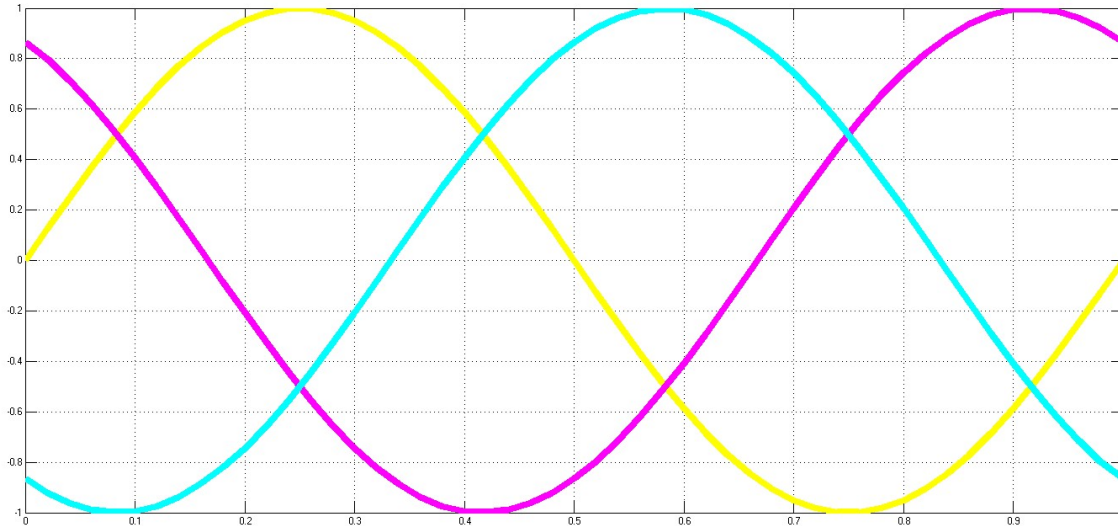
However, our system will be designed to measure line voltages and currents, regardless of the type of connection of the source and load which will make our system more versatile and with the ability to measure the power of any three-phase system.



**Figure 2.2:** Difference between Star and Delta Connections

## 2.2 Three-phase systems conditions

In three-phase systems, the three alternating currents are of the same frequency and reach their instantaneous peak values at different times (Figure 2.3). Taking one conductor as the reference, the other two currents are of the same frequency and delayed in time by one-third and two-thirds of one cycle of the electrical current.



**Figure 2.3:** Symmetrical balanced three phase AC signal.

Assuming a counterclockwise rotating positive-sequence system, a, b, c, the line-to-neutral voltages are as follows:

$$v_a = \sqrt{2}V \sin(\omega t) \quad (2.1)$$

$$v_b = \sqrt{2}V \sin(\omega t - 120^\circ) \quad (2.2)$$

$$v_c = \sqrt{2}V \sin(\omega t + 120^\circ) \quad (2.3)$$

where  $V$  is the rms value of the voltage and  $I$  is the rms value of the current (A).



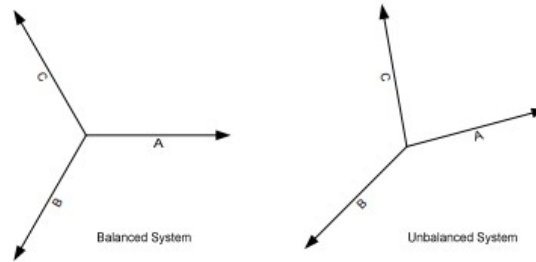
The line currents, assumed lagging the voltage, have similar expressions and they are as follows:

$$i_a = \sqrt{2}I \sin(\omega t - \theta) \quad (2.4)$$

$$i_b = \sqrt{2}I \sin(\omega t - \theta - 120^\circ) \quad (2.5)$$

$$i_c = \sqrt{2}I \sin(\omega t - \theta + 120^\circ) \quad (2.6)$$

When any of the above conditions are not met (different currents or different phase shifts between them), the system is called an *unbalanced system* (Figure 2.4).



**Figure 2.4:** Difference between balanced and unbalanced system

### 2.2.1 Three-phase sinusoidal unbalanced systems

Load imbalance leads to asymmetrical currents that in turn cause voltage asymmetry. There are situations when the three voltage phasors are not symmetrical which leads to asymmetrical currents even when the load is perfectly balanced.

In this case, the three current phasors  $I_a$ ,  $I_b$ , and  $I_c$ , do not have equal magnitudes, and they are not shifted exactly  $120^\circ$  with respect to each other.

The line-to-neutral voltages are as follows:

$$v_a = \sqrt{2}V_a \sin(\omega t + \alpha_a) \quad (2.7)$$

$$v_b = \sqrt{2}V_a \sin(\omega t + \alpha_b - 120^\circ) \quad (2.8)$$

$$v_c = \sqrt{2}V_b \sin(\omega t + \alpha_c + 120^\circ) \quad (2.9)$$

The line currents have similar expressions:

$$i_a = \sqrt{2}V_a \sin(\omega t + \beta_a) \quad (2.10)$$

$$i_b = \sqrt{2}V_a \sin(\omega t + \beta_b - 120^\circ) \quad (2.11)$$

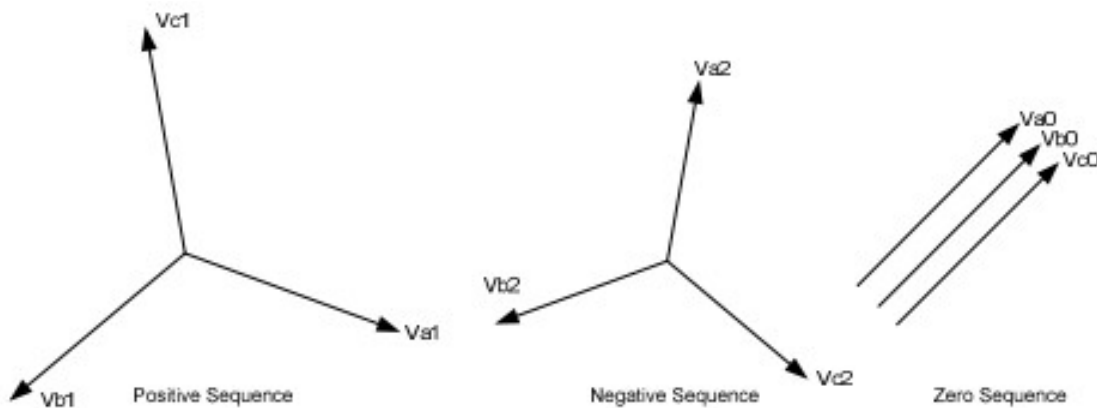
$$i_c = \sqrt{2}V_b \sin(\omega t + \beta_c + 120^\circ) \quad (2.12)$$

In unbalanced networks it may be important to know the sequence of phases, since a change in sequence can result in a distribution of line currents completely different, even when the supply voltages and loads remain the same.

### 2.2.1.1 Symmetric components method

In 1918, Dr. C. L. Fortescue wrote a paper entitled “Method of Symmetrical Coordinates Applied to the Solution of Polyphase Networks” where described how arbitrary unbalanced three-phase voltages (or currents) could be transformed into three sets of balanced three-phase components. He called these components “symmetrical components.” In the paper it is shown that unbalanced problems can be solved by the resolution of the currents and voltages into certain symmetrical relations.

By the method of symmetrical coordinates, a set of unbalanced voltages (or currents) may be resolved into systems of balanced voltages (or currents) equal in number to the number of phases involved (Figure 2.5). The symmetrical component method reduces the complexity in solving for electrical quantities during power system disturbances. These sequence components are known as positive, negative and zero sequence components.



**Figure 2.5:** Difference between balanced and unbalanced system

- The positive sequence corresponds to the power flow that comes from the network to the load, that is, from the generator downstream. The supplied power or energy generated electric has only sequence representation positive, that is, there is no generation of negative sequence homopolar, in symmetric generation systems.
- The negative sequence, is an indication of the existing unbalance measurement in the system, that is the lack of symmetry between voltage phasors at the connection point.
- The presence of homopolar sequence components is linked to the connection regarding ground. Homopolar currents are those that do not close the circuit by the active phases, but they do it for the neutral, or for land, if there was a link Galvanic with the circuit.

Consider the symmetrical system of phasors in Figure 2.5. Being balanced, the phasors have equal amplitudes and are displaced  $120^\circ$  relative to each other. By the definition of symmetrical components,  $\overline{V_{b1}}$  always lags  $\overline{V_{a1}}$  by a fixed angle of  $120^\circ$  and always has the same magnitude. Similarly  $\overline{V_{c1}}$  leads  $\overline{V_{a1}}$  by  $120^\circ$ .

The operator  $a$  is defined as unit vector at an angle of  $120^\circ$ , written as,  $a = 1\angle 120^\circ$ . The operator  $a^2$ , is also a unit vector at an angle of  $240^\circ$ , written  $a^2 = 1\angle 240^\circ$ .

It follows then that:

$$V_{a1} = V_{a1} \quad (2.13)$$

$$V_{b1} = 1\angle 240^\circ V_{a1} = a^2 V_{a1} \quad (2.14)$$

$$V_{c1} = 1\angle 120^\circ V_{a1} = a V_{a1} \quad (2.15)$$

where the subscript (1) designates the positive sequence component. The system of phasors is called positive sequence because the order of the sequence of their maxima occur abc. The subscript (2) designates the negative sequence component and subscript (0) designates zero sequence components. For the negative sequence phasors the order of sequence of the maxima occur cba, which is opposite to that of the positive sequence.

Similarly, for the negative and zero sequence components, we deduce:

$$V_{a2} = V_{a2} \quad (2.16)$$

$$V_{b2} = 1\angle 120^\circ V_{a2} = a V_{a2} \quad (2.17)$$

$$V_{c2} = 1\angle 240^\circ V_{a2} = a^2 V_{a2} \quad (2.18)$$

$$V_{a0} = V_{b0} = V_{c0} = V_{a0} \quad (2.19)$$

In all three systems of the symmetrical components, the subscripts denote the components in the different phases. The total voltage of any phase is then equal to the sum of the corresponding components of the different sequences in that phase. We may further simplify the notation as follows; defining:

$$V_0 = V_0 \quad (2.20)$$

$$V_1 = V_+ \quad (2.21)$$

$$V_2 = V_- \quad (2.22)$$

It is now possible to write our symmetrical components in terms of three, namely, those referred to the  $a$  phase:

$$V_a = V_0 + V_+ + V_- \quad (2.23)$$

$$V_b = V_0 + a^2 V_+ + a V_- \quad (2.24)$$

$$V_c = V_0 + a V_+ + a^2 V_- \quad (2.25)$$

These equations may be manipulated to solve  $V_0$ ,  $V_+$  and  $V_-$  in terms of the line-to-neutral voltages  $V_a$ ,  $V_b$  and  $V_c$ .

$$V_0 = \frac{1}{3}(V_a + V_b + V_c) \quad (2.26)$$

$$V_+ = \frac{1}{3}(V_a + aV_b + a^2V_c) \quad (2.27)$$

$$V_- = \frac{1}{3}(V_a + a^2V_1 + aV_c) \quad (2.28)$$

And the same results for the currents:

$$I_0 = \frac{1}{3}(I_a + I_b + I_c) \quad (2.29)$$

$$I_+ = \frac{1}{3}(I_a + aI_b + a^2I_c) \quad (2.30)$$

$$I_- = \frac{1}{3}(I_a + a^2I_1 + aI_c) \quad (2.31)$$

### 2.2.2 Three-phase non-sinusoidal systems

This section addresses the periodic and nonsinusoidal conditions characterized by distorted voltage and current waveforms. Power system harmonics are defined as sinusoidal voltage and currents at frequencies that are integer multiples of the main generated (or fundamental) frequency. They constitute the major distorting components of the mains voltage and load current waveforms.

Since the waveforms are periodic the time variation of each wave can be expressed by means of a Fourier series. Consider the distorted periodic supply voltage and the distorted periodic load current as follows:

$$v(t) = V_0 + \sum_{h=1}^n \sqrt{2}V_h \sin(h\omega t + \alpha_h) \quad (2.32)$$

$$i(t) = I_0 + \sum_{h=1}^n \sqrt{2}I_h \sin(h\omega t + \beta_h) \quad (2.33)$$

For steady-state conditions, a non-sinusoidal periodical instantaneous voltage or current has two distinct components: the power system fundamental frequency components  $v_1$  and  $i_1$  and the remaining terms  $v_H$  and  $i_H$ , respectively.

$$v = v_1 + v_H \quad (2.34)$$

$$i = i_1 + i_H \quad (2.35)$$

The corresponding rms values squared are as follows:

$$V^2 = V_1^2 + V_H^2 \quad (2.36)$$

$$I^2 = I_1^2 + I_H^2 \quad (2.37)$$

where

$$V_H^2 = V_0^2 + \sum_{h=1}^n V_h^2 \quad (2.38)$$

and

$$I_H^2 = I_0^2 + \sum_{h=1}^n I_h^2 \quad (2.39)$$

The direct voltage and the direct current terms  $V_0$  and  $I_0$ , must be included in  $V_H$  and  $I_H$ . Significant direct current (DC) components are rarely present in alternating current (AC) power systems.

The overall deviation of a distorted wave from its fundamental can be estimated with the help of the total harmonic distortion.

The total harmonic distortion of the voltage is defined:

$$THD_V = \frac{V_H}{V_1} = \sqrt{\left(\frac{V}{V_1}\right)^2 - 1} \quad (2.40)$$

And the total harmonic distortion of the current:

$$THD_I = \frac{I_H}{I_1} = \sqrt{\left(\frac{I}{I_1}\right)^2 - 1} \quad (2.41)$$

Since the last third of the 20th century, the idealized consideration of linear and balanced loads ceased to be valid due to the evolution of technology and the increasing complexity of electrical systems, characterized by the increasingly high use of non-linear loads, power electronic equipment, frequency inverters, distributed generations, single-phase loads, etc., which have led to the constant growth of non-sinusoidal currents with a high content of harmonics.

This is translated into distortion powers and imbalances due to the asymmetry of the voltages and currents in the networks, giving rise to unbalance powers which, in any case, cause problems in the electrical networks and in the charges that are connected to them. The perfect balance is technically unattainable. Only in computer simulations and well equipped laboratories does one encounter absolutely perfect balanced three-phase systems.

As previously said, an adaptable power measurement system will be designed, so that non-sinusoidal and unbalanced conditions will be studied.



## Chapter 3

# Power measurement

The concepts of active power, reactive power, apparent power, and power factor have been used in electrical engineering for almost a century now. For sinusoidal single-phase power systems and sinusoidal balanced three-phase systems they have proved to be very useful and efficient for characterizing the quality of the power transmission, for designing the equipment, for billing purposes, and for compensation. These definitions serve the industry well, as long as the current and voltage waveforms remain nearly sinusoidal and balanced three-phase.

However, important changes have occurred in the last 50 years. The new environment is conditioned by the following facts:

- Power electronics equipment, such as adjustable speed drives, controlled rectifiers, cyclo-converters, electronically ballasted lamps, arc and induction furnaces, and clusters of personal computers, represent major nonlinear and parametric loads proliferating among industrial and commercial customers. Such loads have the potential to create a host of disturbances for the utility and the end-user. The main problems stem from the flow of non-active energy caused by harmonic currents and voltages.
- The traditional instrumentation designed for the sinusoidal 60/50 Hz waveform and balanced systems is prone to significant errors when the current and the voltage waveforms are distorted.
- Microprocessors and minicomputers enable today's manufacturers of electrical instruments to construct new, accurate, and versatile metering equipment that is capable of measuring electrical quantities defined by means of advanced mathematical models.

Therefore the concepts of active, reactive, and apparent powers and the related concept of power factor have to be adapted to the new environment such that measurement algorithms and instrumentation can be designed which give guidance with respect to the quantities that should be measured or monitored for revenue purposes, engineering economic decisions, and the determination of major harmonic polluters.

The measurement, analysis and definition of the different terms of three phase power signal, where voltages and currents are unbalanced and distorted, have been studied in order to standardize the correct indexes that quantify the level of harmonic and distortion.

It is well known that meters that measure energy (kWh) and active power (kW) provide accurate measurements also under nonsinusoidal or unbalanced conditions; nevertheless, meters dedicated to apparent power (kVA) and nonactive power (kvar) measurements are prone to significant errors when the current and voltage waveforms are distorted. The main reason for such uncertainties stems from the inadequate power definitions that dictate the conceptual design of such instrumentation.

In general, quantities used in electrical power systems are defined for sinusoidal conditions. Under non sinusoidal conditions, some quantities can conduct to wrong interpretations. Only the active power has a clear physical meaning since it represents the average value of the instantaneous power over a fix period. Apparent power is the most affected quantity.

Evidently this situation led to the search for a practical solution.

The literature abounds with studies of such conditions presenting approaches that complete or contradict each other. Unfortunately no universal agreement has been achieved yet.

Despite all the existing approaches, the development of this project is based on the IEEE 1459-2010 Standard, which has been chosen because of its objective to propose the concepts and the definitions which may be useful and interesting for the evaluation of the quality of electrical energy transmission, the development of measurement algorithms, and the design of measurement instrumentation.

### 3.1 The IEEE Std 1459-2010

In view of the considerations given in the previous section, and as a result of the work of the "IEEE Working Group in nonsinusoidal situations", the Power System Instrumentation and Measurement Committee of the IEEE Power and Energy Society drafted the IEEE Standard 1459, which was published as a Trial Use Standard in 2000 and confirmed as a full Standard in 2002. A revised and updated version was reconfirmed in February 2010 and it is the one that will be used in this thesis.

This standard includes definitions for the measurements of the power magnitudes of electrical systems in sinusoidal or non-sinusoidal and balanced or unbalanced conditions.

It is meant to serve the user who wants to measure and design instrumentation for energy and power quantification. From the effective apparent power, which is the same as the one introduced by Buchholz in 1950, an elegant decomposition of the terms of power is presented according to the physical phenomena that produce them. The mathematical basis is developed that allows the calculation of the components of the apparent power due to the phenomenon of useful energy transfer, to the phenomenon of phase shift, to the imbalance phenomenon and to the distortion phenomenon. The IEEE Std. 1459 was developed with measurement equipment in mind, this is because it needs at least one complete cycle of the network signal in steady state to achieve correct results.



### 3.2 Background: Power measurement under sinusoidal and balanced conditions

We start assuming a balanced three-phase load supplied with the instantaneous line-to-neutral voltages  $v_a, v_b, v_c$  and line currents  $i_a, i_b, i_c$  as settled in equations from 2.1 to 2.6.

The *instantaneous power*  $p$ , is defined as the product of instantaneous voltage  $v$  and instantaneous current  $i$ .

$$p = v_a i_a + v_b i_b + v_c i_c = p_p + p_q \quad (3.1)$$

It has two components, the instantaneous active power  $p_p$  and the instantaneous reactive power  $p_q$ .

The term  $p_p$  is a constant and represents the average value of the instantaneous power during the measurement time interval  $\tau$  to  $\tau + kT$ . It is called *the average real power* or *the active power*  $P$ , and it indicates the transformation of energy from electric to non-electric form.

$$P = \frac{1}{kT} \int_{\tau}^{\tau+kT} p dt \quad (3.2)$$

The active power in each phase is as follows:

$$P_a = P_b = P_c = VI \cos \theta \quad (3.3)$$

And the total active power in the three-phase system is:

$$P = 3VI \cos \theta \quad (3.4)$$

In this ideal case, due to the voltages and the currents are balanced, the instantaneous power  $p$  is constant and equal to the active power  $P$ .

The *reactive power*  $Q$ , causes the interchange of energy between the source and the load without any net transfer of energy.

The definitions are equal as for the active power in one phase and three phase systems respectively:

$$Q_a = Q_b = Q_c = VI \sin \theta \quad (3.5)$$

$$Q = 3VI \sin \theta \quad (3.6)$$

*Complex power*  $S$  is defined as the complex sum of the average and the reactive power.

$$\mathbf{S}_a = \mathbf{S}_b = \mathbf{S}_c = P_a + iQ_a = \mathbf{VI}^* \quad (3.7)$$

The bold symbols denote complex numbers (phasors) where the current phasor  $\mathbf{I}^* = \mathbf{I}\angle\theta$  is the conjugate of the line current phasor  $\mathbf{I} = \mathbf{I}\angle -\theta$ .

The *apparent power*  $S$ , of a single-phase load can be interpreted as the maximum active power that can be transmitted through the same line while keeping the load rms voltage  $V$  constant and the supplying line power loss constant i.e, the rms current  $I$  constant. The apparent power  $S$ , is the product of the root-mean-square (rms) voltage and the rms current.

$$|S_a| = |S_b| = |S_c| = VI \quad (3.8)$$

The total apparent power is:

$$S = 3VI = \sqrt{P^2 + Q^2} \quad (3.9)$$

To distinguish the three powers  $S$ ,  $P$  and  $Q$ , different units are used for each. Real active power  $P$  is in watts (W), reactive power is in volt-amp reactive (VAR), and apparent power is volt-amperes (VA).

Finally, it is important to define the concept of the *Power Factor PF*. It can be interpreted as the ratio between the energy transmitted to the load over the maximum energy that could be transmitted provided the line losses are kept the same.

$$PF = \frac{P}{S} \quad (3.10)$$

### 3.3 Power measurement under nonsinusoidal and unbalanced conditions

This section deals with the most complex case, three-phase circuits with distorted waveforms, unbalanced loads, and asymmetrical voltages.

#### 3.3.1 Active power

The active power in one non-sinusoidal phase is defined as follows:

$$P_{1\phi} = P_1 + P_H \quad (3.11)$$

where  $P_1$  is the fundamental active power, referred to the fundamental frequency and  $P_H$  the harmonics active power, which contains also components for which  $h$  is not an integer

(i.e., interharmonics and subharmonics).

$$P_1 = \frac{1}{kT} \int_{\tau}^{\tau+kT} v_1 i_1 dt = V_1 I_1 \cos \theta_1 \quad (3.12)$$

$$P_H = V_0 I_0 + \sum_{h \neq 1} V_h I_h \cos \theta_h \quad (3.13)$$

The total active power of a three-phase system is defined as the sum of the active power of each phase  $a, b, c$ :

$$P = P_a + P_b + P_c \quad (3.14)$$

### 3.3.1.1 Positive-, negative-, and zero-sequence active powers

The symmetrical voltage components  $V_+, V_-, V_0$  and current components  $I_+, I_-, I_0$  as settled in equations from 2.26 to 2.31 with the respective phase angles  $\theta_+, \theta_-, \theta_0$  yield the following three active power components:

$$P_+ = 3V_+ I_+ \cos \theta_+ \quad (3.15)$$

$$P_- = 3V_- I_- \cos \theta_- \quad (3.16)$$

$$P_0 = 3V_0 I_0 \cos \theta_0 \quad (3.17)$$

The total active power may also be defined as the sum of the active power of each symmetrical component.

$$P = P_+ + P_- + P_0 \quad (3.18)$$

### 3.3.2 Reactive power

In a similar way it is defined the reactive power.

$$Q_a = \frac{\omega}{kT} \int_{\tau}^{\tau+kT} i_a \left[ \int v_a \right] dt = V_a I_a \sin \theta_a \quad (3.19)$$

$$Q_b = \frac{\omega}{kT} \int_{\tau}^{\tau+kT} i_b \left[ \int v_b \right] dt = V_b I_b \sin \theta_b \quad (3.20)$$

$$Q_c = \frac{\omega}{kT} \int_{\tau}^{\tau+kT} i_c \left[ \int v_c \right] dt = V_c I_c \sin \theta_c \quad (3.21)$$

$$Q = Q_a + Q_b + Q_c \quad (3.22)$$

### 3.3.2.1 Positive-, negative-, and zero-sequence reactive powers

The three reactive powers are as follows:

$$Q_+ = 3V_+I_+ \sin \theta_+ \quad (3.23)$$

$$Q_- = 3V_-I_- \sin \theta_- \quad (3.24)$$

$$Q_0 = 3V_0I_0 \sin \theta_0 \quad (3.25)$$

The total reactive power is

$$Q = Q_+ + Q_- + Q_0 \quad (3.26)$$

### 3.3.3 Apparent power

A geometrical interpretation of arithmetic apparent power  $S_A$  and vector apparent power  $S_V$  is presented in Figure 3.1.

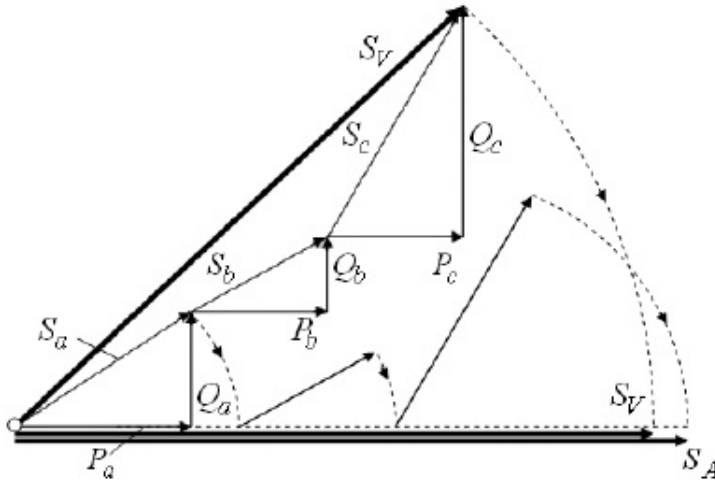


Figure 3.1: Arithmetic and vector apparent powers

#### 3.3.3.1 Arithmetic apparent power

It is defined the total arithmetic apparent power as:

$$S_A = S_a + S_b + S_c \quad (3.27)$$

where the phases apparent power are:

$$S_a^2 = P_a^2 + Q_a^2 \quad S_a = V_a I_a \quad (3.28)$$

$$S_b^2 = P_b^2 + Q_b^2 \quad S_b = V_b I_b \quad (3.29)$$

$$S_c^2 = P_c^2 + Q_c^2 \quad S_c = V_c I_c \quad (3.30)$$

### 3.3.3.2 Vector apparent power

$$S_V = \sqrt{P^2 + Q^2} \quad (3.31)$$

$$S_V = |P + iQ| = |P_a + P_b + P_c + i(Q_a + Q_b + Q_c)| \quad (3.32)$$

$$S_V = |P_+ + P_- + P_0 + i(Q_+ + Q_- + Q_0)| \quad (3.33)$$

It is recommended to renounce these arithmetic and vector apparent power definition, and replace them with the effective apparent power defined below.

### 3.3.3.3 Effective apparent power (VA)

In the past,  $S_e$  was divided into active power  $P$  and nonactive power  $N$  as follows:

$$S_e^2 = P^2 + N^2 \quad (3.34)$$

However, this approach does not separate out the positive-sequence fundamental powers, so it leads to the separation of the rms current and voltage into fundamental and harmonic terms in order to resolve the apparent power as described below.

The real three-phase supplying line carries the current harmonics  $I_{ah}$ ,  $I_{bh}$ , and  $I_{ch}$ . In the case of a four-wire system, there is also a residual or neutral current harmonic  $I_{nh}$ . The actual line and load are replaced with a hypothetical, perfectly compensated system that draws perfectly sinusoidal positive-sequence currents,  $I_e$  and a nil neutral current. The line power loss in the hypothetical system equals the actual power loss causing the same thermal stress. Mathematically this translates in the equality 3.35.

$$\Delta P = 3r_e I_e^2 = r_{dc} \sum_h K_{sh} (I_{ah}^2 + I_{bh}^2 + I_{ch}^2) + r_{ndc} \sum_h K_{snh} I_{nh}^2 \quad (3.35)$$

where  $K_{sh}$  and  $K_{sh}$  are the  $h$ -harmonic order combined skin and proximity effect coefficients for the line conductors and the neutral current path respectively,  $r_{dc}$  and  $r_{ndc}$  are the line and neutral current path dc resistances,  $r_e = K_{s1} r_{dc}$  is the equivalent resistance (i.e., the line resistance measured at fundamental frequency) and  $K_{s1}$  is the combined skin and proximity effect at fundamental frequency (60 or 50 Hz).

Thus, the *equivalent effective current* will have the following expression obtained from 3.35.

$$I_e = \sqrt{\frac{1}{3} \sum_h \left[ \frac{K_{sh}}{K_{s1}} (I_{ah}^2 + I_{bh}^2 + I_{ch}^2) + \frac{K_{snh} r_{ndc}}{K_{s1} r_{dc}} I_{nh}^2 \right]} \quad (3.36)$$

The *rms effective current* can be separated into two components, *the fundamental*  $I_{e1}$  and *the nonfundamental*  $I_{eH}$  as detailed in the equations from 3.37 to 3.40.

$$I_e = \sqrt{I_{e1}^2 + I_{eH}^2} \quad (3.37)$$

$$I_{e1} = \sqrt{\frac{1}{3}[(I_{a1}^2 + I_{b1}^2 + I_{c1}^2) + \rho_1 I_{n1}^2]} \quad (3.38)$$

$$\rho_1 = \frac{K_{sn1} r_{ndc}}{K_{s1} r_{dc}} \quad (3.39)$$

$$I_{eH} = \sqrt{\frac{1}{3}[\sum_{h \neq 1} [K_h(I_{ah}^2 + I_{bh}^2 + I_{ch}^2)] + \rho_h I_{nh}^2]} \quad (3.40)$$

$$K_h = \frac{K_{sh}}{K_{s1}} \quad (3.41)$$

$$\rho_h = \frac{K_{snh} r_{ndc}}{K_{s1} r_{dc}} \quad (3.42)$$

In most practical applications, the ratios  $\rho_1, \rho_h$ , and  $K_h$  are not known. Network topology changes, temperature changes, and seasonal changes in soil humidity and temperature make the estimation of these ratios a very difficult task. Until tools that allow the correct determination of such values will be available, it is recommended to use the values  $\rho_1 = \rho_h = K_h = 1.0$ .

This approach leads to simpler expressions:

$$I_e = \sqrt{\frac{I_a^2 + I_b^2 + I_c^2 + I_n^2}{3}} \quad (3.43)$$

$$I_{e1} = \sqrt{\frac{I_{a1}^2 + I_{b1}^2 + I_{c1}^2 + I_{n1}^2}{3}} \quad (3.44)$$

$$I_{eH} = \sqrt{\frac{I_{aH}^2 + I_{bH}^2 + I_{cH}^2 + I_{nH}^2}{3}} = \sqrt{I_e^2 - I_{e1}^2} \quad (3.45)$$

The effective current may also be referred to the symmetrical components as follows:

$$I_e = \sqrt{I_+^2 + I_-^2 + (1 + 3\rho)I_0^2} \quad (3.46)$$

Note: Consider that for three-wire systems,  $I_{n1} = I_{nh} = 0$  and  $I_0 = 0$ .

The practical expressions for the *effective voltage*,  $V_e$  are obtained in a similar manner. The equivalent voltage is obtained assuming that the active components of the load consist of a set of three equivalent resistances  $R_Y$  connected in Y, supplied by a four-wire line and dissipating the active power  $P_Y$ . The remaining active load consists of three  $\Delta$ -connected equivalent resistances,  $R_\Delta$ , that dissipate the power  $P_\Delta$ .

The power equivalence between the actual and the equivalent system is expressed as follows:

$$\frac{\sum_h(V_{ah}^2 + V_{bh}^2 + V_{ch}^2)}{R_Y} + \frac{\sum_h(V_{abh}^2 + V_{bch}^2 + V_{cah}^2)}{R_\Delta} = \frac{3V_e^2}{R_Y} + \frac{9V_e^2}{R_\Delta} \quad (3.47)$$

And with the notation,

$$\xi = \frac{P_\Delta}{P_Y} = \frac{9V_e^2 R_Y}{R_\Delta 3V_e^2} = \frac{3R_Y}{R_\Delta} \quad (3.48)$$

we can find the effective voltage:

$$V_e = \sqrt{\frac{3(V_a^2 + V_b^2 + V_c^2) + \xi(V_{ab}^2 + V_{bc}^2 + V_{ca}^2)}{9(1 + \xi)}} \quad (3.49)$$

Which referred to the symmetrical components results:

$$V_e = \sqrt{V_+^2 + V_-^2 + \frac{V_0^2}{1 + \xi}} \quad (3.50)$$

Note: In most practical systems,  $V_0$  is negligible.

In case that the value of the ratio  $\xi$  is not known, it is recommended to use  $\xi = 1.0$  that combined with the separation of fundamental components from the harmonics and interharmonics, thus leads to the effective voltage expressions:

$$V_e = \sqrt{V_{e1}^2 + V_{eH}^2} \quad (3.51)$$

$$V_e = \sqrt{\frac{1}{18}[3(V_a^2 + V_b^2 + V_c^2) + V_{ab}^2 + V_{bc}^2 + V_{ca}^2]} \quad (3.52)$$

$$V_{e1} = \sqrt{\frac{1}{18}[3(V_{a1}^2 + V_{b1}^2 + V_{c1}^2) + V_{ab1}^2 + V_{bc1}^2 + V_{ca1}^2]} \quad (3.53)$$

$$V_{eH} = \sqrt{\frac{1}{18}[3(V_{aH}^2 + V_{bH}^2 + V_{cH}^2) + V_{abH}^2 + V_{bcH}^2 + V_{caH}^2]} = \sqrt{V_e^2 - V_{e1}^2} \quad (3.54)$$

and for three-wire systems:

$$V_e = \sqrt{\frac{V_{ab}^2 + V_{bc}^2 + V_{ca}^2}{9}} \quad (3.55)$$

$$V_{e1} = \sqrt{\frac{V_{ab1}^2 + V_{bc1}^2 + V_{ca1}^2}{9}} \quad (3.56)$$

$$V_{eH} = \sqrt{\frac{V_{abH}^2 + V_{bcH}^2 + V_{caH}^2}{9}} = \sqrt{V_e^2 - V_{e1}^2} \quad (3.57)$$

The resolution of the *effective apparent power*,  $S_e$  is separated into two major terms, where  $S_{e1}$  is the *fundamental effective apparent power* and the term  $S_{eN}$  is the *nonfundamental effective apparent power*.

$$S_e^2 = S_{e1}^2 + S_{eN}^2 \quad (3.58)$$

$$S_{e1} = 3V_{e1}I_{e1} \quad (3.59)$$

$$S_{eN}^2 = S_e^2 - S_{e1}^2 = D_{eI}^2 + D_{eV}^2 + S_{eH}^2 \quad (3.60)$$

The *nonfundamental effective apparent power*,  $S_{eN}$  has three components:

The current distortion power, usually the largest component of  $S_{eN}$

$$D_{eI} = 3V_{e1}I_{eH} \quad (3.61)$$

the voltage distortion power,

$$D_{eV} = 3V_{eH}I_{e1} \quad (3.62)$$

and the effective harmonic apparent power which is characterized by the *total harmonic active power*,  $P_H$  (Equation 3.13) and the *harmonic distortion power*,  $D_{eH}$ .

$$S_{eH} = 3V_{eH}I_{eH} \quad (3.63)$$

$$S_{eH}^2 = P_H^2 + D_{eH}^2 \quad (3.64)$$

$$D_{eH} = \sqrt{S_{eH}^2 - P_H^2} \quad (3.65)$$

The components of  $S_{eN}$  can be expressed in function of the equivalent total harmonic distortions  $THD_{eV}$  for voltage and  $THD_{eI}$  for current.

$$THD_{eV} = \frac{V_{eH}}{V_{e1}} \quad (3.66)$$

$$THD_{eI} = \frac{I_{eH}}{I_{e1}} \quad (3.67)$$



It results as follows:

$$S_{eN} = S_{e1} \sqrt{THD_{eI}^2 + THD_{eV}^2 + (THD_{eI}THD_{eI})^2} \quad (3.68)$$

$$D_{eI} = S_{e1}THD_{eI} \quad (3.69)$$

$$D_{eV} = S_{e1}THD_{eV} \quad (3.70)$$

$$S_{eH} = S_{e1}THD_{eV}THD_{eI} \quad (3.71)$$

$$(3.72)$$

The *power factor, PF* follows its classical definition:

$$PF = \frac{P}{S_e} = \frac{P_1 + P_H}{S_e} \quad (3.73)$$

The most important powers are the fundamental positive-sequence active and reactive powers  $P_1^+$  and  $Q_1^+$ , respectively.

They are tied to the fundamental positive-sequence apparent power

$$(S_1^+)^2 = (P_1^+)^2 + (Q_1^+)^2 \quad (3.74)$$

with the fundamental power factor

$$PF_1^+ = \frac{P_1^+}{S_1^+} \quad (3.75)$$

The unbalance fundamental power

$$S_{U1} = \sqrt{S_{e1}^2 - (S_1^+)^2} \quad (3.76)$$

is introduced to allow the positive-sequence powers  $P_1^+$  and  $Q_1^+$  separation from  $S_{e1}$ .  $S_{U1}$  includes the contributions of the 50/60 Hz negative- and positive-sequence powers and gives a crude indication about the degree of load imbalance.



## Chapter 4

# Modeling and simulation

Modeling and simulation of designed systems is a very valuable tool for several reasons:

- Total manipulation of the parameters of the model.
- Variation of the temporal horizons of the model, from a few milliseconds to years.
- Measurement with graphical and numerical tools with which to analyze the results of the simulation.
- Real-time monitoring of model values.
- Isolation of variables, being able to modify the parameters of the components individually, without affecting the rest of the model.
- Elimination of the risk of damaging the components.
- Reduction of project costs when performing a detailed analysis.

Prior to the construction of the real physical circuit, and in accordance with the power calculation techniques described in the previous chapter, a digital model will be built, using the Simulink tool of MATLAB.

### **Matlab & Simulink**

MATLAB, acronym for MATrix LABoratory, arises in 1984, created by Cleve Moler, a mathematician and American programmer specialized in numerical analysis.

It is a fourth-generation programming language and numerical analysis environment whose uses include matrix calculations, developing and running algorithms, creating user interfaces (UI) and data visualization. The multi-paradigm numerical computing environment allows developers to interface with programs developed in different languages, which makes it possible to harness the unique strengths of each language for various purposes.

MATLAB is used by engineers and scientists in many fields such as image and signal processing, communications, control systems for industry, smart grid design, robotics as well as computational finance thanks to one of its greatest virtues, its modularity that creates an infinite number of modules to deal with a wide variety of problems in different areas. These

modules are grouped into libraries called toolboxes. Nowadays we are at a point where there are more than 50 toolboxes, however, undoubtedly the one that has had the most impact on the evolution of MATLAB applications is Simulink.

Simulink is a program for simulation (modeling and analysis) of nonlinear dynamic systems which appearance was linked to the first version of MATLAB for Windows, in May 1994, in what was incorporated the option to install the toolbox separately.

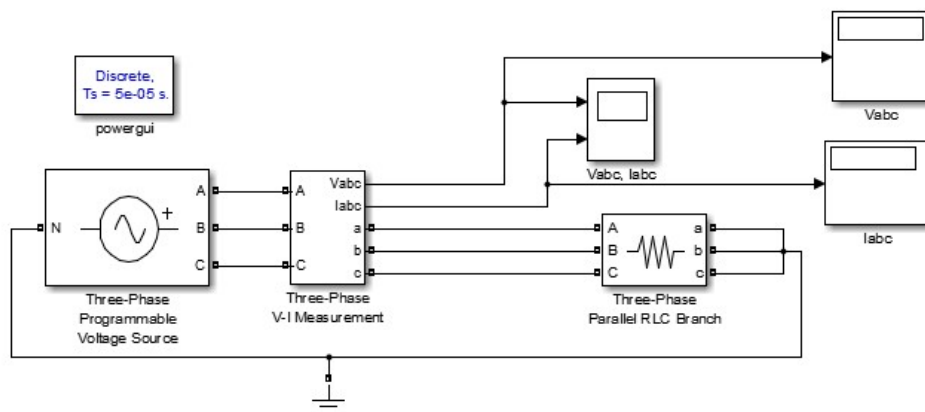
Simulink offers a graphical environment of windows, in which the user can define the systems by means of a block diagram, in a much more intuitive way than other simulation programs. Using Simulink, the user can create their models in the form of high-level block diagrams starting from libraries of basic components, use toolbox components for specific applications, and program their own blocks for, establishing the appropriate connections and parameterizing the model properly, to greatly reduce the development time if the model was created by conventional programming languages.

Once the model is created, Simulink also allows to execute its analysis, being able to choose different integration methods.

## 4.1 Starting point

As initial contact, modeling of a three-phase sinusoidal and balanced system will be carried out (Figure 4.1) in order to learn how to measure the corresponding currents and voltages correctly.

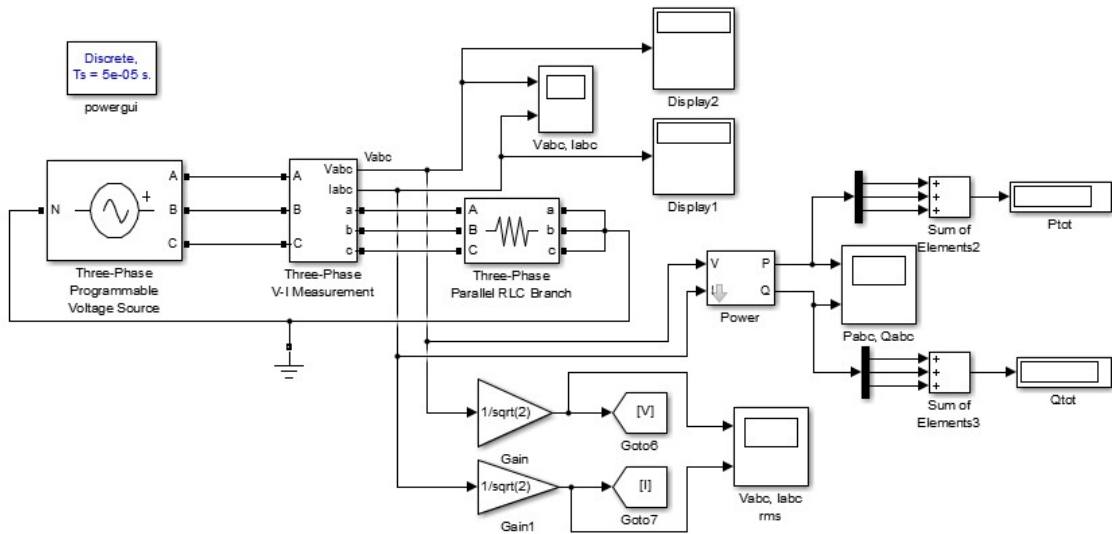
As a physical electrical system will be modeled, it is required the use of the Simulink toolbox, *Simscape*. In the following figure, all the blocks referring to the electrical circuit or to the measurement of electrical signals come from this library.



**Figure 4.1:** Balanced and sinusoidal system model.

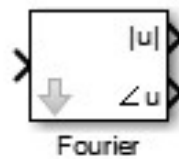
The electrical circuit consists on a three-phase source supplying a three-phase load. In this case, the voltage values are phase to phase  $230 V_{rms}$  in  $50 Hz$  of frequency. The load consists on a resistance of  $1000 \Omega$ . The *Three-phase V-I Measurement* block represents the voltage and current sensors.

## 4.2 Common model



**Figure 4.2:** Electrical system design

After understanding the measurement blocks operation, the first step is to prepare the measured current and voltage signals for the power calculation. As deduced in the previous chapter, rms values will be used, so the read signals must be divided by  $\sqrt{2}$ . Subsequently the magnitude and phase of these signals must be calculated for the application of the power formulation. In Simulink, the way to calculate these magnitudes is through the *Fourier* block (Figure 4.3). It must be configured by setting parameters such as the fundamental frequency of the input signal, the harmonic number for which these quantities are calculated, the initial magnitude and phase of the output signal if known, and the sample time of the block.

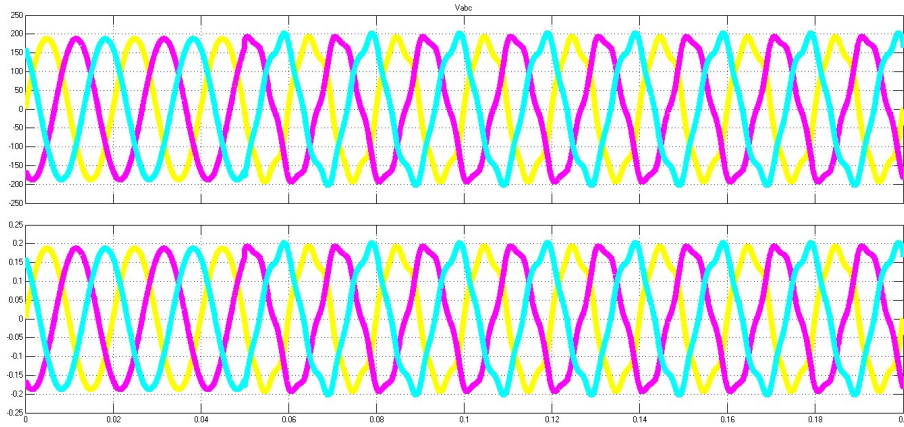


**Figure 4.3:** Fourier block

The next step is to generate some harmonics at the supplying voltage so that the system becomes the case of our study.

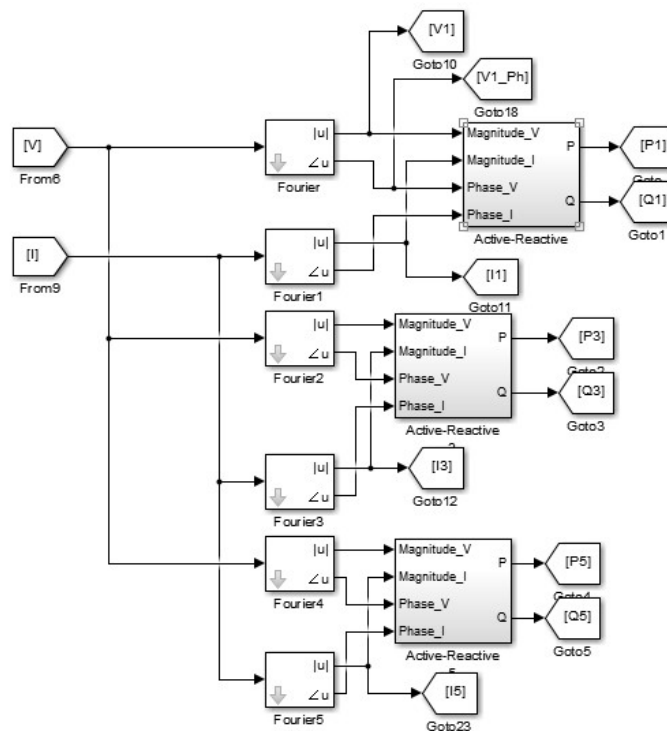
The *Three-Phase Programmable Voltage Source* block has been chosen because of its ability to generate harmonics, which will allow us to simulate a real non-sinusoidal system.

Same voltage supplying signals are generated as in the preceding section with the difference that after 0.05 seconds of simulation, the distortion starts due the generation of the third and fifth harmonics (Figure 4.4).



**Figure 4.4:** Distorted supplying voltage

Pursuing the objective of this project and based on the formulation described above, the necessary operations are performed to calculate the power. Firstly, the active and reactive power of the fundamental frequency and of each harmonic are determined (Figures 4.5 and 4.6).



**Figure 4.5:** Block diagram for active and reactive powers calculation.

At this point, it has been estimated the fundamental and harmonic power for every phase. The next step is to add them to obtain the powers of each phase. At last, these phase powers are added to obtain the total active and reactive power of the system (Figure 4.7).

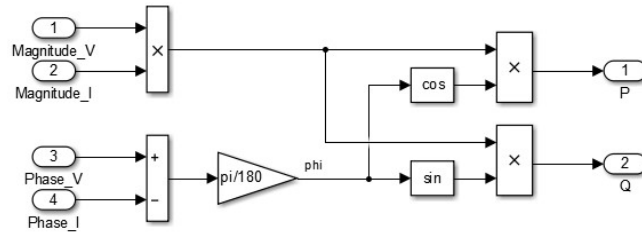


Figure 4.6: Active-Reactive subsystem.

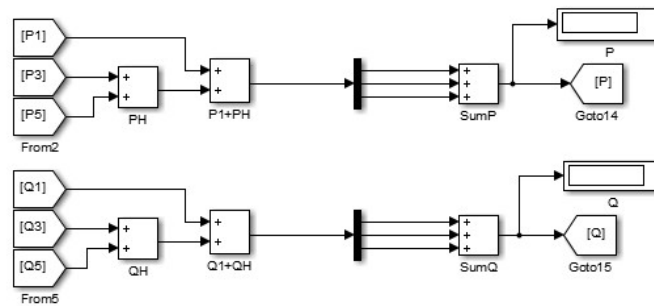


Figure 4.7: Block diagram for total active and reactive powers calculation.

In the same way that was deduced, these powers may be calculated based on the sequence of phases (Figures 4.8 and 4.10)

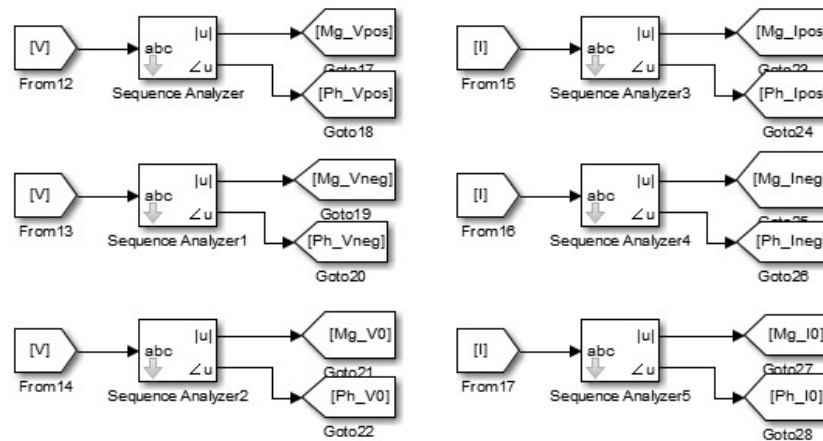


Figure 4.8: Symmetrical components calculation.

In this case, for obtaining the symmetrical components of voltage and current, the *Sequence analyzer* block is used (Figure 4.9). Its mode of operation is similar to that of the Fourier block described above, with the addition of being able to specify the sequence component the block outputs.

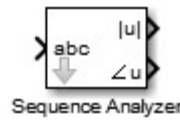


Figure 4.9: Sequence analyzer block.

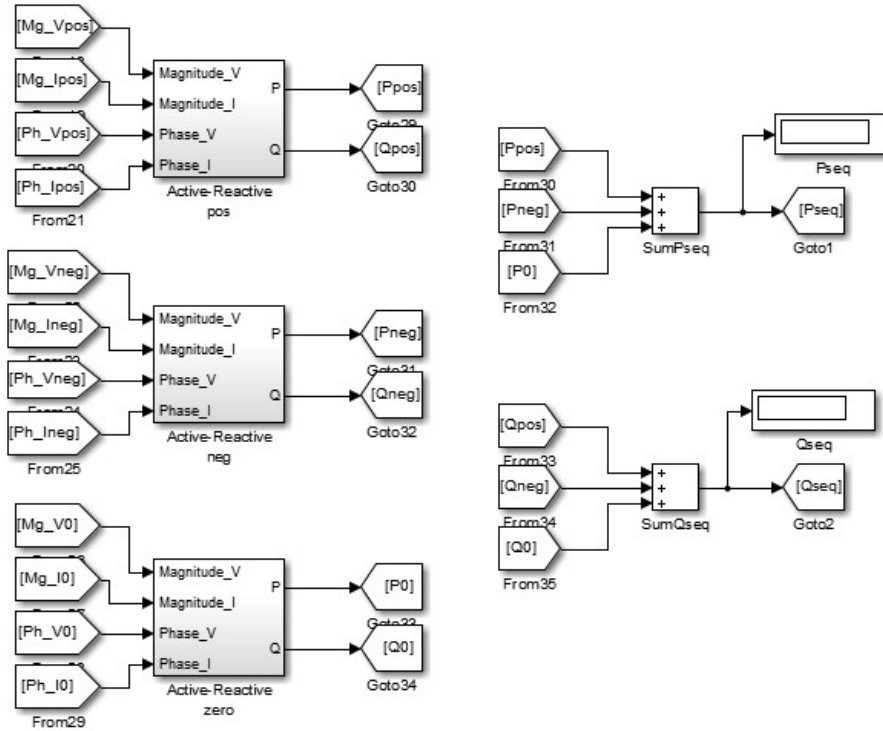


Figure 4.10: Total active and reactive powers based on positive, negative and zero powers.

According to the recommendation of the IEEE 1459 Standard, as far as the apparent power is concerned, the effective apparent power will be calculated.

Beginning, the effective current of both forms is calculated, decomposing it into the fundamental and non fundamental effective current (Equations from 3.36 to 3.40) (Figure 4.11) and referring it to the symmetrical components (Equation 3.37) (Figure 4.12) so that the correct deduction of both can be verified.

Secondly, the effective voltage is calculated as in the equation 3.50 concerned to the symmetrical components calculation. The fundamental effective voltage comes from the equation 3.56 based on the phase to phase voltages at this frequency, and the nonfundamental effective voltage from the quadratic difference in 3.57.



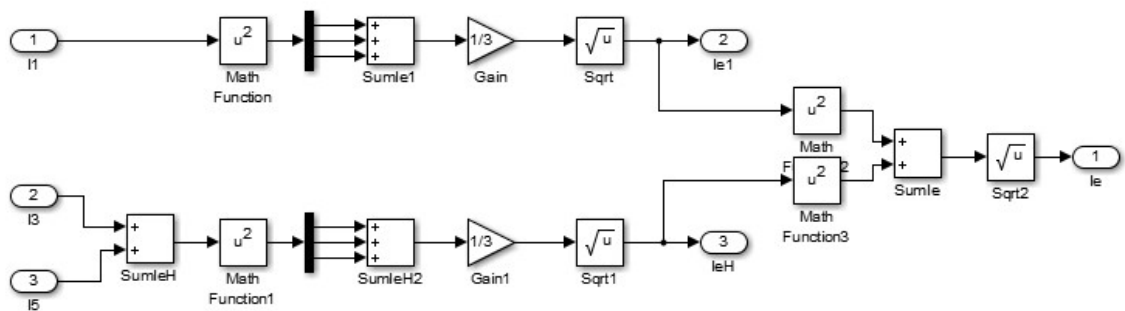


Figure 4.11: Fundamental and nonfundamental frequency effective current decomposition.

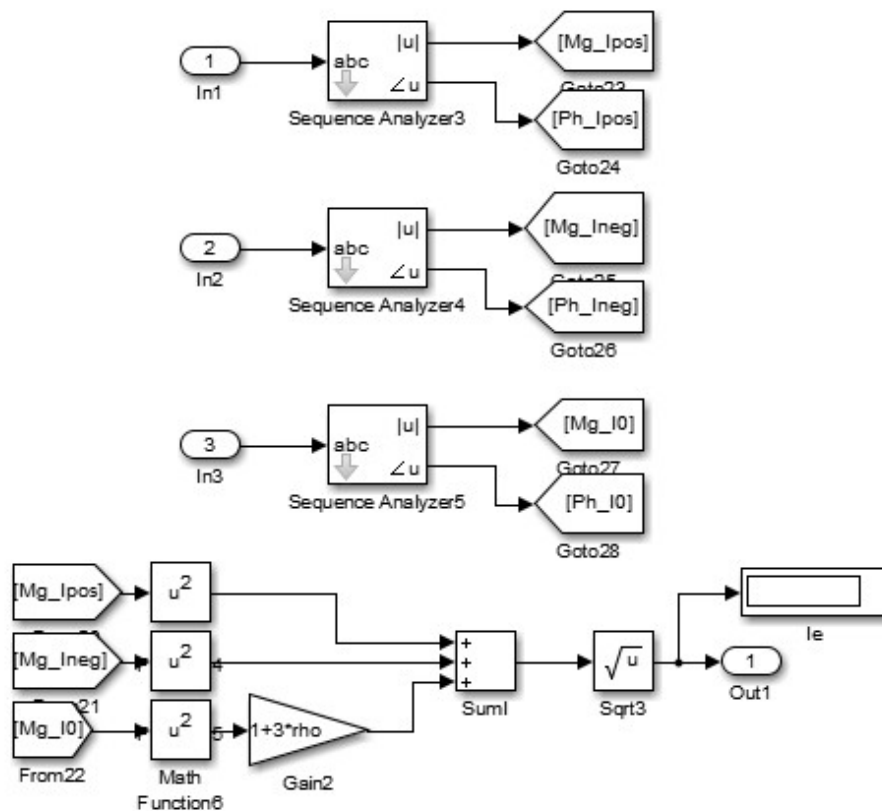


Figure 4.12: Effective current calculated from the symmetrical components.

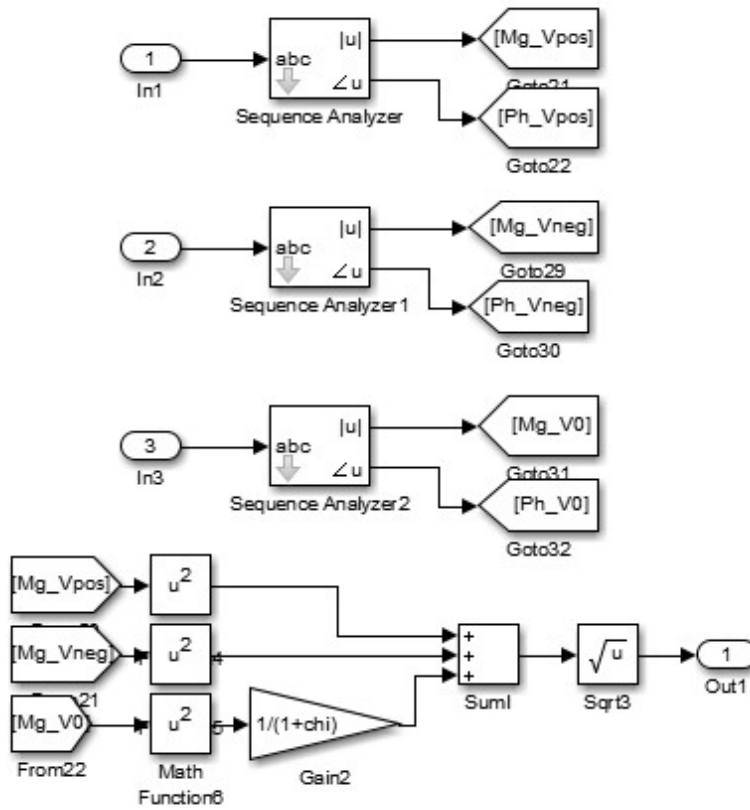


Figure 4.13: Block diagram for  $V_e$  calculation.

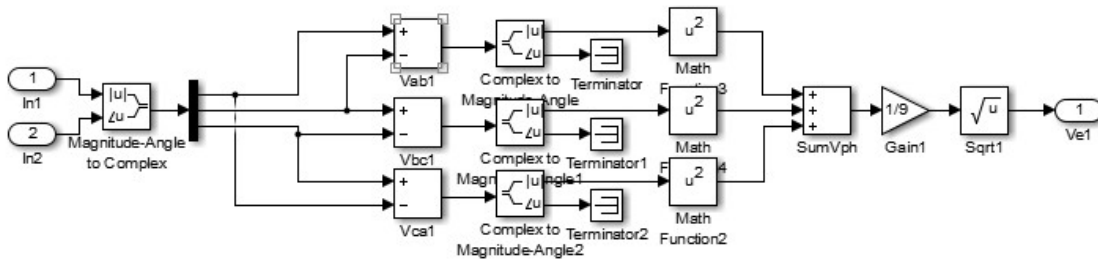


Figure 4.14: Fundamental frequency effective voltage.

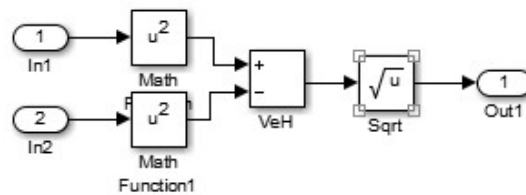


Figure 4.15: Nonfundamental frequency effective voltage.

With all these values we proceed to the modeling of the effective apparent power calculation following the equations 3.58 to 3.63 as shown in figure 4.16

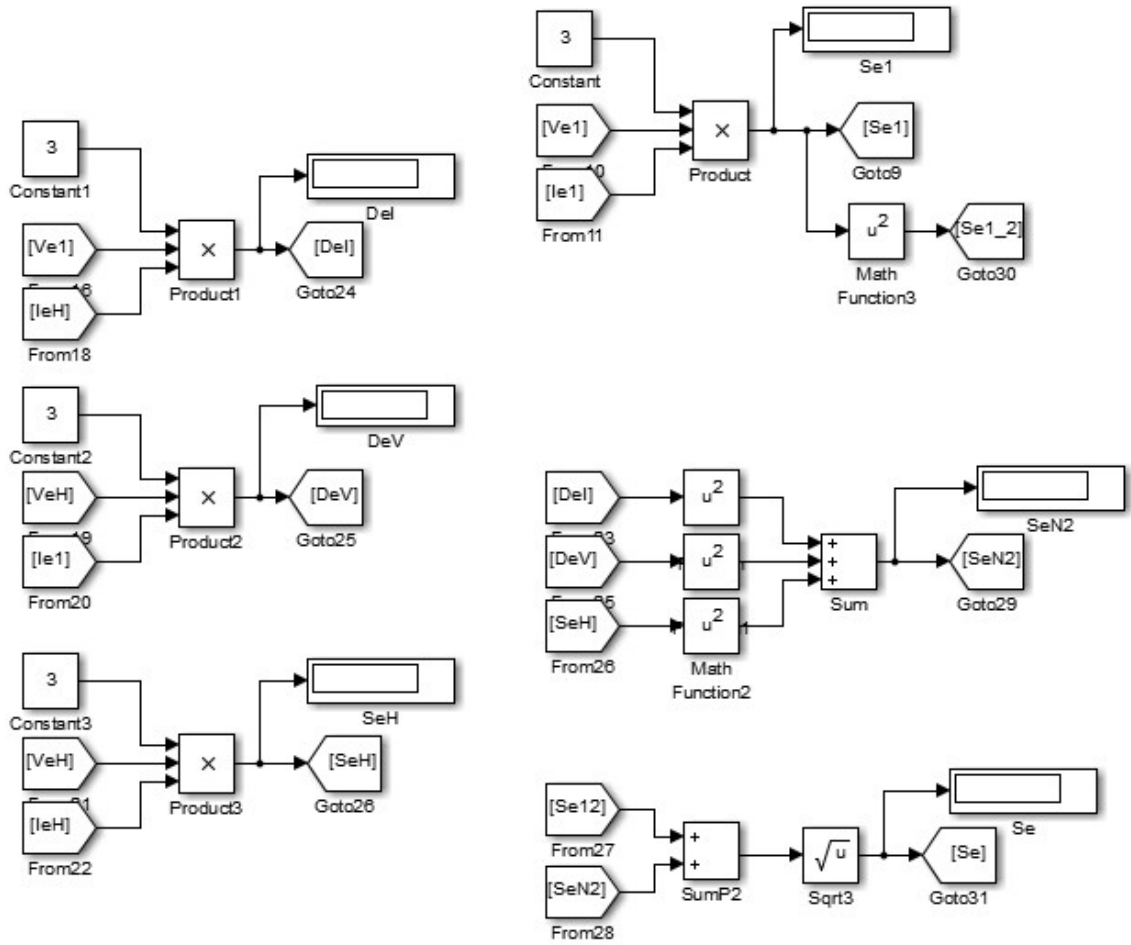


Figure 4.16: Calculation of effective apparent power.

To conclude, the values that measure the system distortion and the power ratio between the energy transmitted to the load over the maximum energy that could be transmitted provided the line losses are kept the same are modeled(Figure 4.17).

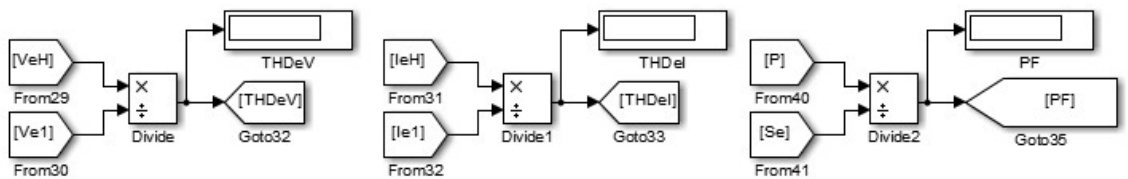


Figure 4.17: System distortion values.

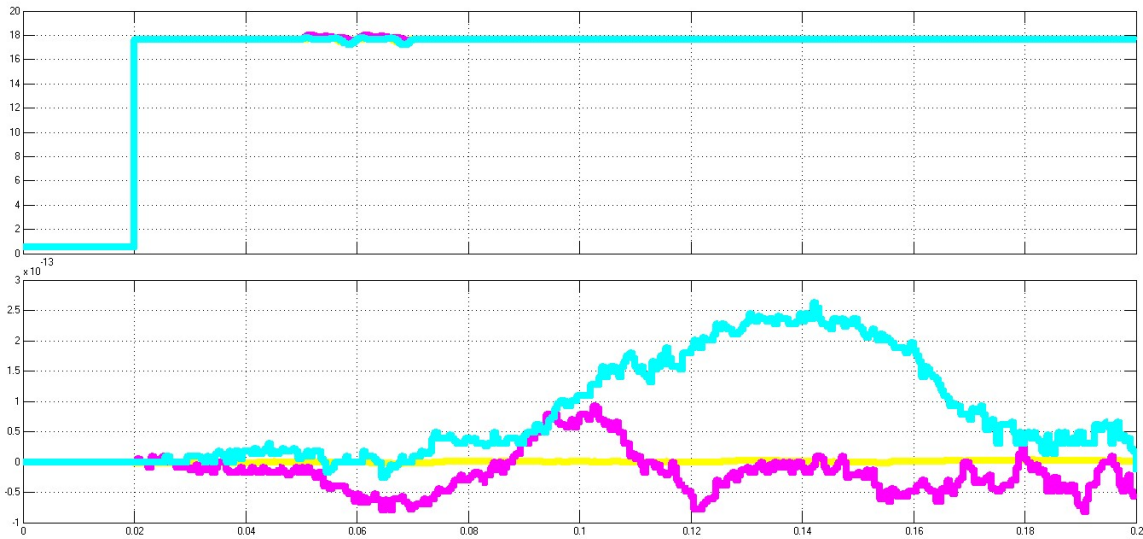
### 4.3 Simulation results

A simulation of 0.2 seconds is performed.

As explained throughout the previous section, in this simulation, a 230  $V_{rms}$  and 50  $Hz$  generator with contents of the third and fifth harmonics, will feed a pure resistive load composed by a resistance of 1000  $\Omega$ .

Furthermore, a discretization time  $T_s$  of  $5 * 10^{-5}$  has been chosen.

The results obtained from the modeling of the deduced equations are compared with those given by the *Power* block of the Simscape library, developed by mathworks, which results can be observed in the figure 4.18. For active power a value of 52.76 W is obtained. On the other hand, and as expected for a resistive circuit, the reactive power is almost zero ( $1 * 10^{-14}$  var).



**Figure 4.18:** Active and reactive power results from *Power* block.

The result of our model is established in 53.17W for the active power, and also nil for the reactive( Figures 4.19 to 4.21). As can be seen, the power is disturbed with the introduction of the harmonics at 0.05 seconds, later finding the balance.

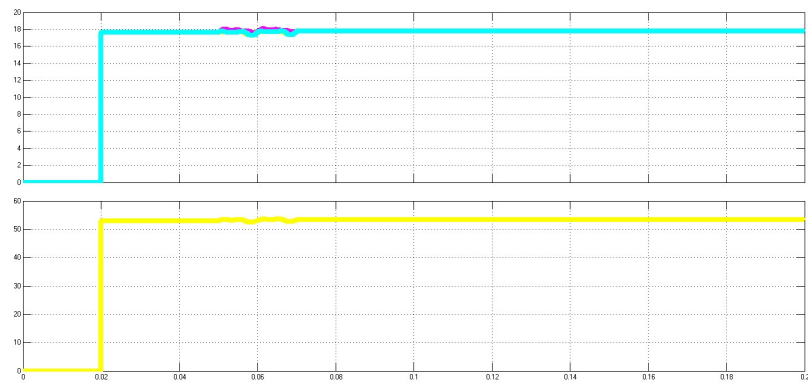


Figure 4.19: Active power result.

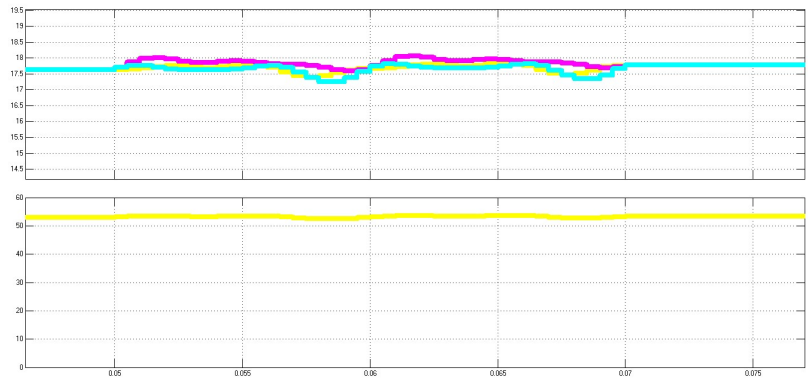


Figure 4.20: Active power result detailed at distortion.

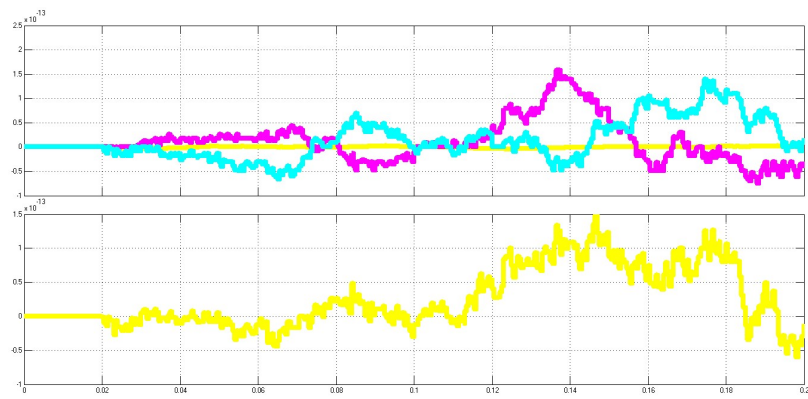
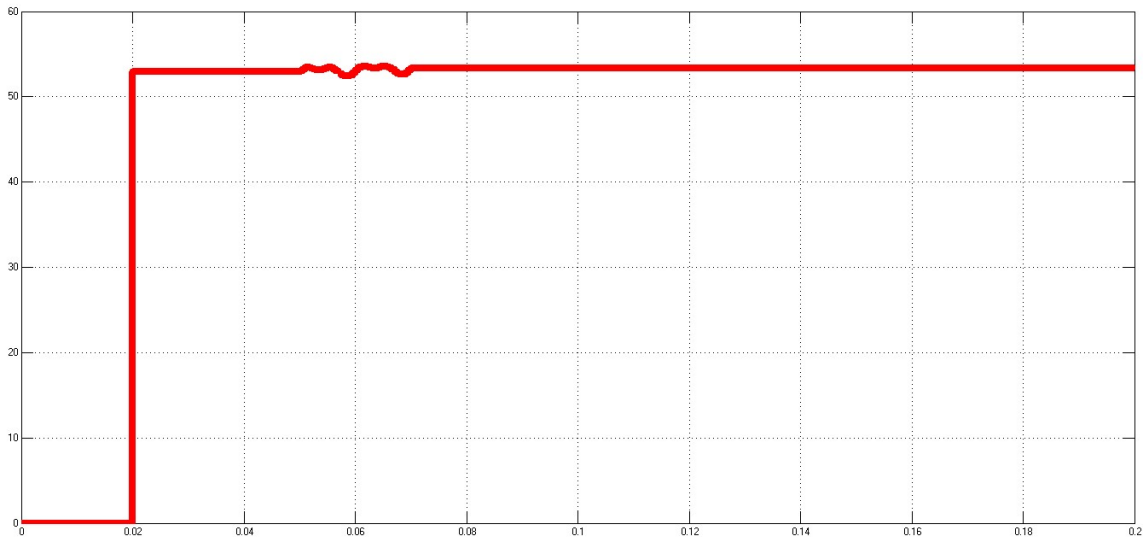


Figure 4.21: Reactive power result.

A similar result is obtained for the apparent power, in which the power is set at 53.16 VA (Figure 4.22)



**Figure 4.22:** Effective apparent power result.

As conclusion of these data, we can conclude that the use of the IEEE 1459 standard estimation is appropriate and that a correct modeling of the power measurement has been made.

## Chapter 5

# Physical implementation

After having made the modeling of the measurement system, it begins the physical implementation.

It is intended to program the microcontroller so that, read and processed the voltage and line current signals, calculate the power values in the same way as in the simulation of the previous chapter.

The first step is to program "the brain" of our system. This is referred to the microcontroller ATmega 2560, present on the Arduino Mega 2560 board. It is a board based on the microcontroller ATmega2560 which has been chosen due to the following fundamental reasons:

- The possibility of developing microcontroller programming through graphic blocks using the MATLAB / Simulink tool.
- Its low economic cost, which makes it accessible for all user levels.
- It is capable of receiving analog signals, which allows measuring voltages and currents.
- According to the objective of our project, we want to calculate the power in real time, and that is the main reason to chose this board over the other ones due to it allows to use the Simulink program in *External mode*, which makes possible to load the program into the device for execution thus allowing to perform calculations and operations in parallel, reducing the computational load and the weight to occupy in the microcontroller's data memory.

This objective will be done by programming the Arduino to perform the calculation on its own, while a program for receiving the calculated power data is running from Simulink at the same time.

## Arduino

Arduino, based on the description offered on its official website, is "an open-source electronic prototype platform based on flexible and easy-to-use hardware and software". It is a simple, flexible and economical tool to implement digital programs in real systems, using a wide range of sensors and actuators.

The Arduino platform has its origin in 2005, at the Interactive Institute of Ivrea (Italy), in which students used the BASIC Stamp microcontroller, at a cost of about 100 Dollars, which is too high for students. From the need of a platform of low cost and without problems of compatibilities between operating systems, Arduino project arises, whose name derives from King Arduino of Italy, between the years 1002 and 1014. The institute was closed in 2006, having been previously released the patent of the project, to avoid that it was seized, and thus allowing the project was not forgotten, and developers around the world had the opportunity to evolve the project.

An Arduino device consists of a printed circuit board in which various elements are integrated, being the most important for the purposes of the application of the present project the microcontroller, the input and output ports and the interface with the computer.

In the case of the present work, as already mentioned, the Arduino board selected is an Arduino Mega 2560 (Figure 5.1).

The Arduino Mega 2560 has 54 digital input/output pins (of which 14 can be used as PWM outputs), 16 analog inputs, 4 UARTs (hardware serial ports), a 16 MHz crystal oscillator, a USB connection, a power jack, an ICSP header, and a reset button. It contains everything needed to support the microcontroller; simply connect it to a computer with a USB cable or power it with a AC-to-DC adapter or battery to get started.

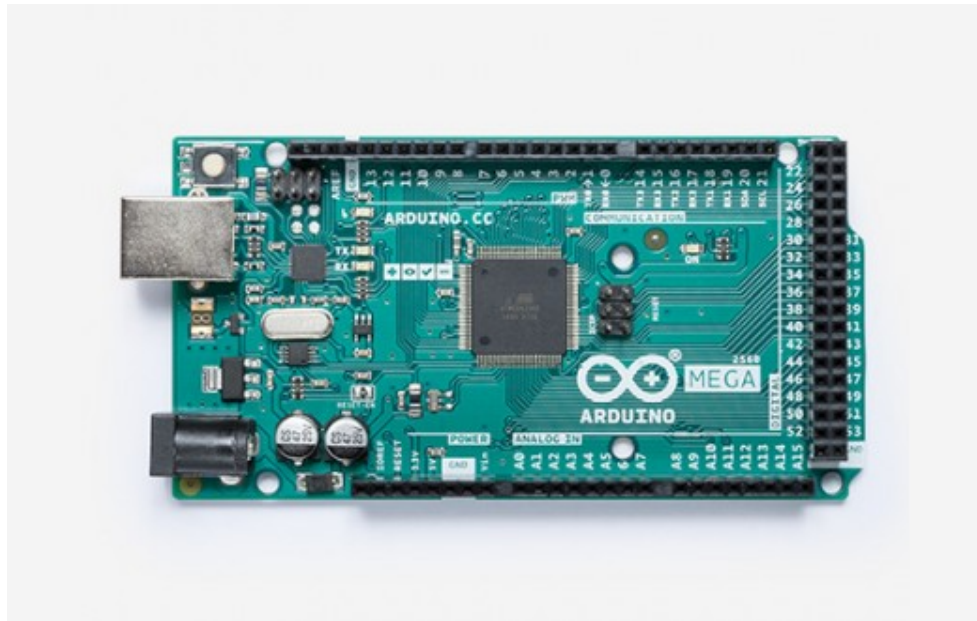
In our case, it will be powered by USB as this port will also be used for communication as it will be seen below.

Regarding the software, Arduino has its own IDE (Integrated Development Environment), called Arduino Development Environment, open source and simple operation. However, Arduino allows replacing this IDE, in which programming is done by code, by MATLAB / Simulink, a graphical development environment in which the programming of the Arduino is done by functional blocks. Knowing the intention of Mathworks to develop a wide variety of toolboxes to expand the range of MATLAB applications, the simplicity and intuitiveness of its way of developing graphical models using Simulink blocks and the wide dissemination that Arduino has at the global level, it was a matter of time before both platforms were integrated.

This happens from the year 2012, with the appearance of two support packages for Arduino hardware, *Matlab support package for Arduino*, and *Simulink support package for Arduino*. Depending on the needs of each project, it will be interesting to download one or another software package, depending on what you want to develop and the use you want to give the Arduino board within the model.

In the sphere of this thesis, the interesting thing will be to use a Simulink model that can

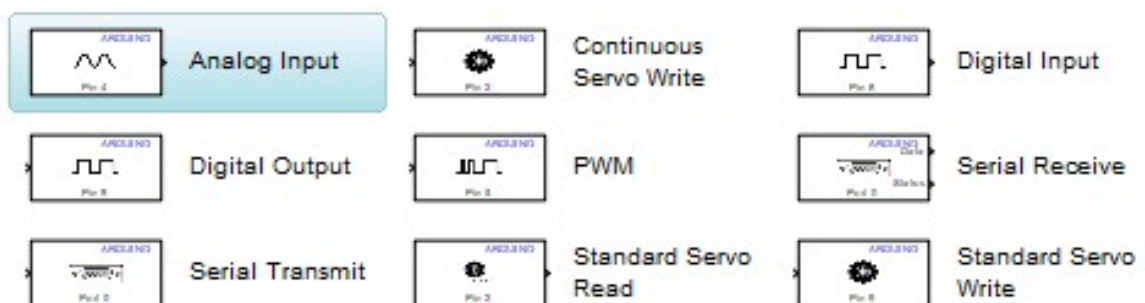




**Figure 5.1:** Arduino Mega 2560 board.

be executed embedded in the Arduino board, so the package to be installed will be that of Simulink for Arduino.

To install the support toolbox for Arduino in MATLAB, which version used for the present project is r2014a, the manufacturer's instructions have been followed, displaying the Add-Ons menu in the MATLAB console. Specifically, the Simulink package for Arduino is included in the hardware support packages, among which we must select the type of hardware that is available (in the case of this work, Arduino Mega). Following the installation procedure, the program will be ready to create models in Simulink executable in Arduino. The modules belonging to the Simulink support package for Arduino are on the 'Simulink Support Package for Arduino' route, combined with the conventional Simulink blocks. The Simulink support package for Arduino contains the blocks shown in Figure 5.2.



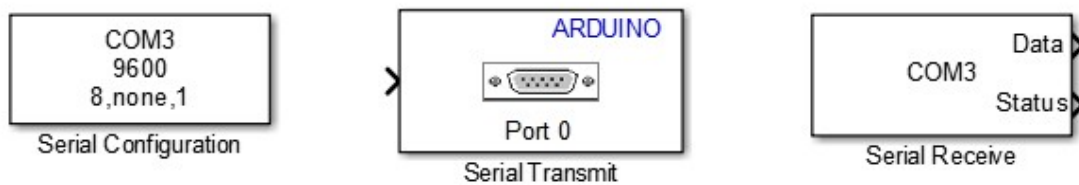
**Figure 5.2:** Blocks of the Simulink package for Arduino.

## 5.1 Simulink-Arduino communication

The Simulink support package for Arduino has blocks that allow the sending and reception of data by serial communication, being possible to send and receive information between the computer and the Arduino.

Different models have been created, covering the needs that were arising until reaching the final model that will be explained in detail below. The final model allows communication with Arduino in *Normal mode*. This mode has been chosen over the *External mode* which is used to deploy the calculation program on the target and could seem more appropriate to be a type of simulation in real time, because in our case, in real time we must do the signal processing and calculation of the power, while the results can be sent to Simulink with a certain delay.

Three blocks make our communication possible: *Serial Configuration*, *Serial Recive* and *Serial Send* (Figure 5.3). These blocks use the Universal Serial Port (USB), for sending and receiving data, and this port is Half-Duplex type, which means that only can receive or send data at the same time, that is, to send or receive data, the port must be free of traffic.



**Figure 5.3:** Blocks of the Simulink package for Arduino for Serial Communication.

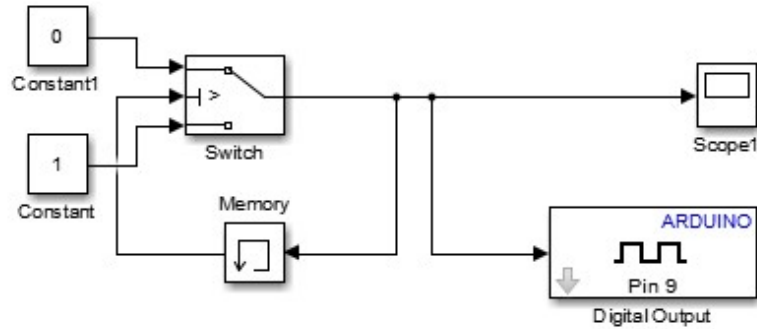
Serial Configuration block configures the parameters of a serial port that can be used to send and receive data. Values must be given to all its parameters before placing a Serial Send or a Serial Receive. Through Serial Transmit block it is sent buffered data to the specified serial port. Serial Receive configures and opens an interface to a remote address specified using the Serial Protocol. Configuration and initialization occurs once at the beginning of the simulation. The block acquires data during the execution time of the model.

### 5.1.1 Signal processing

Among the objectives of this project, it is the calculation in real time of the power and its sending to the computer for its visualization and analysis.

Different models and block diagrams have been created in order to know the maximum frequency at which the microcontroller could generate or receive an analog signal.

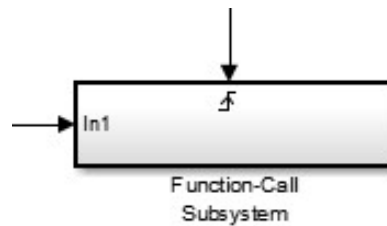
In the figure 5.4, the block diagram used for the generation of a periodic square wave is represented, by means of which it is tried to find out the frequency to which our card can process signals. It has reached the value of 5 KHz, corresponding to a period of 0.2 ms,



**Figure 5.4:** Generation of a square wave.

more than enough for the signals that are intended to process.

Although high frequency value is needed for real-time signal processing and power calculation, the results can be sent at a lower frequency for visualization so that the microcontroller memory is not overloaded. With this objective, the *Function Call Subsystem* block (Figure 5.5) will be used, which enables the code generated within it every certain time programmed by the designer.



**Figure 5.5:** Function-Call Subsystem block with a data entry and a clock for its activation by rising edge.

### 5.1.2 Software adaptation

In order to adapt the software proposed in the previous chapter, a new model of a single-phase system is made. It is carried out for the purpose of facilitating the processing of two signals, one current and one voltage, compared to the six present in the three-phase system (Figure 5.6). In this model, two signals generated with a signal generator are introduced into analog pins A0 and A4 although any of the A0 to A15 could be used. These signals will represent a voltage signal of  $5 V_{pp}$  and  $2.5 V$  offset and a current signal of  $2 V_{pp}$  and  $1 V$  offset. Both will be generated in the first tests with a frequency of  $5 \text{ Hz}$ .

A microcontroller has no capacity to work with analog signals, so it is needed to convert the analog signals into digital signals in order to work with them (Figure 5.7). In the Arduino Mega, the analog digital internal converter has a resolution of 10 bits, this means that the analog input voltage is converted into a numeric value between 0 and 1023, and a reference voltage of  $5 \text{ V}$  since it is TTL technology.

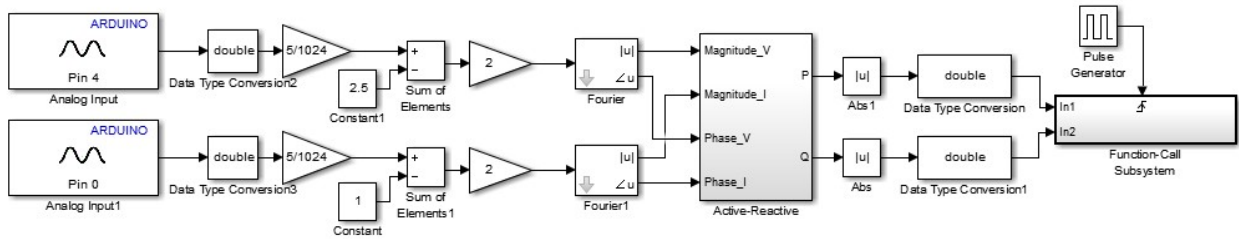


Figure 5.6: Block diagram of the single-phase system.

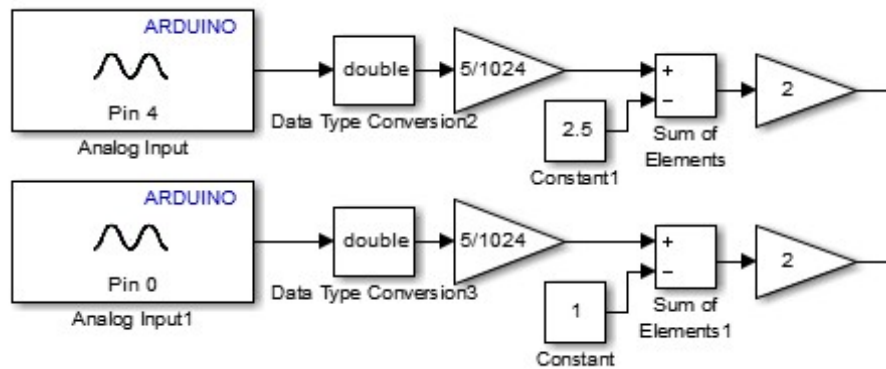


Figure 5.7: Adaptation of the analog signal.

Then, the magnitude and phase of the signals are calculated and the power calculation is carried out.

Finally, the calculated power values must be adapted so that they can be sent by the serial port. This communication based on simulink has the limitation of 8 bits per data (uint8), and our results are 16 bits (uint16). The solution proposed is that they will be sent as a vector composed of 2 data of 8 bits preceded and terminated by 2 limiting data without valuable information (Figure 5.10). The limiting data for each signal have been selected from the terminator character of Matlab. Otherwise, as stated above, the results of the power calculation will be sent less frequently due to the limitations that the serial port has on it.

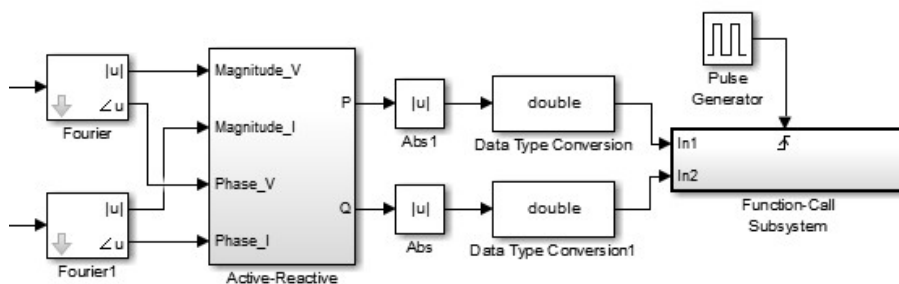


Figure 5.8: Adaptation of the power results.

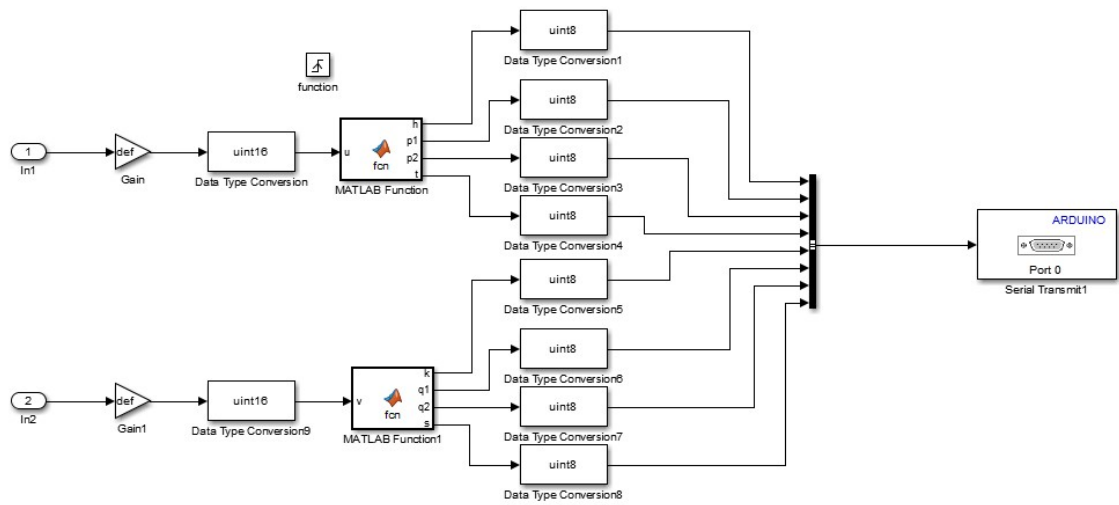


Figure 5.9: Adaptation of the power results for its transmission by the serial port.

```

function [h,p1,p2,t]= fcn(u)
    %#codegen
    ul6=uint16(u);
    x=typecast(ul6,'uint8');

    h = 150;
    p1 = x(1);
    p2 = x(2);
    t = 170;

function [k,q1,q2,s]= fcn(v)
    %#codegen
    vl6=uint16(v);
    y=typecast(vl6,'uint8');

    k = 50;
    q1 = y(1);
    q2 = y(2);
    s = 70;

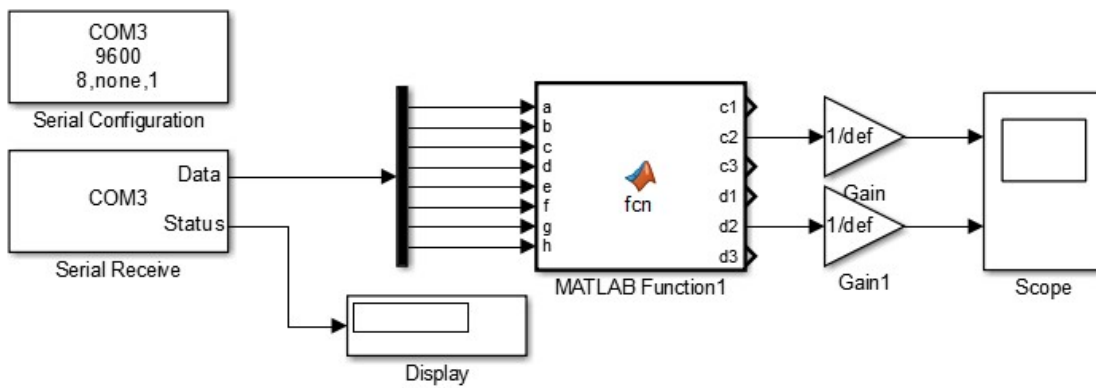
```

Figure 5.10: Matlab Function for data vector generation.

For this test, a sampling time  $T_s$  of  $5 * 10^{-4}$  has been chosen more than enough for the representation of the described signals and the delivery of calculated power is made every  $T_s * 10 = 5 * 10^{-3}$ .

All this adaptations that the divided data is being sent, must be taken into account in the reading of data so that it must be generated again as a single data uint16 to facilitate its understanding and representation (Figure 5.12).

While the script that contains the model of Figure 5.6, will be installed in External mode in the microcontroller so that it works autonomously, the one that contains the model of Figure 5.11 will run infinitely in Normal mode through the Simulink environment to receive the power values.



**Figure 5.11:** Block diagram for results reading.

```

function [c1, c2, c3,d1,d2,d3]=fcn(a, b, c, d, e, f, g, h)
%#codegen
    persistent C1
    if isempty(C1)
        C1 = 0;
    end
    persistent C2
    if isempty(C2)
        C2 = 0;
    end
    persistent C3
    if isempty(C3)
        C3 = 0;
    end

    persistent D1
    if isempty(D1)
        D1 = 0;
    end
    persistent D2
    if isempty(D2)
        D2 = 0;
    end
    persistent D3
    if isempty(D3)
        D3 = 0;
    end

    vett1=uint8([0 0]);
    vett2=uint8([0 0]);

    if a==150 && d==170
        c1=uint16(a);
        vett1(1)=uint8(b);
        vett1(2)=uint8(c);
        v1_16 = typecast(vett1,'uint16');
        c2= uint16(v1_16(1));
        c3 = uint16(d);
        C1=double(a);
        C2=double(c2);
        C3=double(d);
    else
        c1=uint16(C1);
        c2=uint16(C2);
        c3=uint16(C3);
    end

    if e==50 && h==70
        d1=uint16(e);
        vett2(1)=uint8(f);
        vett2(2)=uint8(g);
        v2_16 = typecast(vett2,'uint16');
        d2= uint16(v2_16(1));
        d3 = uint16(h);
        D1=double(e);

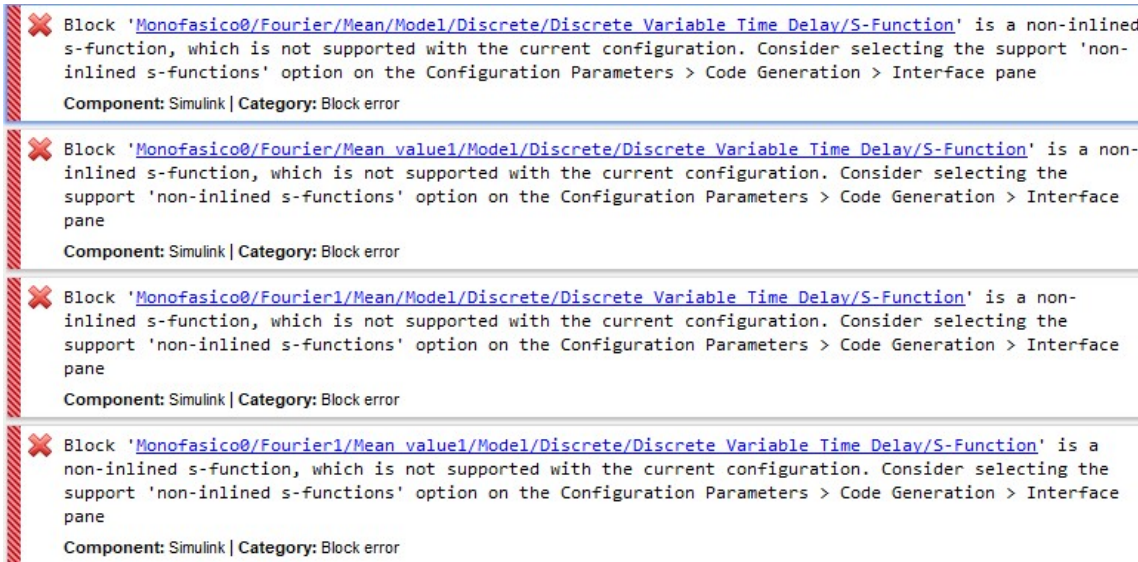
```

Figure 5.12: Matlab function for results reading adaptation.



Once all this theoretical adaptation has been carried out, the microcontroller is engraved with the code of the Figure 5.6, resulting in an unsolvable error as in Figure 5.13.

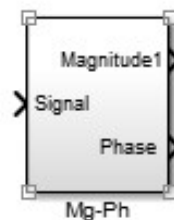
It deals with the incompatibility of the block for the magnitude and phase calculation , *Fourier*, of the library Simscape with the language accepted by the Arduino.



**Figure 5.13:** Error due to Fourier block.

Investigating the possible causes of the error, we conclude that it is due to the *S – function* `sfun_discreteVariableDelay` developed as Discrete Variable Time Delay at the *Mean* block. This function uses internal parameters of the library itself, which means that it is not valid to be used autonomically by the Arduino.

As a solution to this problem, the design of a new *Fourier* block based on other blocks not belonging to the library Simscape is proposed ( Figure 5.14).



**Figure 5.14:** New "Fourier" block.

The Fourier block performs a Fourier analysis of the input signal over a running window of one cycle of the fundamental frequency of the signal.

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} [a_n \cos(n\omega t) + b_n \sin(n\omega t)] \quad (5.1)$$



where  $n$  represents the rank of the harmonics. ( $n = 1$  corresponds to the fundamental component.) The magnitude and phase of the selected harmonic component are calculated by these equations:

$$|H_n| = \sqrt{a_n^2 + b_n^2} \tag{5.2}$$

$$\angle H_n = \arctan 2 \frac{a_n}{b_n} \tag{5.3}$$

and

$$a_n = \frac{2}{T} \int_{t-T}^t f(t) \cos(n\omega t) dt \tag{5.4}$$

$$b_n = \frac{2}{T} \int_{t-T}^t f(t) \sin(n\omega t) dt \tag{5.5}$$

$$T = \frac{1}{f_1} \tag{5.6}$$

$$f_1 = \text{FundamentalFrequency} \tag{5.7}$$

It is designed analogously to the existing one avoiding problematic blocks.

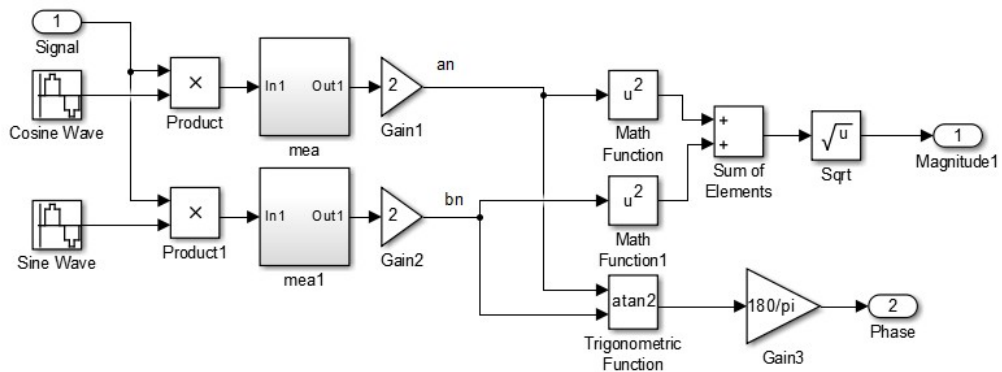


Figure 5.15: Block diagram of the new block for magnitude and phase measurement

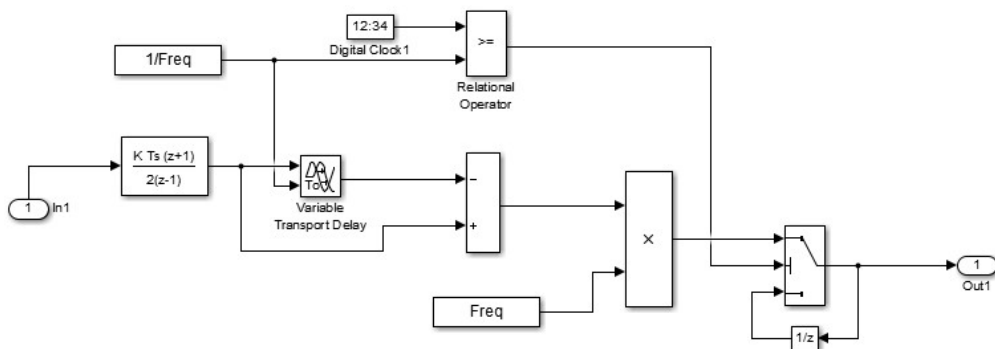


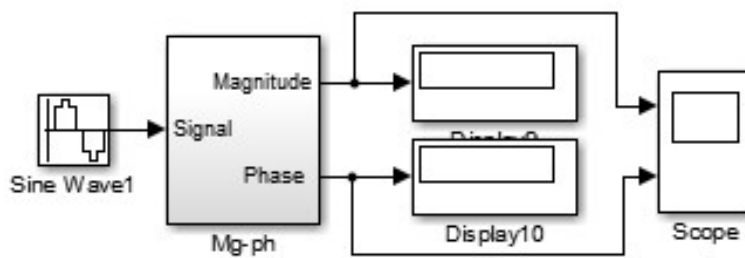
Figure 5.16: Block diagram of the new "Mean" block.

With this new design it is possible to solve the compilation problem and the Arduino admits the new block.

However, another problem arises due to the *Mean* block for the calculation of the coefficients  $a_n$  and  $b_n$  due to the variable delay.

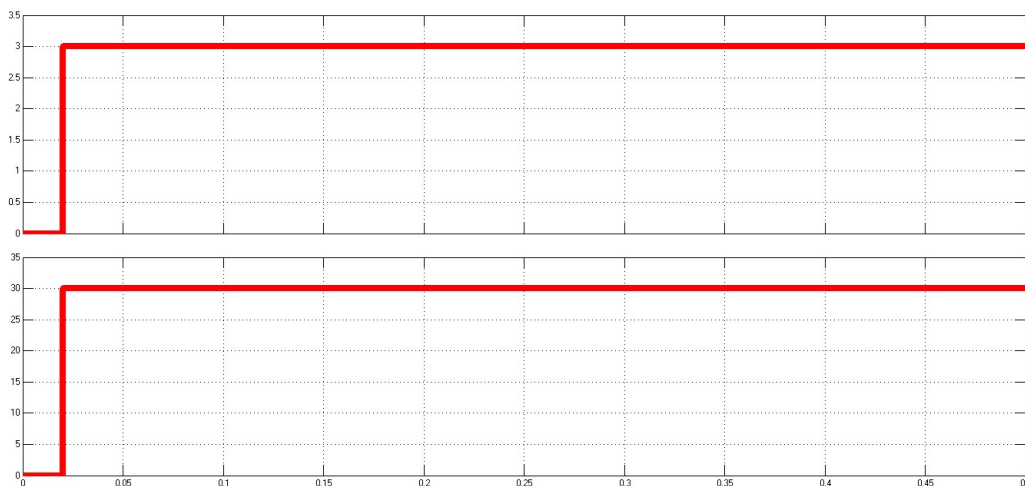
This new obstacle is not detected due to any compilation error, but by performing tests varying the frequency of the signals from which we want to analyze the magnitude and phase.

For the attempt to solve this error, a new simulation model 5.17 has been carried out in which it is only sought to find the magnitude and phase of different signals and on which the possible causes of the mismatch have been investigated.

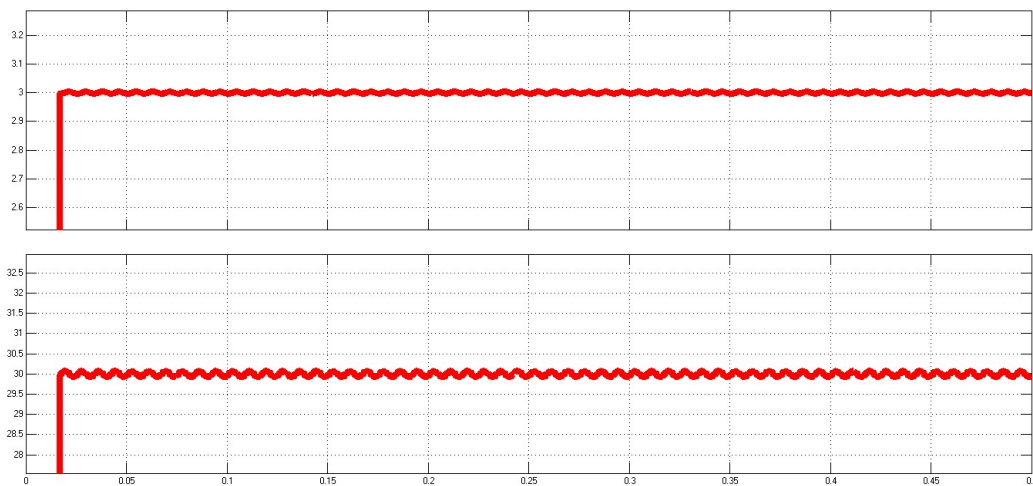


**Figure 5.17:** Block diagram for frequency variation tests.

After having carried out the necessary tests, it has been discovered that the error persists for those frequencies whose period is an irrational number. For example, a sinusoidal signal of 3 V of amplitude and  $30^\circ$  of phase shift is generated. In case the fundamental frequency is 50 Hz, its period is 0.2 and the operation of the block is correct (Figure 5.18). On the other hand, for 60 Hz, the period would be 0.0166666667 and the oscillations can be appreciated (Figure 5.19).



**Figure 5.18:** 50Hz signal magnitude and phase.



**Figure 5.19:** 60Hz signal magnitude and phase.

It has been concluded that these distortions are due to the internal decimal approximations of the Matlab tool and unfortunately, an acceptable solution for this error has not been found.

In addition, this error can not be assumed for the precise calculation of power that we tried to calculate, so it has not continued with the development of the project in this line.



# Conclusions

At this point and just before finalizing the project, it is necessary to assess whether the work has been satisfactory, analyzing whether it meets the objectives and guidelines that were originally set.

The main objective of the project, which consisted in the construction of a system for calculating the power of a three-phase system under unbalanced and non-sinusoidal conditions, has not been completed due to incompatibilities of the tools proposed for its development.

However, following the lines proposed for its improvement, other objectives have been achieved.

- An exhaustive study has been made of the available formulation for the the real power calculation of a system under distorted conditions.
- It has been executed a good adaptation of the interface, getting communication between Arduino and Simulink so it has been able to acquire and process signals.
- It have been known the compatibility limitations between two great tools such as Arduino and Matlab.

Despite all this, having failed to the main objective, it can not be considered that the project has been totally satisfactory.

## Future developments

As future lines of development, three cases are proposed:

- Pursue the main objective of power calculation under distorted conditions, using other programming software such as the IDE, typical of Arduino.
- Pursue this same objective using a more powerful microprocessor such as Raspberry pi.
- In depth research on the incompatibility to perform the measures described above.



# Bibliography

- [1] Arduino Mega 2560.
- [2] Armónicos en las redes eléctricas.
- [3] CAPITULO 10 : CIRCUITOS TRIFASICOS. pages 1–23.
- [4] Potencia instantánea en sistemas trifásicos de cuatro hilos.
- [5] Marcos A De Armas, Julio R Gómez, and Percy R Viego. Análisis de sistema de potencia desbalanceado con el empleo de herramientas estadísticas , coeficientes complejos y redes neuronales artificiales. XXIX(1):1–6, 2008.
- [6] En Base and Al Microcontrolador. Medidor De Factor De Potencia Para Ondas Distorsionadas. 1994.
- [7] S Canturk, M E Balci, and M H Hocaoglu. On the Definition of Apparent Power. XVIII(2):1–10, 2015.
- [8] Harvey L Curtis. Definitions of Power and Related Quantities. 1935.
- [9] R Diabi and R Hamdaoui. REACTIVE POWER DIFFICULTIES MEASUREMENT UNDER NON SINUSOIDAL CONDITIONS. pages 1–7.
- [10] A E Emanuel. APPROACH FOR NONSINUSOIDAL AND UNBALANCED SYSTEMS. 13(2), 1998.
- [11] Alexander Eigeles Emanuel. *POWER DEFINITIONS AND THE PHYSICAL MECHANISM OF POWER FLOW*. 2010.
- [12] Pedro Ángel Blasco Espinosa. DESEQUILIBRIO., FORMULACIÓN DE LA POTENCIA DE SINUSOIDALES, APLICACIÓN A REDES ELÉCTRICAS DESEQUILIBRADAS. 2015.
- [13] Magnago Fernando, Reineri Claudio, and Lovera Santiago. Power Quality Measurement Under Non-Sinusoidal Condition. (June 2014), 2011.
- [14] Gustavo L. Ferro. El método de las Componentes Simétricas. 2015.
- [15] C L Fortescue. Method of Symmetrical Co-Ordinates Applied to the Solution of Polyphase Networks. 1918.

- [16] Alberto G. Martínez. Metodo de las componentes simetricas. pages 1–35, 2014.
- [17] Stephen E Marx. Symmetrical components. 2012.
- [18] Andrés Pavas. Potencia en sistemas trifásicos y evaluación de la asimetría en condiciones sinusoidales. pages 1–12, 2011.
- [19] Jos Arrillaga University and Neville R. Watson. *Power system harmonics*. 2003.
- [20] Michel Veysiere. Energy Measurement Techniques for Non- Sinusoidal Situations. (January 2014), 2003.
- [21] IEEE Worksgroup. *IEEE Standard Definitions for the MEasurement of Electric Power Quantities Under Sinusoidal, Nonsinusoidal, Balanced or Unbalanced Conditions*. 2010.