


Article

Coordination between High School and University Teachers in Spain to Reduce Mistakes in Calculus

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Abstract: In level tests carried out in recent years to evaluate the competence acquired by calculus students enrolled in the Computer Software Engineering degree at the University of Oviedo, it has been observed that a significant percentage of students make very similar conceptual errors. This article describes the research undertaken by a working group of teachers called BACUNIMAT, currently made up of two university professors and five high school professors. The name BACUNIMAT is the acronym in Spanish for the High School and University Teachers Working Group. The aim of this research group was to analyze the main deficiencies in mathematical knowledge that students possess upon arrival at university. The analysis was performed in order to propose solutions to alleviate these deficiencies. The research proposes how to focus mathematical teaching in secondary schools in order to better prepare students for university.

Keywords: mathematical education; engineering education; mathematical errors; secondary school mathematics; undergraduate mathematics

1. Introduction

At the Conference of Rectors (CRUE), many of its members asked that communication with secondary education be “more intense and fluid” so that the “bridge” between baccalaureate and university studies “will be much safer and the jump less great” [1]. This work is in line with the strategic objective of the University of Oviedo, which demands “greater collaboration with secondary education” [2], since the participation of high school teachers has been fundamental in its development. Please note that, in Spain, the baccalaureate is formed of two academic years that students must pass before starting their university degrees.

CRUE, established in 1994, is a non-profit association formed by a Spanish both public and private universities. It is involved in the development of all the normative dimensions that affect higher education in Spain. It plays an important role in the promotion of different activities that foster relations with society and industry in Spain.

Another of the strategic objectives of the University of Oviedo is to “improve the linguistic competences of the students”. This problem has been tackled in the course of this work since, in the process of teaching and learning mathematics, the oral and written expression of the processes carried out and the reasoning followed acquire special importance, as they help to formalize thought. It is a fact that many students who enter university have serious difficulties [3] in expressing a simple definition with mathematical (and non-mathematical) language.

The problems linked to the transition from high school to university are well-known and common world-wide. In fact, nowadays, there is a vast amount of literature dealing with how support and orientation policies can improve the performance of university students [4,5]. In the case of mathematics, several authors have stressed the importance of the problem of adaptation to university [6,7]. It seems

clear for many authors, and we agree with them, that the transition from secondary [7] to tertiary mathematics studies involves many adjustments.

Previous work by various authors has shown that certain errors are committed by a high percentage of students, which has also been detected in this work. This circumstance seems to be widespread not only in Spain [8–11], but also in several European countries [12,13]. In the case of engineering students, several researchers [14–16], and also educational and professional institutions [17,18], have found prior mathematical training to be deficient.

There are studies concerning which factors are those that influence success in mathematics of first-year students [19] that have come to the conclusion that motivation is the most influential factor related to the level of success. Despite this, the studies in question highlighted the need to explore the potential mismatch between lectures and student expectations in mathematics subjects at university. Another study [20] examined the academic performance of students in a first-year undergraduate mathematics course at the University of the Sunshine Coast in Australia. They found that the most important variable for the prediction of the success of students in mathematics at university was the amount and level of mathematics courses taken at secondary school.

The reasons why students make errors in mathematics can be diverse, but when these errors are very similar among very different students, coming from different schools and without having had previous contact, the reason or reasons for these errors can be more limited. This work deals with the study of these errors.

In general, during high school years, the relationships between teachers and students are closer than at university. In the Spanish education system, lesson attendance is compulsory at high school but not at university, where, in general, only attendance of practical sessions is mandatory. This can be a problem for students, since they sometimes do not attend lessons due to the lack of compulsory attendance.

There have been reports concerning problems at Irish universities since the mid-1980s showing dissatisfaction with the mathematical ability of new students [21,22]. The authors presented a critical vision of secondary school mathematics in Ireland, remarking that the Second Level Education System is extremely exam-oriented, which results in an approach where pupils and teachers focus on learning those concepts more likely to be required to pass the exams.

From our point of view, the situation in Spain is very similar to the Irish case. This fact is well-known among the educational community and has led to the implementation of different improvement strategies. A study published in 2015 [23] analyzed the strategies that are implemented nowadays at 34 of the public universities in Spain. The most common activities engaged in are as follows: bridging training courses for new students, personalized follow-ups, and training courses for university teachers in order to have a better understanding of new students. In the case of bridging courses, they are sometimes tailored to the students' characteristics according to their age or previous studies.

The profile of most of the students who enter into the bachelor's degree in Computer Software Engineering of the University of Oviedo is that of a student who has completed the Baccalaureate in the modality of Sciences, with certain mastery of some systematic processes of mathematics, but with significant deficiencies in conceptual aspects of the discipline; in other words, a skilled sciences student, but with an average profile among science high school graduates in Spain.

In Spain, for university degrees, it is not as common as either at high school or in universities in other countries to have compulsory homework assignments for students. Additionally, from our point of view, it is remarkable that students in general rarely seek the help of the teacher to understand unclear concepts; instead, they ask their peers for help via social networks. This kind of behavior does not help to solve the main conceptual errors. Therefore, a closer relationship between teachers and students is required.

It is necessary to bear in mind that "mathematical learning does not consist of a process of incorporating data, rules, etc., into a blank mind, but implies a dialogue (implicit or explicit) between the student's previous knowledge and the new ones, which the teacher tries to teach him" [24]. Bearing

in mind that in order to learn mathematics there must be a dialogue between prior knowledge and new knowledge, the hypothesis that poorly grounded knowledge can lead to common errors among students gains strength.

What are the reasons for poor prior knowledge? Some of these reasons, from the point of view of the authors, could be the following:

- Textbooks that are not very rigorous or have conceptual errors: Analyzing textbooks from different publishers, we have observed a lack of rigor and, on occasion, errors in the conclusions reached, which can lead to conceptual errors in students [25];
- Systematization of methods can lead to carelessness in understanding what is done, why it is done, and for what purpose: “The affirmation to the student that there is an automatic method to establish a family of results, even if it is true, tends to relieve the student of the fundamental responsibility of controlling intellectual work, blocks the transmission of the problem, which often causes the activity to fail” [26];
- During the second year of the baccalaureate, the pressure of the compulsory Evaluation Test of the High School Students for University Access, called EBAU, the breadth of the subject matter, and the reduced teaching time can lead teachers to systematize methods to speed up the acquisition of basic knowledge that will allow students to pass the tests;
- Errors due to language difficulties [27]: Learning mathematical symbols and vocabulary is a problem similar to that of learning a foreign language for many students. A lack of semantic comprehension of mathematical texts is a source of error. During the secondary stage, the use of mathematical symbology is not uniform between schools or between teachers; and
- Errors due to poor learning of facts and skills: These types of errors include all deficiencies in content and specific procedures for performing a mathematical task [28].

The specific objective pursued with this research work has been to detect the possible conceptual errors in mathematics among students from high school and those who study the subject of calculus in the Computer Software Engineering degree at the University of Oviedo. In the authors’ opinion, these results can be extrapolated to all students of similar degrees who take mathematics-based subjects as a compulsory part of their degree, but are not pursuing a Bachelor degree in Mathematics. To the best of the authors’ knowledge, the situation in Spain is similar to other Western countries. We would like to remark that this is the first time that this kind of initiative has been performed by teachers of the University of Oviedo and high schools of the Principality of Asturias, which is a region located on the north coast of Spain.

Finally, it is also clear that the pre-eminent position of mathematics has been challenged, particularly in Western countries. Its position has been overtaken by computing and computer-based technology. In other words, nowadays, a significant proportion of those students that some decades ago used to study mathematics at higher levels in secondary school, now study other subjects due to competition for time in the secondary curricular with other technological subjects. However, the number of students studying mathematics as part of a degree is increasing [29], especially in the case of statistics and mathematical computing.

2. Materials and Methods

In addition to the consultation of high school textbooks in order to review the way in which the concepts that present the greatest difficulties for students are explained and the sharing of the experience acquired by the teachers participating in this work, both a level survey and a survey of students’ perception of their own level and of the difficulty of the subjects were applied to the participating students. These surveys were carried out by the teachers who are part of the BACUNIMAT working group. The name BACUNIMAT is the acronym in Spanish for the High School and University Teachers Working Group.

During the academic years 2017–2018, we encouraged our team of secondary school and university teachers to meet periodically to discuss mathematical concepts and how they are perceived by students. We were not only able to contrast impressions between teachers of both academic levels and with different degrees of experience and backgrounds, but the work group also analysed objective information collected in the tests proposed to the students.

Meetings took place once or twice a month, sometimes at the university and other times at different high schools in the towns of Gijón and Avilés (Spain). The team was chaired by Manuel José Fernández Gutiérrez, who has more than 35 years of experience teaching different engineering degrees and during the last 10 years, has also been teaching the Master’s degree in Teacher Training in Secondary and Upper-Secondary Education and Vocational Training. All the decisions in the group were made either by voting or by consensus.

The level survey (Table 1) consists of a series of multiple-choice questions relating to mathematical concepts that should be known by students who have completed the baccalaureate. It consists of eight questions with four possible answers, only one of which is correct. If the answer is correct, it is counted as 1 and if it is not, it is counted as 0. The right answers for the level survey in Table 1 are as follows: a for question 6; b for questions 2, 3, 4, and 5; and c for questions 1, 7, and 8.

Table 1. Level survey.

Site of origin:			
1) The number 0.090909090909 ...			
a) it's irrational	b) it's integer	c) it's rational	d) it's roman
2) $\log_2(1/4) =$			
a) 2	b) -2	c) 1/2	d) it doesn't exist
3) We have $f(x) = \sqrt{x}$ if $x \geq 0$, $g(x) = \sqrt{x}$ if $x > 0$, $l = \lim_{x \rightarrow 0} f(x)$ and $m = \lim_{x \rightarrow 0} g(x)$. Then:			
a) $l = 0$, it doesn't exist m	b) $l = m = 0$	c) it doesn't exist l and m	d) $m = 0$, it doesn't exist l
4) We have $f(x) = \frac{x^3}{x^2+1}$. Point out the false statement:			
a) f is an odd function	b) f is delimited in R	c) f is a rational function	d) f is continuous in R
5) The number of real square roots in the equation $x^3 + x - 5 = 0$ is			
a) 0	b) 1	c) 2	d) 3
6) We have $f(x) = x^2$ defined in the closed interval $[-2, 1]$. The absolute maximum of $f(x)$ is			
a) 4	b) -2	c) 1	d) there is no maximum
7) If f is continuous in $[a, b]$, the area of the flat region delimited by the curve $y = f(x)$, vertical straight lines $x = a$, $x = b$ and the abscissa axis, is given by			
a) $\int_a^b f(x)dx$	b) $\left \int_a^b f(x)dx \right $	c) $\int_a^b f(x) dx$	d) none of the above
8) The indefinite integral of $\frac{x}{1+x^2}$ is			
a) $2 \ln(1 + x^2) + C$	b) $\ln(1 + x^2) + C$	c) $(\ln(1 + x^2)/2) + C$	d) $\arctg(x) + C$
ln = Neperian logarithm.			

This level survey was answered by the students of calculus studying for the Software Engineering degree in the first sessions of the 2017–2018 course, without subsequently being informed of the correct answers. In addition, they responded again to the same survey during one of the evaluation tests of the subject. The present study only compares the results obtained by the 52 persons that resulted from eliminating the student body that did not respond on both occasions and the persons whose origin in that year was not from a secondary school, that is, repeaters or other faculties. The questions used are included in Table 1.

The purpose of the survey of students’ perception of their own level and of the subject (the questions of which are presented in Tables 2 and 3) is to ascertain their opinion of mathematics in the second year of the baccalaureate, their perception of the mathematical knowledge they have on completing the baccalaureate, and the difficulty of mathematics in the first year of university. Table 2 shows those questions that are linked to the perception that students have of their own level and of the subject. In Table 3, questions are about the same issue, but in this case students have to rate some

phrases that are proposed to them. This survey was answered by 300 students studying different degrees in engineering from the University of Oviedo who were taking the subject of calculus in the first four-month period of the academic year 2017–2018.

Table 2. Survey of student’s perception of their own level and of the subject. Part I.

1. Gender	<input type="checkbox"/> Male	<input type="checkbox"/> Female
2. You did your 2nd High School year in a	<input type="checkbox"/> Public	<input type="checkbox"/> Private/subsidized Center
3. I received support classes (“private class” of the subject of Mathematics II ...	<input type="checkbox"/> never	<input type="checkbox"/> rarely
	<input type="checkbox"/> occasionally	<input type="checkbox"/> frequently
		<input type="checkbox"/> very often
4. My final grade in Mathematics II (2°Bach) has been:		
5. The part of Mathematics II that has been the most difficult for me:		
6. With what grounding do you think you have come to the University to study the subject of Calculus?	<input type="checkbox"/> none	<input type="checkbox"/> little
	<input type="checkbox"/> average	<input type="checkbox"/> good
		<input type="checkbox"/> very good
7. The current requeriment in Calculus compared to that of Mathematics in the 2nd year of High School is	<input type="checkbox"/> much less	<input type="checkbox"/> less
	<input type="checkbox"/> similar	<input type="checkbox"/> greater
		<input type="checkbox"/> much greater
8. You thinks the rigor of the subject of Mathematics in the Baccaulaureate, has been ...	<input type="checkbox"/> nil	<input type="checkbox"/> scant
	<input type="checkbox"/> sufficient	<input type="checkbox"/> quite adequate
		<input type="checkbox"/> very adequate

Table 3. Survey of student’s perception of their own level and of the subject. Part II. Phrase rating.

9. The teacher of Mathematics II in 2nd year of Baccaulaureate should have been more rigorous in mathematical concepts”	<input type="checkbox"/> strongly disagree	<input type="checkbox"/> disagree	<input type="checkbox"/> I don’t agree or disagree	<input type="checkbox"/> agree	<input type="checkbox"/> totally agree
10. “Upon arrival at the University, the Calculus teacher believes that we have more initial mastery over the subject matter than we actually have”	<input type="checkbox"/> strongly disagree	<input type="checkbox"/> disagree	<input type="checkbox"/> I don’t agree or disagree	<input type="checkbox"/> agree	<input type="checkbox"/> totally agree
11. “A bridge course is necessary at the beginning of my University course to review and clarify some concepts that were studied in high school”	<input type="checkbox"/> strongly disagree	<input type="checkbox"/> disagree	<input type="checkbox"/> I don’t agree or disagree	<input type="checkbox"/> agree	<input type="checkbox"/> totally agree
12. “When I finished high school, I had the feeling that I was well prepared in mathematics and that I mastered the subject perfectly”	<input type="checkbox"/> strongly disagree	<input type="checkbox"/> disagree	<input type="checkbox"/> I don’t agree or disagree	<input type="checkbox"/> agree	<input type="checkbox"/> totally agree
13. In the Baccaulaureate the subject Mathematics II is explained too quickly so it does not allow time to go into the subject in depth and that is a burden for my university course ...	<input type="checkbox"/> strongly disagree	<input type="checkbox"/> disagree	<input type="checkbox"/> I don’t agree or disagree	<input type="checkbox"/> agree	<input type="checkbox"/> totally agree

3. Results

Of the 52 respondents of the level test, 13 (25%) were women and 39 (75%) were men. In addition, 36 (69%) came from public education and 16 (31%) came from a private or subsidized school. The average score obtained by students at the beginning of the 2017–2018 academic year (September 2017) was 3.23 points, with a standard deviation of 1.46 and a median of 3. When subjected to the same test in January 2018, this group of students obtained an average score of 4.62, with a standard deviation of 1.39 and a median of 4. Note that the maximum score that can be obtained in this test is eight points. The box diagram in Figure 1 shows the distribution of the scores obtained by the students on the two occasions they took the test. In this figure, it is observed that 50% of the results (interquartile range) are between two and four points in the first sitting of the test and between four and six points, in the second, with medians of three and four points, respectively.

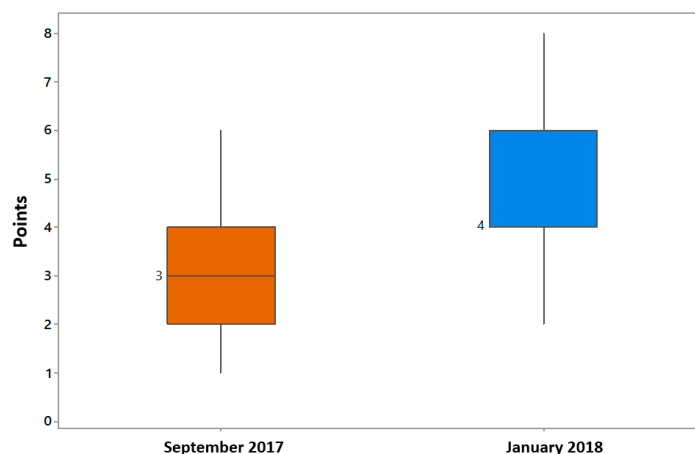


Figure 1. Boxplot diagram of student scores in September 2017 and January 2018.

When analysing the results obtained by the students both in the overall subject and in the tests carried out in September 2017 and January 2018 according to the centre of origin (public versus private or subsidized centres), no statistically significant differences were observed between groups, either in the test carried out in September or January or in the overall result obtained in the subject. On the other hand, if the results obtained in the two tests are analysed, as well as the final grade of the subject according to gender (Figure 2), despite the fact that no statistically significant differences were found between groups either in the test carried out in September or in January, a statistically significant difference can be observed in relation to the final grade of the subject, with women obtaining a better grade: median value of 6.3 and a mean value of 6.1 as opposed to a value of 4.1 for men for the median and 4.5 on average.

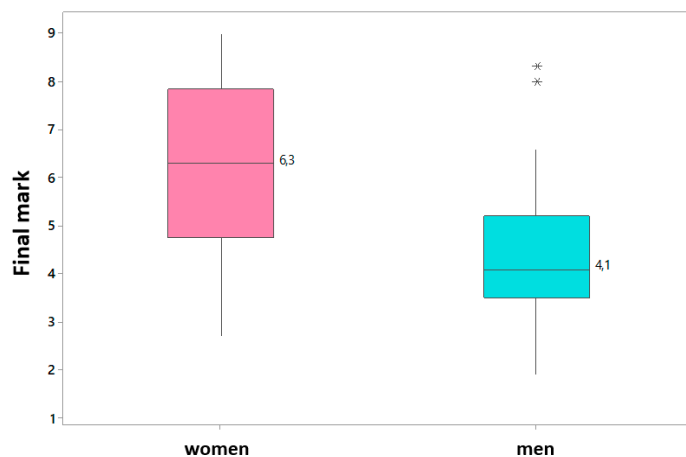


Figure 2. Boxplot of the scores obtained by students in the subject according to their gender.

Figure 3 shows the percentage of hits for each of the eight questions in September 2017 and January 2018. In the case of Question 1, which had a 90% success rate in September 2017, in January 2018, the percentage dropped to 85%. For the rest of the questions, the percentage of hits increased, with the largest increase corresponding to Question 7 (42%), followed by Question 8 (29%), and Question 2 (23%).

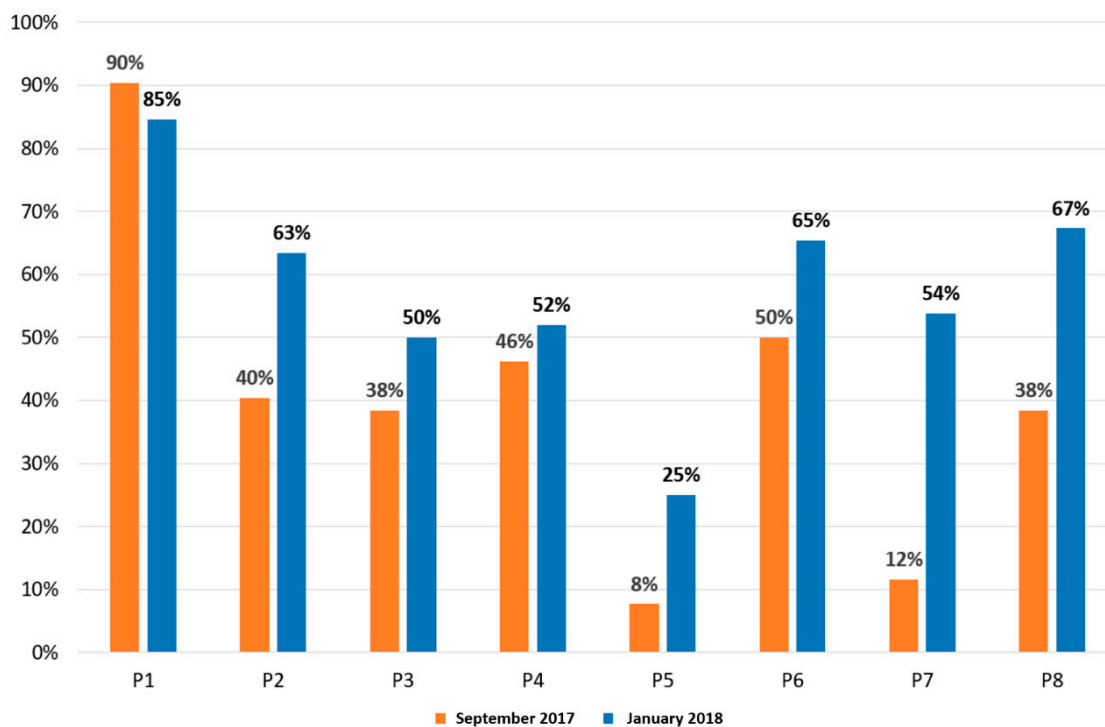


Figure 3. Percentage of hits on each of the questions in September 2017 and January 2018.

3.1. Results of the Level Test

In this section, we will present an analysis of the results obtained by the students in the level test. We would like to remark that as students are answering a test, one cannot infer what mistakes were made from multiple-choice questions. In order to save such limitations, the answers provided by students were analysed by the team, taking into account their teaching experiences.

The vast majority of students who took the test got Question 1 right. Regarding Question 2, the difficulty could be linked to how to express $\frac{1}{4}$ to the power of 2 and also to the fact that logarithms are mainly studied in the first year of high school, but not in the second year.

The difficulty of Question 3, which addresses the concept of the limit of a function at a point, could be related to the fact that a function is not required to be defined at a certain point in order to have a limit. In most secondary schools, limits are not taught in great detail. Question four is again a theoretical question, in which students are able to deduce the correct answer if they know the definitions of each of the concepts.

Question number five was the most difficult question for the students. In our experience, many students confuse real irrational roots with complex (not real) ones. From a theoretical point of view, and to provide a correct answer to this question, it would be particularly useful for the students to know Bolzano's theorem and a consequence of Rolle's theorem, but from a practical point of view, in order for them to give the correct answer, it is sufficient to study the derivative of the proposed function, since every polynomial equation of an odd degree has at least one real root. In spite of this and taking into account the experience of the secondary school teachers consulted, many students consider that this issue is merely an algebraic problem.

Regarding Question 6, students tend to confuse the absolute maximum of a function (if it exists) with the point in the domain where it is reached. In addition, it should be borne in mind that the functions proposed in the baccalaureate are not normally defined in a closed interval.

In Question 7, concerning the calculation of the area of a flat region, quite a few students propose that the area coincides with the absolute value of the integral value of the function. From our point of view, this mistake may be owing to two fundamental reasons: on the one hand, the lack of mastery of mathematical language and, on the other, the fact that the absolute value of a number is always

a quantity greater than or equal to zero. It is not difficult to graph functions that change sign in an interval $[a, b]$ in such a way that the absolute value of the defined integral does not coincide with the area enclosed by the graph of the function and the abscissa axis.

Finally, the proposed integral, in Question 8, is easily resolved if the students are familiar with the concept of a primitive function. They could even derive the chosen function and, thus, corroborate the result.

3.2. Results of the Perception Survey

The survey of students' perception of their own level and of the difficulty of the subject was completed by a total of 300 Calculus students of various engineering degrees. Thirty-three percent of the students surveyed (Figure 4) stated that they had received frequent or very frequent support classes throughout the academic year in order to pass the subject. In comparison with this data, 34.7% of the students stated that they had never had them.

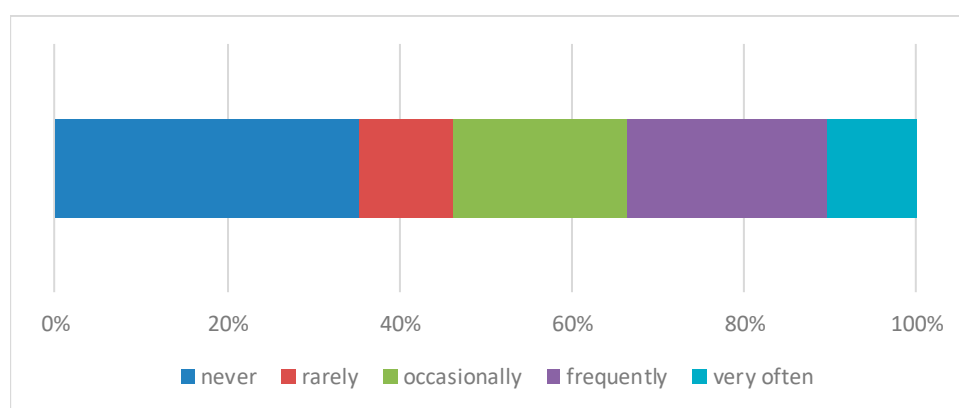


Figure 4. Reception of support classes for the subject.

When asked about which area of the mathematics subject in the second year of high school they considered to be more difficult (Figure 5), 26.7% of the students stated analysis, while for 22.7%, it was geometry. For 20.7%, all areas of the subject presented equal difficulty, while the area that was considered to be the most difficult by the lowest number of students was probability and statistics (10%).

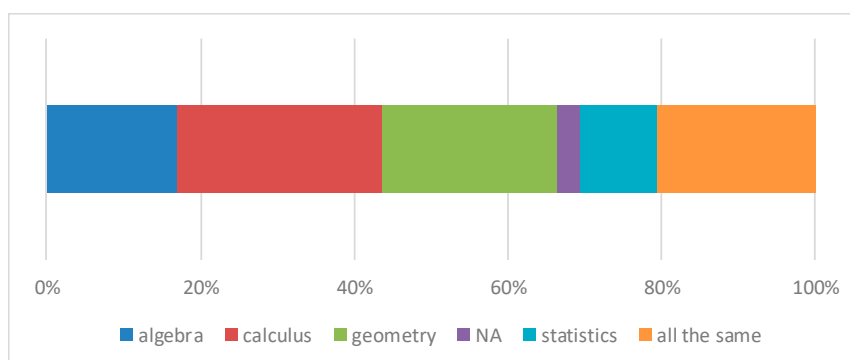


Figure 5. Areas of mathematics of the second year of the baccalaureate that are most difficult for students.

When asked about what mathematical basis they felt they had to approach mathematics at a university level (Figure 6), 45% of respondents felt that theirs was good or very good. In spite of this, when comparing the perceived requirement, 87.6% of the students stated that for mathematics subjects at university, it was either greater or much greater than that of the baccalaureate.

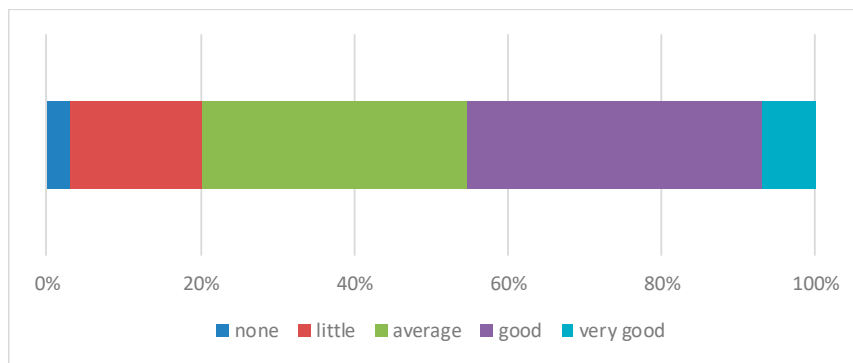


Figure 6. Perception of the prior mathematical basis for approaching university mathematics.

From the point of view of the majority of students surveyed (Figure 7), there is no lack of rigor in the way in which pre-university mathematics teaching is approached, given that only 15.7% considered it to be scant. In spite of this, 34.7% of those surveyed agreed or fully agreed with the statement that their high school mathematics teacher should have been more rigorous in their approach to the subject.

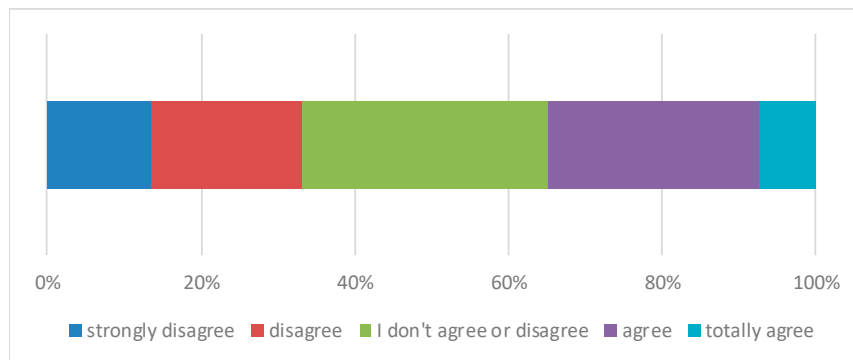


Figure 7. Rigor in the approach to pre-university mathematics.

There is also a widespread perception among students (67.7%) that their calculus teacher considers that they have a greater mastery of the subject than they actually have (Figure 8). In addition, just over half of the students (52.3%) agreed or fully agreed with the need for a bridging course before entering university (Figure 9).

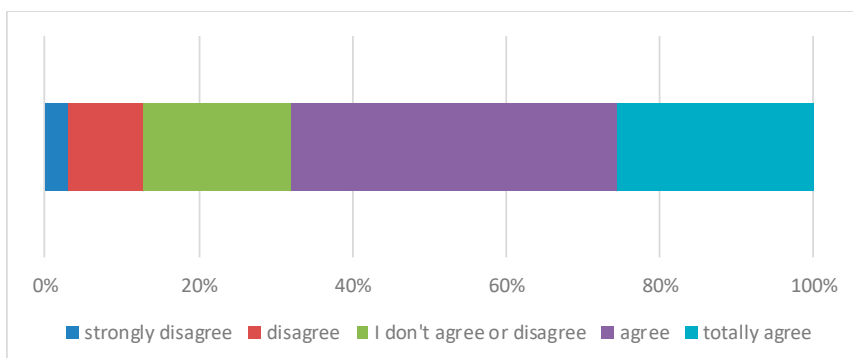


Figure 8. Degree of knowledge of mathematics that the calculus teacher assumes that the student has with respect to the reality.

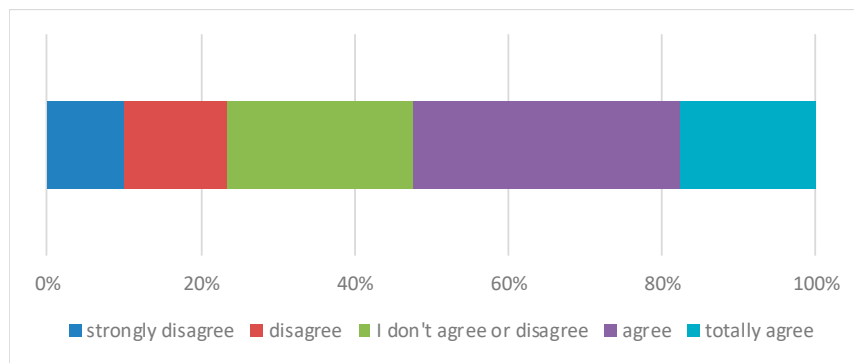


Figure 9. Need for a bridging course before entering university.

Furthermore, just over half of the students (51%) agreed or totally agreed (Figure 10) that upon completion of their baccalaureate studies, they considered themselves well-prepared in mathematics and believed that they mastered the subject completely. Finally, 54.3% of the students (Figure 11) agreed or completely agreed that the subject of mathematics in the second year of the baccalaureate was too broad for the time available.

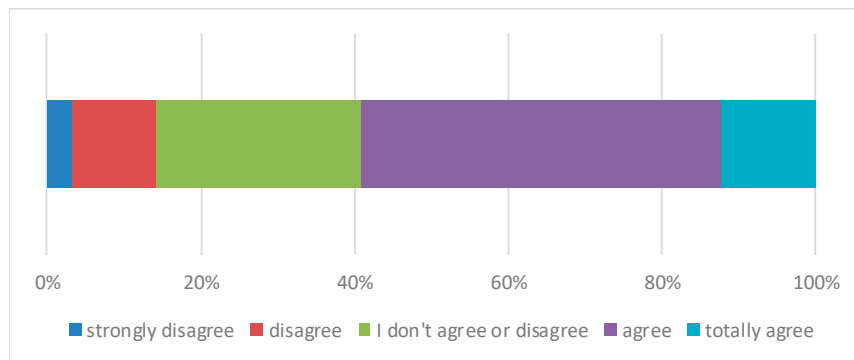


Figure 10. At the end of the second year of the baccalaureate, the student considers him/herself sufficiently prepared in the subject of mathematics.

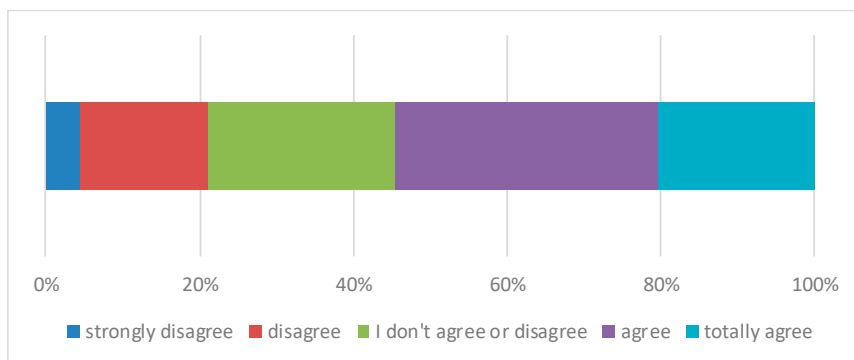


Figure 11. Responses to the statement that the subject matter of the second year of the baccalaureate is too broad.

4. Discussion

With regard to the reception of support classes in the second year of the baccalaureate, it was found that one-third of the students surveyed received them frequently or very frequently, while another third never received them. From our point of view, we do not believe that having support classes can be linked to success in the subject. In spite of this, we do not have information in this

respect, as there is no field in both surveys that relates the responses of individuals to the level test with the perception survey.

Additionally, in our opinion, the kind of support classes that students receive often lack quality, as they are given by non-professional teachers, or even colleagues. There is no easy solution to this problem beyond encouraging students to make use of academic tutoring. From our point of view, there is a research study that we consider to be of great interest.

The results of the survey show very similar values of the perceived difficulty in the four modules that the subject of Mathematics II has in the second year of the baccalaureate (analysis, algebra, geometry, and statistics), with the exception of statistics, which is only the most difficult module for 10% of the students.

The existence of an important difference in the depth and difficulty of the contents taught in the baccalaureate subjects with respect to first-year engineering subjects is clearly shown in view of the answers to the questions relating to the perception of the level of preparation, as well as the difficulty of the university subjects of mathematics. In other words, approximately half of the students surveyed considered themselves to be well-prepared and when they arrived at university, their perception was that mathematics subjects were much more difficult than they had previously thought.

The results obtained from our questionnaire are in line with those problems perceived in higher education in the United Kingdom, according to a report prepared in 1995 [30]. The referred report showed unprecedented concern about the mathematical preparedness of first-year undergraduates. Some of the most serious problems perceived by those in higher education were related to the lack of ability to perform algebraic calculation, the decrease in analytical abilities and the change of the perception of what mathematics are and the importance of proof and precision.

A third of the students surveyed consider that the mathematics teacher should have been more rigorous in the approach to the subject, probably because they find that they are assumed to have a greater mastery than they actually have, while slightly more than half of the students are in favour of a bridging course before starting university. This fact should give us food for thought. There is evidently a problem for students in moving from high school to university.

Regarding the implications of the results, the study performed led us to think that a more intensive coordination among teachers from both educational levels is required for the creation of the subject contents. Although it is true that there is coordination to prepare the entrance exam to the university (EBAU), it is no less true that in the mathematics subjects of the baccalaureate, most of the aspects of the curriculum that are directly related to the entrance exam are linked to the correct resolution of certain mechanical exercises. This means that, in general, we can find that students lack a theoretical mathematical background.

The requirement of better co-ordination is not a new idea. In a 1998 work [31], the authors put forward some ideas related to this issue:

- Establish a better dialogue between secondary and tertiary teachers, as is done in our working group;
- Provide students with orientation activities. These could start in secondary school and would help students to choose their university degrees;
- Provide students with individual help. A non-compulsory initiative similar to this has been implemented at our university; and
- Disseminate information about “success stories” that have helped first-year students to pass mathematics-based subjects.

However, the problem presented in the present research is quite complex and no easy solution can be found. From our point of view, there are certain steps that could be taken and would contribute to improve the current situation. Firstly, the mathematical contents proposed for the second year of the baccalaureate are too broad. One of our proposals consists of the reduction of the contents of statistics and probability. Secondly, we propose to reduce the load of long routine exercises with the help of

mathematical online platforms that would speed up operations and allow both teachers and students to focus on mathematical concepts. Please note that we are not thinking of introducing software such as OCTAVE [32] or MACSYMA [33]. We suggest the use of online platforms such as GEOGEBRA [34] or Wolfram Alpha [35], without a learning curve.

Finally, more than half of the students surveyed consider that the subject matter of Mathematics II in the second year of the Baccalaureate is too broad. If we add to this the fact that half of the students do not consider themselves to be sufficiently prepared by the end of the baccalaureate, it could be concluded that courses subsequent to the second year of the baccalaureate and prior to the first year of the baccalaureate were lacking. Taking into account what has been discussed thus far in this paper, it may be convenient to rethink the global content of mathematics in the two baccalaureate courses.

5. Conclusions

The first-year students come from the Baccalaureate centres with some lack of conceptual knowledge concerning mathematical analysis. This work, found that these insufficiencies are reduced after the first semester, when they take the subject of calculus in the Oviedo School of Computer Engineering. In addition, no significant differences were found based on the type of centre previously attended, although this did depend on the gender.

Although, given the anonymity of the responses to both surveys, it was not possible to relate the level of mathematical knowledge to the students' perception of their knowledge of the subject and due to the difficulty of this, we consider that the perception survey reaffirms our initial thesis. The subject matter of Mathematics II is too broad and this forces us to look for shortcuts that allow a wide variety of typical problems to be resolved, but at the cost of a certain amount of rigor. This makes the student focus on learning methods rather than on the study of mathematical concepts.

The results of the studies relayed in the introduction basically refer to procedural errors, whereas in this project, we referred fundamentally to errors of concept. What remains to be solved is to attract more students to actively participate in the subject matter of mathematics and, thus, make it possible to improve the percentage of passes, which is traditionally low in the subject of calculus.

From the authors' point of view, the experience of the present work has been very positive in everything related to teacher coordination, since it has propitiated that teachers of secondary and university education meet periodically to talk about mathematical concepts and how these are perceived by the students.

Furthermore, this work can be considered as the basis to continue with this collaboration through the development of teaching material that tries to clarify the concepts analysed. It should also be pointed out that, within the framework of this project, it has not only been possible to contrast impressions between teachers of both academic levels and with different degrees of experience, but has also been possible for the working group to analyse objective information gathered in the tests and questionnaires that were proposed to different groups of students and that were processed with the help of different statistical methodologies.

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