Broadband Shaped-Beam Reflectarray Optimization Using Support Vector Machines

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***Abstract*—In this work, SVMs are employed to accelerate the optimization of a 1-meter contoured-beam reflectarray antenna for direct broadcast satellite application in a 15% bandwidth in dual-linear polarization. A method of moments based on local periodicity is used to obtain samples to train the SVMs for each frequency. The surrogate model is then used for a design at central frequency, that is later used as starting point for a broadband optimization procedure that is accelerated more than an order of magnitude without a significant loss of accuracy. The minimum copolar gain in the coverage zone is improved more than 10 dB at the upper frequency while maintaining a computationally efficient design procedure.**

***Index Terms*—Support Vector Machine (SVM), broadband reflectarray antenna, contoured-beam, Direct Broadcast Satellite, Intersection Approach**

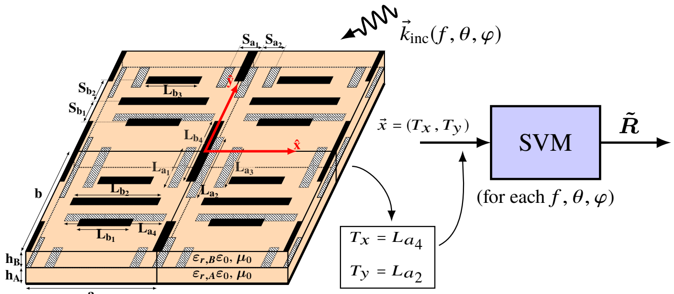

Fig. 1. Diagram of the reflectarray element based on two sets of parallel dipoles and its relation with the SVM training of the $\boldsymbol{R}$ matrix.

I. Introduction

The main drawback of printed reflectarrays is their inherent narrow bandwidth, that is primarily attributed to two factors: the poor bandwidth of narrowband resonant elements, which is usually around 3%-5% and the differential spatial phase delay [1], [2]. The first problem may be solved by employing wide-band printed elements which introduce several resonances [3], [4]. Also, the use of sub-wavelength elements may improve the bandwidth [5], although at the expense of reducing the phase-shift range, limiting the design of shaped-beam reflectarrays. The second factor may be overcome by adjusting the geometry of the unit cell at several frequencies [3], [6], using true time delay reflectarray elements [7], increasing the f/D ratio [2] or using curved [8] or faceted [9] reflectarrays.

Machine learning techniques such as Artificial Neural Networks have been employed for the analysis [10] and design [11] of reflectarrays. Support Vector Machines (SVMs) have also been used to accelerate the analysis of reflectarrays [12], achieving an acceleration factor greater than three orders of magnitude. SVMs have also been used for direct optimization of reflectarrays [13], but only at a single frequency, still obtaining a narrowband antenna.

In this work, we use a surrogate model of the reflectarray element based on SVMs to perform a broadband direct optimization or a shaped-beam reflectarray for Direct Broadcast Satellite (DBS) application with a European coverage. A Method of Moments based on Local Periodicity (MoM-LP) is employed to generate samples of the electromagnetic behaviour of the unit cell to train the SVM, as well as to validate the final solution. The optimal SVM Gaussian kernel (γ) and soft margin (C) are found with an efficient grid search that greatly accelerates the training process. The obtained surrogate model is compared with simulations from MoM-LP, showing a high degree of accuracy. Then, the generalized Intersection Approach (IA) is employed to perform a broadband direct optimization of the reflectarray antenna using the SVMs. The optimized layout practically fulfils requirements in a 15% bandwidth, demonstrating the capabilities of the proposed technique.

II. SVM Model of the Reflectarray Unit Cell

For each frequency, the feed generates an incident field ($\vec{E}_{\text{inc}}(f)$) on the reflectarray surface. Then, the reflected tangential field ($\vec{E}_{\text{ref}}(f)$) is related to the incident field through the matrix of reflection coefficients for a given unit cell:

$$\boldsymbol{R}(f) = \begin{pmatrix} \rho_{xx}(f) & \rho_{xy}(f) \\ \rho_{yx}(f) & \rho_{yy}(f) \end{pmatrix}. \qquad (1)$$

This matrix is computed using a full-wave analysis tool, in the present case the MoM-LP described in [14], which analyses the unit cell shown in Fig. 1. ρ_{xx} and ρ_{yy} are known as the direct coefficients and mainly control the copolar pattern for each linear polarization (X and Y, respectively). ρ_{xy} and ρ_{yx} are the cross-coefficients and mainly contribute to the cross-polar pattern. Thus, when performing copolar only synthesis, assuming $\rho_{xy} = \rho_{yx} = 0$ is a good approximation [15]. Once the reflected tangential field has been obtained, the far field is easily computed [1].

The reflectarray element is shown in Fig. 1. It is made up of two sets of four parallel dipoles. Each set controls the phase-shift for a linear polarization by tuning the dipole lengths. Thus, the width of the dipoles will be fixed to 0.4 mm and the separation between dipoles to 4 mm. In addition, the substrate has $h_A = 2.363$ mm, $h_B = 1.524$ mm, $\varepsilon_{r,A} = 2.55 - j0.0023$ and $\varepsilon_{r,A} = 2.17 - j0.0020$.

The goal of the SVM is to obtain surrogate models of the complex reflection coefficients of (1) for each reflectarray cell and each frequency. Following the approach described in [12], we consider two input variables for each SVM, T_x and T_y, that are related to the dipole lengths as follows:

$$L_{a_4} = T_x; \quad L_{b_1} = L_{b_3} = 0.63T_x; \quad L_{b_2} = 0.93T_x$$
$$L_{b_4} = 0.95T_y; \quad L_{a_1} = L_{a_3} = 0.58T_y; \quad L_{a_2} = T_y. \qquad (2)$$

T_x and T_y allow to control the phase-shift for linear polarizations X and Y, respectively.

We model separately the real and imaginary parts of the direct reflection coefficients to extract their phase, as well as their magnitude. In this way, the surrogate models are more accurate [12]. Thus we model $N_c = 6$ real-valued functions per unit cell. In addition, we consider a discrete set of $N_a = 52$ angles of incidence to the reflectarray cells formed by combinations of:

$$\begin{aligned} \theta &= \quad [5, 15, 25, 30]^\circ, \\ \varphi &= \pm[10, 30, 50, 70, 90, 110, 130, 150, 170]^\circ, \end{aligned} \qquad (3)$$

which are reduced to $N_a = 26$ using symmetries and modelled independently. Finally, we also model separately each considered frequency (10.95 GHz, 11.40 GHz, 11.85 GHz, 12.30 GHz and 12.75 GHz), which yields $N_f = 5$. Thus, a total of $N_c \times N_a \times N_f = 780$ surrogate models are generated.

Obtaining each surrogate model involves the training of a SVM. By following the guidelines presented in [12], a mean training time of 38 seconds is achieved in an Intel Core i7-5600U CPU at 2.6 GHz, having a mean error of -39.4 dB for all surrogate models, which ensures a high degree of accuracy. Fig. 2 shows the phase and magnitude of ρ_{xx} for oblique incidence ($\theta = 30^\circ$, $\varphi = 50^\circ$) and the five frequencies. As it can be seen, the SVM provides accurate results with regard to the MoM-LP tool. The mean absolute deviation for all the phase-shift curves shown in Fig. 2 is 2.25°, while for the magnitude is -56.8 dB. Similar results were obtained for other coefficients and angles of incidence.

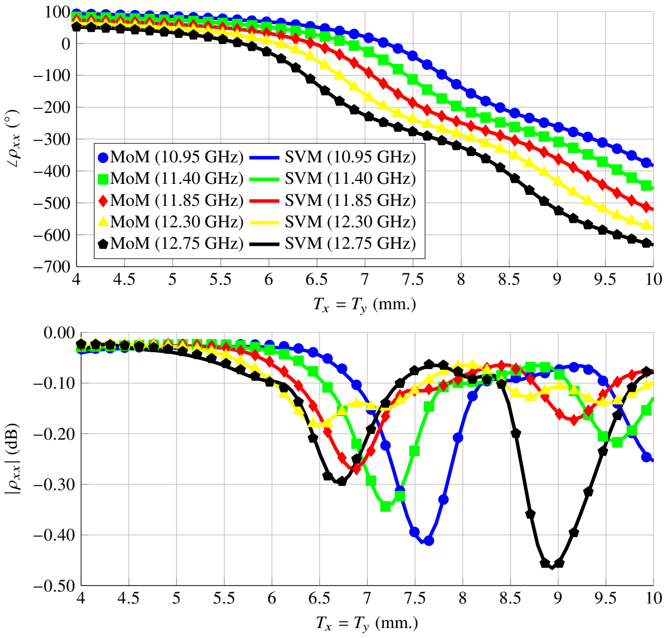

Fig. 2. For the direct reflection coefficient ρ_{xx} with an oblique angle of incidence $(\theta, \varphi) = (30^\circ, 50^\circ)$, comparison at five different frequencies between MoM-LP simulations and the SVM surrogate model for phases (top) and magnitudes (bottom).

III. Broadband Reflectarray Optimization

A. Central Frequency Design

A rectangular printed reflectarray in single-offset configuration is considered [6]. The reflectarray is comprised of 74×70 elements (5180 in total), with a periodicity of 14 mm in both dimensions. The feed is modelled as a $\cos^q \theta$ function, selecting the q such that the feed generates an illumination taper of -14.8 dB, -17.0 dB, -18.5 dB, -22.3 dB, and -25.3 dB at 10.95 GHz, 11.40 GHz, 11.85 GHz, 12.30 GHz and 12.75 GHz, respectively. In addition, the feed phase center is placed at (-358, 0, 1070) mm with regard to the center of the reflectarray.

The same European coverage as in [6] is considered, corresponding to a satellite in geostationary orbit at 10° E longitude. The goal is to achieve a minimum copolar gain of 28 dBi in a 15% frequency band (10.95–12.75 GHz) .

First, the generalized Intersection Approach (IA) at central frequency is applied for a phase-only synthesis. The required phase-shift for both linear polarizations is obtained such that the radiated far field fulfils the requirements. Then, by using a zero-finding routine, the values of T_x and T_y are sought for each reflectarray element such that they match the required phase-shift at central frequency. Fig. 3(b) shows the radiation pattern for Y polarization at central frequency (11.85 GHz). It fully complies with the specifications at that frequency. However, as it can be seen in Figs. 3(a) and 3(c), it is far from complying at extreme frequencies (10.95 GHz and 12.75 GHz).

B. Broadband Optimization

For the broadband optimization, the generalized IA is also employed, but this time using the SVM models to account for the frequency behaviour of the reflectarray element. In addition, the optimization variables are T_x and T_y, thus having a total of 10360 degrees of freedom. The design at the central frequency is used as starting point for this optimization, which is done in several steps, increasing progressively the number of optimizing variables to improve convergence [16]. This is done by selecting reflectarray elements from the center outwards in concentric circles. In the last step, all elements are optimized at the same time. Fig. 3 shows the initial and optimized radiation patterns for Y polarization at central (11.85 GHz) and extreme frequencies (10.95 GHz and 12.75 GHz). The layout was simulated with both MoM-LP using the real angle of incidence at each reflectarray element

Table I
MINIMUM COPOLAR GAIN FOR BOTH LINEAR POLARIZATIONS (X AND Y) AT THE FIVE FREQUENCIES OF INTEREST COMPARING SIMULATIONS WITH THE MOM-LP TOOL (USING THE REAL ANGLES OF INCIDENCE AT EACH REFLECTARRAY ELEMENT AND THE SAME ANGLES USED BY THE SVM) AND SVM.

	Tool	**10.95 GHz**		**11.40 GHz**		**11.85 GHz**		**12.30 GHz**		**12.75 GHz**	
		X	Y	X	Y	X	Y	X	Y	X	Y
Initial (in dBi)	**MoM-LP (real ang. inc.)**	25.99	25.94	28.79	28.54	30.11	30.06	26.03	28.21	15.15	23.69
	MoM-LP (SVM ang. inc.)	26.10	25.93	28.75	28.53	29.88	30.10	26.38	28.24	19.82	23.91
	SVM	26.08	25.96	28.75	28.56	29.90	30.10	26.38	28.22	19.93	23.85
Optimized (in dBi)	**MoM-LP (real ang. inc.)**	27.75	27.84	28.39	28.67	28.33	28.81	28.64	29.08	26.75	28.11
	MoM-LP (SVM ang. inc.)	27.72	27.81	28.32	28.65	28.02	28.87	28.44	29.13	27.41	28.19
	SVM	27.69	27.84	28.31	28.69	28.01	28.88	28.44	29.14	27.46	28.16

(solid lines) and SVM with the discretized angles (dashed lines). As it can be seen, the SVM predicts the radiation pattern with a high degree of accuracy, as it was expected from the results of the reflection coefficients shown in Section II. For these three frequencies, the minimum copolar gain is 27.84 dBi, 28.81 dBi and 28.11 dBi at 10.95 GHz, 11.85 GHz and 12.75 GHz, respectively. Although at 10.95 GHz it does not achieve a minimum copolar gain of 28 dBi, it fulfils specifications in 90.3% of the coverage area.

Table I summarizes the results for both linear polarizations at the five frequencies for the initial and optimized layouts. The Table includes simulations of the layouts with both MoM-LP and SVM to assess the accuracy of the surrogate models. In addition, the MoM-LP simulations were performed for two different cases for the angles of incidence: the real angles at each reflectarray element and the discretized angles of incidence employed by the SVM (which are given in (3)). As it is shown, the minimum gain predicted by the SVM is close to the one computed using MoM-LP, and more similar results are obtained when both tools employ the same angles of incidence. This is consistent since the error of the surrogate models is very low, as shown in Section II, and it translates to a good prediction of the radiation patterns.

Regarding the optimized layout, it completely fulfils specifications at 11.40 GHz, 11.85 GHz and 12.30 GHz, and also at 12.75 GHz for polarization Y. At 10.75 GHz it is close to fulfil the 28 dBi requirement. It is noteworthy to remark the improvement at 12.75 GHz, since the minimum copolar gain has improved more than 10 dB and 4 dB for polarization X and Y, respectively. In the case of 10.95 GHz, the improvement is better than 1.5 dB for both polarizations. It has been checked that the reflectarray fulfils the 28 dBi in the range 11.05 GHz–12.50 GHz in dual-linear polarization, which corresponds to a 12.2% bandwidth. This has been achieved by only employing one degree of freedom per cell and polarization and it is expected to improve if more degrees of freedom are employed.

Finally, the broadband optimization was carried out in an Intel Xeon E5-2630 v4 CPU at 2.2 GHz. While using the MoM-LP tool each iteration took a mean time of 735.1 s (more than 12 min), using the SVM it was reduced to 34.96 s per iteration: a speed-up larger than one order of magnitude (acceleration factor of 21). Taking into account that the optimization took close to 500 iterations, the total time savings were more than 90 h (from 102 h using MoM-LP to 4.8 h using SVM), while keeping a high degree of accuracy by using SVM models of the reflectarray unit cell.

IV. CONCLUSIONS

In this work, Support Vector Machines (SVMs) have been employed to perform a broadband optimization of a 1-meter shaped-beam reflectarray for Direct Broadcast Satellite (DBS) application. The SVMs are employed to obtain surrogate models of the electromagnetic behaviour of the reflectarray element, in particular to predict the values of the reflection coefficients at different frequencies and angles of incidence. The SVM training takes a mean time of 38 s per surrogate model, while obtaining a high degree of accuracy when compared to MoM-LP simulations. A layout obtained at central frequency is used as starting point for a broadband optimization in the band 10.95 GHz–12.75 GHz considering five equispaced frequencies. After the optimization, the reflectarray completely fulfils the 28 dBi requirements for the European coverage in a 12.2% bandwidth, and it is close to fulfil requirements in a 15% bandwidth. This has been achieved using only one degree of freedom per unit cell and polarization. The improvement in minimum copolar gain at 12.75 GHz is more than 10 dB and 4 dB for polarizations X and Y, respectively. Finally, the optimization algorithm is sped up more than one order of magnitude using SVMs instead of MoM-LP for the computations, saving a considerable amount of time, more than 3.5 days in the present case, while obtaining a high degree of accuracy when compared to MoM-LP simulations.

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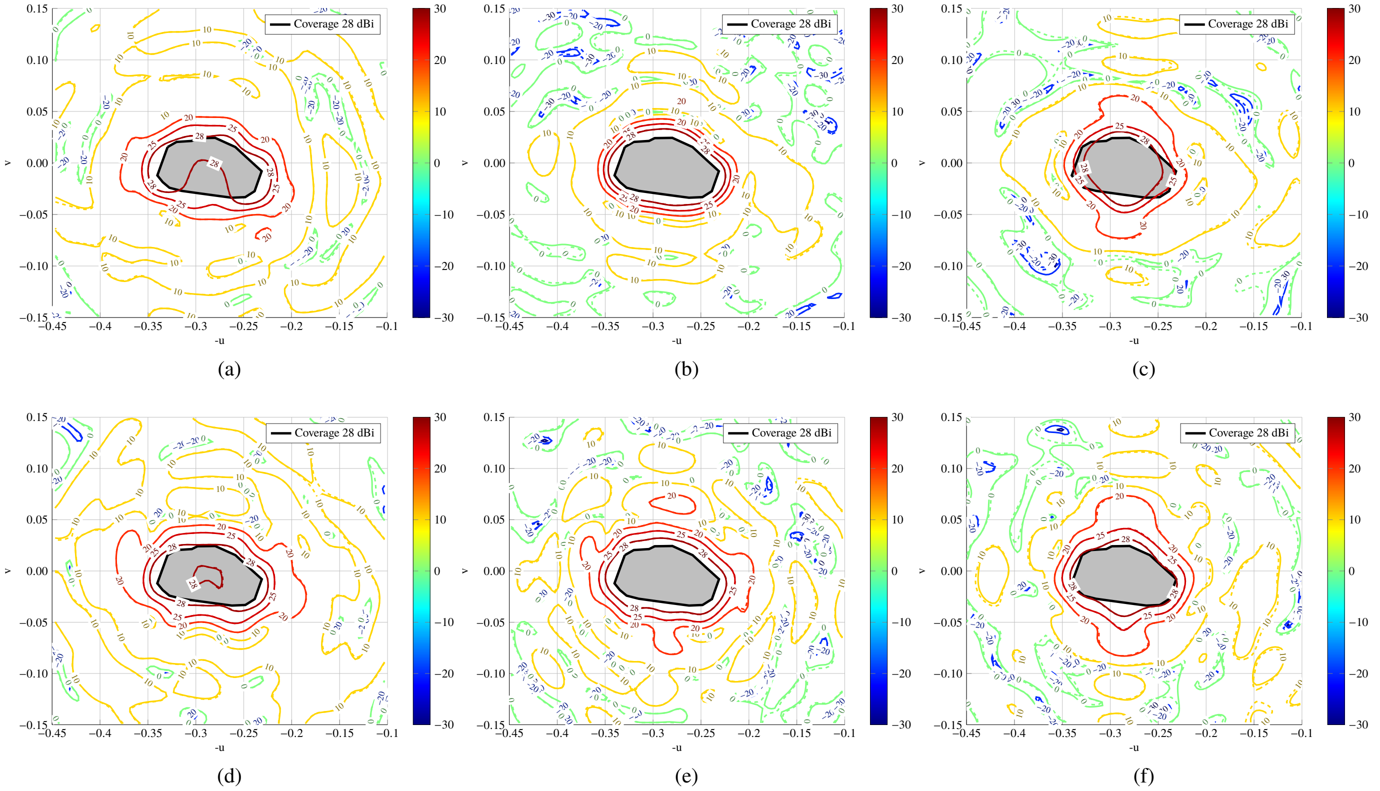

Fig. 3. Initial (top) and optimized (bottom) radiation patterns for Y polarization at (a), (d) 10.95 GHz; (b), (e) 11.85 GHz; and (c), (f) 12.75 GHz simulated with MoM-LP (solid lines) and SVM (dashed lines).

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