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An agent-based simulator for quantifying the cost of uncertainty in production systems

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ABSTRACT

Product-mix problems, where a range of products that generate different incomes compete for a limited set of production resources, are key to the success of many organisations. In their deterministic forms, these are simple optimisation problems; however, the consideration of stochasticity may turn them into analytically and/or computationally intractable problems. Thus, simulation becomes a powerful approach for providing efficient solutions to real-world product-mix problems. In this paper, we develop a simulator for exploring the cost of uncertainty in these production systems using Petri nets and agent-based techniques. Specifically, we implement a stochastic version of Goldratt's PQ problem that incorporates uncertainty in the volume and mix of customer demand. Through statistics, we derive regression models that link the net profit to the level of variability in the volume and mix. While the net profit decreases as uncertainty grows, we find that the system is able to effectively accommodate a certain level of variability when using a Drum-Buffer-Rope mechanism. In this regard, we reveal that the system is more robust to mix than to volume uncertainty. Later, we analyse the cost-benefit trade-off of uncertainty reduction, which has important implications for professionals. This analysis may help them optimise the profitability of investments. In this regard, we observe that mitigating volume uncertainty should be given higher consideration when the costs of reducing variability are low, while the efforts are best concentrated on alleviating mix uncertainty under high costs.

1. Introduction

Manufacturing flexibility, which may be understood as "the ability of the firm to manage production resources and uncertainty to meet customer requests" ([1], p. 174), has become strategically crucial for enhancing the competitive position of companies in most industries nowadays. One of its critical dimensions is *product-mix flexibility*, which captures the capability of the manufacturing system to produce a number of product lines and/or several variations within a line [2]. This property enables organisations to quickly adapt

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to the fast-changing needs of customers and exploit emerging opportunities in the current, intensively competitive business scene [3–5].

Companies with such flexibility need to address a key decision in their resource-constrained production systems: how much to manufacture each product line. This is often named the *product-mix problem* and represents a classical question in the operations area (e.g. [6–12]). Because resources are costly and products generate different incomes, product-mix decisions have a major impact on firms' financial results and strongly contribute to achieving organisational goals. For example, Morgan and Daniels [13] and Wang et al. [14] discussed the relevance of this problem for companies in the automotive and electronics sectors, respectively.

1.1. Background: deterministic and stochastic product-mix problems

In their most basic form, product-mix problems can be modelled as relatively simple optimisation problems that can be solved through exact methods, such as linear programming (e.g. [15]). These are generally small-scale, static problems with a relatively small number of products and only one bottleneck, assuming that the system operates under deterministic conditions. As the number of products and constraints grows, the complexity increases significantly. Product-mix problems then become mid- or large-scale problems that may be solved via heuristic or metaheuristic methods [16,17]. For example, Fredendall and Lea [18] employed a constructive heuristic for achieving near-optimal solutions in multiple constraint problems, while Onwubolu and Mutingi [19] developed a metaheuristic genetic algorithm. Both solutions were built using principles of the Theory of Constraints, which poses an interesting perspective to analyse product-mix problems based on taking decisions according to the bottleneck of the production system[1] [20].

Most research efforts in this discipline leave uncertainties out of the scope of the analysis, as highlighted by several authors, such as Hasuike and Ishii [21], Linhares [22], Feşel [23], and Hilmola and Gupta [24]. The reason behind the deterministic assumption is that such uncertainties enormously increase the analytical and computational complexity of product-mix problems. However, ignoring uncertainties may decrease the practical utility of the proposed solutions in modern business environments, which are often characterised by multiple sources of uncertainties (e.g. [25]). In this regard, Linhares [[22], p. 128] claimed that "there is still a clear need for further research of cases [product-mix decisions] with imperfect and non-deterministic information". Next, we briefly review those works that investigated product-mix decisions in uncertain, dynamic environments.

The paper written by Hodges and Moore [9] is often cited as one of the seminal works in this area. They considered demand uncertainty in a product-mix problem. Given that linear programming becomes insufficient, they develop a multi-item newsvendor optimisation problem with additional constraints affecting the decision variables. Kasilingam [26] also explored product-mix decisions with uncertain demand by modelling the problem as an objective function together with a set of constraints. Specifically, this study followed a non-linear integer programming approach with a two-phase solution procedure. First, the problem is reformulated, and second, the problem is relaxed through the Lagrangian method. The works by Hasuike and Ishii [8,21] also deserve mention, as they considered product-mix problems in which the returns, defining the individual profit generated by each product, are stochastic. To this end, they transformed the original problem with uncertainty into a deterministic equivalent model through stochastic optimisation techniques (e.g. see [27]).

Modelling and exploring stochastic product-mix problems by means of optimisation methodologies undoubtedly provides meaningful insights and industrially relevant solutions. However, at the same time, this approach oversimplifies the problem from some important angles, as it uses an input-output representation that ignores the inner workings of the production system. Importantly, optimisation-based studies do not account for the intrinsic complexity of managing the production system when uncertainty is present. In this regard, it is convenient to underline that uncertainties often trigger several inefficiencies in the flow of materials that have a significant impact on the economic performance of the system. For example, we may refer to the need for internal safety stock buffers when customer requirements are variable over time and the lead times are not negligible. In the light of this, Mabin and Davies [[28], p. 678] concluded that, in product-mix problems, the "inevitable ambiguities and complexities that arise in the real world" make that the "search for a 'best fit' frame should be recognized and abandoned".

This perspective highlights the value of simulation-based approaches to explore product-mix problems. These techniques allow scholars and practitioners to better perceive and control the dynamics emerging in the flow of materials in production and distribution systems; see the discussions in Holweg and Bicheno [29], Kleijnen [30], and Ponte et al. [31]. In addition, this approach would facilitate the development of 'what-if' analyses to evaluate the performance of the strategies provided by optimisation studies and compare them against other alternatives in uncertain, real-world environments, as proposed by Al-Aomar [32] and Hilmola and Gupta (2011). All in all, we can claim that optimisation-based approaches to exploring product-mix problems need to be complemented with simulation tools that allow professionals to effectively design and implement effective decision-making mechanisms for such production systems.

1.2. Research objective, methodological approach, and contribution to the literature

Following from the previous discussion, an important gap in the relatively small literature exploring stochastic product-mix problems arises from the lack of consideration of the operational and economic impact of uncertainties on the production system.

[1] Sobreiro and Nagano [89] review and compare several methods based on the Theory of Constraints to optimise product-mix decisions.

From this perspective, prior studies have not considered potential inefficiencies emerging in the flow of materials when inventory control mechanisms are employed in the presence of uncertainties.

To narrow this gap, we focus on uncertainty in customer demand in this paper. In this sense, we examine the cost of demand uncertainties in real-world production systems that face product-mix decisions. Specifically, we explore the mathematical relationship between the net profit of production systems and the relevant parameters that characterise demand uncertainty, which has not been studied by prior works and would offer relevant implications for professionals. This allows us to explore the trade-off between the benefits and the cost of variability reduction, which helps practitioners embrace effective investments in such initiatives. To this end, we consider two dimensions of demand uncertainty that arise naturally in product-mix settings: *demand volume* (that is, the overall demand faced by the organisation) and *demand mix* (that is, the relationship between the demands of the different product lines).

To measure the operational and economic effects of variability in the volume and mix of customer demand, we develop our study in the context of a well-known product-mix problem, commonly labelled as the *PQ problem*. This defines the model of a production system that elaborates two products from three raw materials and a purchased part with four workstations in a deterministic scenario. This was introduced by Goldratt [33] and has been often used —sometimes slightly modified— in subsequent works (e.g. [34,35]) as well as in recent studies (e.g. [7,24]). In this way, we enlarge the traditional, deterministic PQ problem by modelling the aforementioned sources of demand variability. To manage the flow of materials in the production system exposed to demand uncertainties, we apply the Theory of Constraints to design a Drum-Buffer-Rope scheduling mechanism (e.g. [36,37]). This is focused on protecting the throughput of the production system at a reasonable cost in terms of inventory holding.

The main results, findings, and implications of this paper that contribute to the knowledge in this area can be summarised as follows:

(1) *We provide a mathematical model that expresses the net profit in production systems with product flexibility as a function of the key parameters that define uncertainty in the volume and mix of demand. This describes the curve through which demand uncertainties induce a meaningful decrease in the net profit, and we show that this occurs due to the combined effect of a throughput decrease and an operating expense increase.*
(2) *We show that the Drum-Buffer-Rope scheduling mechanism provides the system with enough robustness to preserve the net profit under a certain degree of demand variability. In this regard, we reveal that the system is more robust to variability in the mix of demand than to that in the overall volume of demand.*
(3) *We suggest that companies should pursue their optimal level of demand variability by analysing the trade-off between the benefits and the costs derived from undertaking initiatives aimed at uncertainty reduction. In this regard, we reveal that, when the costs are low, reducing volume uncertainty should be given the highest consideration, while, under high costs, efforts are best concentrated on alleviating mix uncertainty.*

In line with the previous methodological considerations, we note that we use simulation techniques — the stochastic product-mix problem under study may become intractable through analytical approaches. In particular, we model the production system through agent-based techniques (e.g. [38]). They represent an emerging modelling approach that is well suited for studying complex dynamic systems that operate in fast-changing environments[2]; see Jennings et al. [39]. In this sense, we also aim to illustrate how an agent-based decision support system can be built to capture a sufficient level of detail that allows for substantial improvements in the real-world system. This may be interpreted as a contribution of this paper of a methodological nature. Finally, we employ regression modelling techniques for accurately estimating the impact of demand volume and mix uncertainties on the economic performance of the organisation considered in the stochastic PQ problem.

From a wider viewpoint, our work is well-aligned with the increasingly important concept of Industry 4.0 (I4.0). I4.0, termed the fourth industrial revolution, has enabled companies to digitize and integrate horizontally across the entire organization and vertically across the supply chain. This has allowed them to become part of digital networks and ecosystems spanning the globe while retaining their distinct regional footprint [40]. The digital transformation facilitates data-driven workflows and collaboration at multiple levels (machine to machine, business to manufacturing, and business to business) to achieve the promised gains. Nonetheless, it also poses enormous challenges in modelling and instrumenting special characteristics of I4.0, such as self-optimization, self-control, networking capability, or decision-making ability, which are required to achieve the much-desired intelligent and flexible production [41]. In this sense, work on how to cope with the complex, dynamic and stochastic nature of I.40 manufacturing is still sparse. From this perspective, our work also aims to contribute to this discipline by showing how advanced simulation techniques may catalyse the execution of I4.0 in organisations.

1.3. Structure of the article

In the remainder of this article, *Section 2* describes the production system. We first depict the original PQ problem, and we later explain how we model the new sources of uncertainty and how we measure the economic performance of the system. *Section 3* details the agent-based model and its implementation. First, we offer an overview through a turtle diagram. Subsequently, we discuss how we model the system dynamics through a coloured Petri net, and we explain how the model was developed with agents in NetLogo.

[2] Accordingly, the literature provides evidence of an increasing interest in the agent-based paradigm for organisational decision making. We refer readers to the article written by North and Macal [90] on the value of this methodology for business-related applications.

Finally, we clarify its verification and validation. *Section 4* presents the study through which we evaluate the consequences of demand uncertainties on the economics of the production system. We plan the experimentation process, we present and discuss the results, and finally reflect on their implications by looking at the trade-off between the benefits and the costs of variability reduction. Finally, *Section 5* concludes and proposes interesting avenues for future research derived from this work.

2. Production system

Eliyahu M. Goldratt exposed his view on managing organisations in a series of books. In them, he made use of several illustrative examples to convey key, often counterintuitive, lessons, through which he explained the rationale behind the innovative Theory of Constraints [20]. In one of these books, "*The Haystack Syndrome: Sifting information from the data ocean*", Goldratt [33] developed a simple but forceful exercise to show how classical management principles can lead to inefficient solutions. This exercise, later referred to as the PQ problem [42], was further extended to explore other supply chain issues, such as the link between operations [35] and purchasing and the make-or-buy decision [43].

2.1. The PQ problem

Let us consider a production system that manufactures two final products: P and Q. The former, P, requires two different raw materials, RM1 and RM2, specifically one unit of each, plus a purchased part, PP4. The latter, Q, needs a unit of RM2 together with a unit of a different raw material, RM3. At the same time, the production system includes four workstations, labelled from A to D. Each of the raw materials needs to be processed in two different workstations in the following order: RM1 at A (processing time: 15 min) and C (10 min); RM2 at B (15 min) and C (5 min); and RM3 at A (10 min) and B (15 min). Later, the assembly of intermediate products takes place in workstation D, where P and Q require 15 and 5 min per unit of final product, respectively.

In the PQ problem, it is known that the selling prices of P and Q are \$90 and \$100 (per unit), respectively; while the cost of RM1, RM2, and RM3 is the same, specifically \$20 (per unit), and the cost of PP4 is \$5 (per unit). In addition, it is assumed that the production system needs to incur a fixed operating expense of \$6000 per week.

The key question here involves determining the optimal quantities of P and Q to be manufactured in order to maximise the net profit. In this regard, it is considered that the weekly demand for products P and Q is 100 and 50 units, respectively. The dilemma stems from the inability of the production system to completely satisfy the overall demand, as it is assumed that it operates in a capacity-constrained setting. In particular, each workstation operates 8 h per day for 5 days per week, which results in 2400 min per week. Note that completely satisfying the weekly demand of the market would entail 3000 min at B; that is, 150 units of RM2 (used for P and Q) × 15 min, plus 50 units of RM3 (only for Q) × 15 min. Workstation B then becomes the bottleneck of the system. To sum up, Fig. 1 summarises all the previous information, thus demarcating the production problem.

The problem can be easily solved through simple optimisation techniques, e.g. the simplex algorithm for linear programming [44], or by applying the Theory of Constraints' principles [20]. In any case, the solution is the same: even though Q generates a higher unit margin (Q: \$100 - \$40 = \$60; P: \$90 - \$45 = \$45) and requires less time within the workstations (Q: 20 + 25 + 5 = 50 min; P: 25 + 20 + 15 = 60 min), manufacturing P should be prioritised.[3] The explanation is that P involves 15 min of the bottleneck per unit, i.e. workstation B, while Q entails 30 min of this resource. Therefore, the economic margin per unit of bottleneck is larger for Q (Q: \$45 / 15 min = \$3 / min; P: \$60 / 30 min = \$2 / min). For more detail on the PQ problem, the optimal solution, and its interest, the in-depth discussion by Youngman [45] is highly recommended.

2.2. Introducing stochasticity in customer demand

The original PQ problem defines a perfect-information, deterministic scenario in the sense that all the relevant parameters (times, costs and prices, demand, and capacity constraints) are assumed to be fixed and known. In this paper, we enrich this context by considering uncertainty in customer demand, which brings it closer to real-world scenarios. We note that uncertainty can be defined as "any deviation from the unachievable ideal of completely deterministic knowledge of the relevant system" ([46], p. 5), while we chose to model demand uncertainty, as it is an essential driving force of the current business context [47]. Therefore, it needs to be appropriately addressed by managers' decision-making processes [48].

We model the uncertain nature of the demand for products P and Q by considering variability in two complementary dimensions: demand volume and demand mix. Bernstein and DeCroix [49] argued that these two dimensions of demand arise naturally in a variety of real-world settings. They discussed that in some practical contexts demand volume uncertainty prevails, while others are mainly characterised by demand mix uncertainty. Then, considering both provides a rich picture of the dynamic behaviour of the production system. We note that this approach differs from previous efforts to model demand uncertainty in product-mix problems (such as those works discussed in Section 1), allowing us to capture the economic impact of two conceptually different sources of uncertainty.

The nature of uncertainty is often classified as epistemic or aleatoric [50]. The former is due to limited data or knowledge, and thus

[3] Prioritising P allows the system to completely meet the demand of this product (100 units per week) and a portion of the demand of Q, 30 units per week, due to the bottleneck restrictions. This would generate a profit (= throughput – operating expense) of (100 × \$45 + 30 × \$60) – (\$6,000) = \$300 per week. In contrast, prioritising Q would entail satisfying the entire weekly demand of this product, i.e. 50 units, and 60 units of P (out of the 100 units required by customers). The profit would be (60 × \$45 + 50 × \$60) – \$6,000 = -\$300 per week.

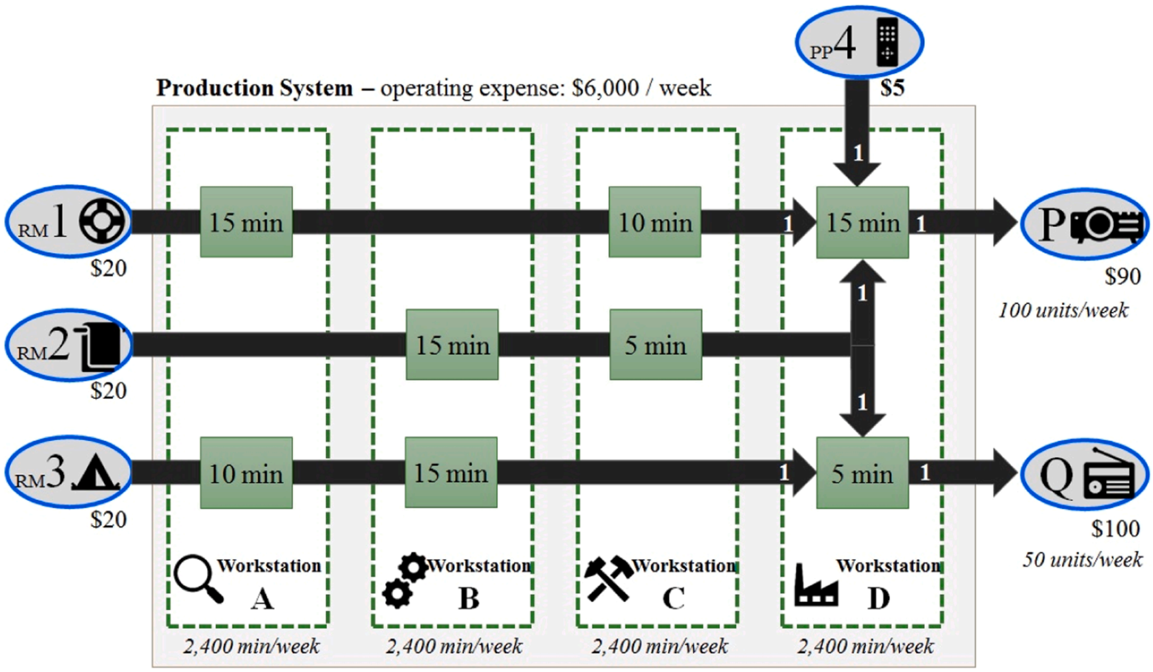

Fig. 1. Schematic representation of the PQ production system in the deterministic context.

it may be mitigated (e.g. [51]). In contrast, the latter refers to variability due to inherently random effects; thus, it needs to be accommodated by the system (e.g. [52]). In our work, we model demand uncertainty as an aleatoric uncertainty. In this sense, the demand volume and mix have been modelled through independent and identically distributed (i.i.d.) random variables that follow the statistical distributions that are described and justified in the following paragraphs.

Volume variability refers to the stochastic nature of the overall demand that the production system needs to attend to. To model this source of uncertainty, we employ a gamma distribution. Importantly, this statistical distribution is defined only on non-negative values and takes different forms depending on the setting of the relevant parameters. In this fashion, one can model a wide range of real-world demand shapes; see Snyder [53].

In this work, we assume that the volume of demand, ρ, is an i.i.d. random variable that follows a gamma statistical distribution, $\rho \sim \Gamma(k_\rho, \theta_\rho)$. We adopt the classical parametrisation of this distribution, where k_ρ is denoted as the shape parameter and θ_ρ is the scale parameter. The mean of ρ is $\mathrm{E}[\rho] = k_\rho \theta_\rho$, and the variance becomes $\mathrm{var}[\rho] = k_\rho {\theta_\rho}^2$. To keep the deterministic PQ problem as the benchmark, we will explore different values of k_ρ, adjusting θ_ρ so that $\mathrm{E}[\rho] = 150$ (i.e. $\theta_\rho = 150/k_\rho$). In this sense, k_ρ determines the probability density function (PDF) of the distribution, and thus the variance of ρ, $\mathrm{var}[\rho] = \mathrm{E}[\rho]^2/k_\rho$. The coefficient of variation then becomes $\mathrm{cv}[\rho] = \sqrt{\mathrm{var}[\rho]}/\mathrm{E}[\rho] = 1/\sqrt{k_\rho}$, that is, $\mathrm{cv}[\rho]$ is inversely proportional to the square root of k_ρ. By way of illustration, Fig. 2(a) provides the PDF of the demand volume for three values of the shape parameter, $k_\rho = \{2, 20, 200\}$. Notice that as k_ρ grows, the volume of demand becomes less uncertain.

Mix variability refers to stochasticity in the relationship between the demand for both products. We model it through the mix ratio, λ, which is the ratio of the demand for P and the overall demand (i.e. the sum of the demand for P and Q). In this case, we used a beta distribution, as it is defined on the interval [0,1]; e.g. Ahrens and Dieter [54].

Thereby, the mix ratio, λ, is assumed to be an i.i.d. random variable whose behaviour is defined by a beta statistical distribution, $\lambda \sim \mathrm{B}(\alpha_\lambda, \beta_\lambda)$. In this classical parametrisation of the beta distribution, α_λ and β_λ are two different shape parameters, which regulate the form of the PDF, with a mean of $\mathrm{E}[\rho] = \alpha_\lambda/(\alpha_\lambda + \beta_\lambda)$ and a variance of $\sigma[\lambda] = \alpha_\lambda \beta_\lambda / [(\alpha_\lambda + \beta_\lambda)^2 (\alpha_\lambda + \beta_\lambda + 1)]$. Again, to study the effects of demand uncertainties in relation to the deterministic PQ problem, we will investigate different levels of α_λ, adjusting β_λ so that $\mathrm{E}[\lambda] = 100/150 = 2/3$ (i.e. $\beta_\lambda = \alpha_\lambda/2$). This results in a coefficient of variation that decreases as α_λ grows according to the following relationship: $\mathrm{cv}[\lambda] = \sqrt{\mathrm{var}[\lambda]}/\mathrm{E}[\lambda] = \sqrt{\beta_\lambda}/\sqrt{\alpha_\lambda(\alpha_\lambda + \beta_\lambda + 1)} = 1/\sqrt{2 + 3\alpha_\lambda}$. Fig. 2(b) depicts the PDF of the demand mix for three values of the shape parameter, $\alpha_\lambda = \{2, 20, 200\}$. From inspection of this figure, one can see that as α_λ increases, the mix of demand becomes less variable, and consequently, the demand mix is less uncertain.

Thus, the demand for P and Q at time t can be expressed as $d_{Pt} = \lambda_t \rho_t$ and $d_{Q_t} = (1 - \lambda_t)\rho_t$, respectively. It can be easily shown that $d_{Pt} + d_{Q_t} = \rho_t$, representing the overall demand volume (as defined before). In addition, as λ varies within the interval [0,1], $0 \leq \{d_{Pt}, d_{Q_t}\} \leq \rho_t \ \forall t$. Finally, due to $\mathrm{E}[\rho] = 150$ and $\mathrm{E}[\lambda] = 2/3$, the long-term mean of the individual demands for products P and Q will be 100 and 50, respectively.

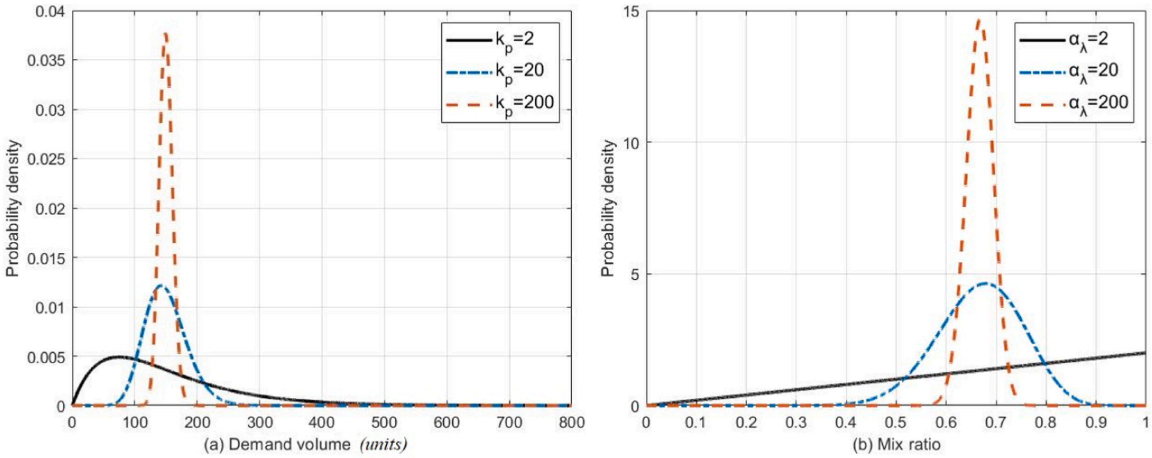

Fig. 2. Probability density function of the demand volume (a) and mix ratio (b) for three shape parameters.

2.3. The economics of the production system

Finally, some considerations about the economic performance of the production system need to be made. In the context of the PQ problem, this performance is generally measured in terms of net profit. Following guidelines from the throughput accounting [24], the net profit for period t, NP_t, can be calculated as the throughput of the production system, T_t, minus the operating expense, OE_t, i.e. $NP_t = T_t - OE_t$[55]

The throughput in period t can be obtained by $T_t = (90s_{Pt} - 45o_{Pt}) + (100s_{Q_t} - 40o_{Q_t})$,[4] where: 90 and 100 are the selling prices of P and Q, respectively; 45 and 40 are the unit costs of P and Q, respectively; s_{Pt} $(\leq d_{Pt})$ and $s_{Q_t}(\leq d_{Q_t})$ are the sales (that is, the satisfied demand) of P and Q, respectively; and o_{Pt} and o_{Q_t} are the production orders issued for manufacturing P and Q, respectively. Note that, like in the deterministic problem, we assume a lost-sales inventory system [56], i.e. customer demand that cannot be fulfilled on time is cancelled.

On the other hand, in the deterministic context, the operating expense is defined as fixed, $OE_t = 6,000$ (see Section 2.1). However, this approach requires adaptation to the stochastic context, as uncertainties will unavoidably result in an increase in the costs of managing and keeping the inventory. We capture this effect by using the expression, $OE_t = 6,000 + IC_t$, where IC_t accounts for the extra inventory-related costs derived from the stochasticity of the system. Such inventory-related costs cover different sources of costs, such as storage, obsolescence, insurance, and/or the opportunity cost of the money 'tied up' as inventory; see the discussion in Slack et al. [57].

All in all, uncertainty may impact the net profit from two angles. First, uncertainty decreases the throughput. This occurs when the production order is higher than the actual demand ($o_{Pt} > d_{Pt}$; $o_{Q_t} > d_{Q_t}$), or when the satisfied demand is less than the actual demand ($s_{Pt} < d_{Pt}$; $s_{Q_t} < d_{Q_t}$). Second, uncertainty increases the operating expense, as holding a greater amount of inventory becomes necessary. Both effects will be clarified in the following section, where we provide detail on how the agent-based production system operates in the stochastic context.

3. Agent-based model and simulation

Agent-based techniques are a relatively new approach to modelling that is especially suited for investigating complex systems that are strongly built on interdependencies [58]. This paradigm represents a bottom-up approach, in which a system is created from the interactions of its basic units, agents, in a time-changing environment, which facilitates a natural recreation of real-world systems [59]. Also, this facilitates the implementation of complex behaviours [79] and has some important advantages in terms of scalability, modularity and incrementality [60].

As discussed by Epstein and Axtell [61], the agent-based approach allows researchers to break disciplinary boundaries and explore problems that would be difficult to study through other methodological approaches. In a similar line of argument, Lättilä et al. [62], p. 7969–7970] underlined that the agent-based approach "is expected to have comprehensive effects on the way that businesses use computers to support decision-making". Accordingly, an increasing number of studies explore the potential of agents for building model-driven decision support systems in different fields; see the recent papers by Ponte et al. [63], Chica and Rand [64], Drakaki et al. [65], Zaffar et al. [66], Fernández-Isabel et al. [67], and Zhang et al. [68], amongst many others. Interested readers are referred to the articles by Chatfield et al. [69], Nilsson and Darley [70], and Groves et al. [71] for further detail on how decision-making procedures

[4] In the deterministic context, this simplifies to $T_t = 45o_{P_t} + 60o_{Q_t}$, where 45 and 60 are the unit margins provided by P and Q, respectively; e.g. de Souza et al. (2013). This occurs as, in the absence of uncertainties, orders directly translate into sales, i.e. $S_{P_t} = o_{P_t}, S_{Q_t} = o_{Q_t}$.

aided by agents can be helpful in manufacturing and logistics settings.

For all the above reasons, we used agents to develop a simulator of our stochastic variant of the PQ problem. The operations of the production system were modelled through a coloured Petri net, a common alternative for modelling the dynamics of agent-based systems [72,73]. The agents and Petri nets were later implemented using the functionalities of the NetLogo environment [74]. In addition, we note that the simulation system incorporates a discrete-event engine to manage the simulation clock in a more precise and time-efficient manner. In the following subsections, we describe in detail the agent-based simulator that we built. A similar approach can be followed for building intelligent decision-making support tools for product-mix problems in real-world settings that are characterised by other sources of uncertainty.

3.1. High-level overview of the production system

Fig. 3 offers a general understanding of the production system in the form of a turtle diagram.[5] This displays the most relevant process characteristics: (transformable) inputs, outputs, resources (both infrastructure and human), methods, and targets. In this way, this representation provides a scheme for modelling and exploring the system.

We note that *inputs* and *outputs* encompass not only the materials flow but also the information flow. In terms of the former, the system transforms the raw materials (RM1, RM2, RM3) and the purchased part (PP4) into the final products P and Q. As regards the latter, customer demand is both an input, through sales orders that impact the system operations, and an output, through the record of sales that are considered for production planning purposes.

The top part of the diagram shows the key resources. First, we see the *infrastructure*. This mainly refers to the workstations (A, B, C, D) and the shop floor facilities, modelled through the size of the buffers that will be implemented to protect the production system against uncertainties. Second, the *human actors*. They include the workforce that performs the required activities to transform inputs into outputs, together with fundamental stakeholders that significantly impact the production system, such as the customer and the supplier.

Finally, the *methods* and *targets* can be seen at the bottom of the diagram. As we will explain later, the flow of materials operates according to a Drum-Buffer-Rope scheduling mechanism [37]. In addition, the prioritisation policy defines the responsedecision-makers when the production system cannot completely satisfy the demand for both products. Finally, and in line with the original PQ problem, the net profit (*NP*) is understood as the main economic performance indicator. This is calculated by measuring the throughput (*T*) and the operating expense (*OE*) of the production system (specifically, $NP = T - OE$; see Section 2.3).

3.2. Modelling and implementation of the Petri net with agents

In the development of our agent-based simulator, we followed a standard procedure based on two stages: modelling and implementation. In the modelling stage, we used Petri nets to model the operations of the production system over time. We selected Petri nets, amongst other options, due to their precise, compact, graphical, and intuitive nature of presentation [75]. They also allowed us to later build the simulator in an incremental way, thus being more efficient and flexible. Indeed, Petri nets are frequently used in I4.0 environments, as this approach can be particularly useful to model and instrument the I4.0-induced concepts, tools and methods and, therefore, to understand and leverage I4.0 technologies (see [76–78]).

A Petri net is a collection of transitions and places connected by direct arcs [79]. *Transitions* (represented by bars) refer to events that may occur, while *places* (circles) are conditions that hold tokens. Transitions have: (i) one or more input places, which enable the transitions when they hold tokens that satisfy a rule defined by the connecting arcs; and (ii) one or more output places, to where tokens move from the input places, according to the arc weights and the places' capacities, as soon as enabled transitions fire. In this sense, the allocation of tokens to the different places defines the state of the Petri net at any time. In our case, we are modelling an extended coloured Petri net. Specifically, we use two colours for the tokens: white ones for the flow of information, and black ones for the flow of materials (e.g. [80]). Also, we employ inhibitor arcs to implement the prioritisation policy. As these arcs disable transitions when the input place is marked with a token, and thus put additional constraints to the firing of transitions, ours may be interpreted as an extended Petri net [81].

Our coloured Petri net can be mathematically described as $CPN = (S, T, F, M_0, \Sigma, C, N, E, \rho)$, where: S and T are the disjoint sets of places and transitions, respectively; $F : (S \times T) \cup (T \times S) \rightarrow \mathrm{N}$ is the multiset of arcs, including the weights; $M_0 : S \rightarrow \mathrm{N}$ is the initial marking that assigns a number of tokens to each place, such that for all $t \in T$ the set $\{s \mid F(s,t) \rangle 0\}$ is finite and non-empty, and the set $\{s \mid F(t,s) \rangle 0\}$ is finite; Σ is the set of colours; C is the colour function; N is the node function; E is the arc expression function; and $\rho \subseteq T \times T$ is the priority relation, where $(t_1, t_2) \in \rho$ means that t_1 has lower priority than t_2. In this sense, ρ allows us to define each run the product that is prioritised (P or Q) in the flow of black tokens that run through the production system. Last, we note that places are n-bounded, where n is a positive integer that makes that $M(s) \leq n$ for all possible markings M. This introduces the capacity limitations in the production system, which is also required to implement the prioritisation policy, together with the inhibitor arcs.

In the implementation stage, we used the functionalities of the NetLogo programming environment to achieve, with agents, the

[5] A quality management tool that is commonly used to visually display high-level information that assists in the effective modelling and improvement of business processes; see e.g. Russell [91].

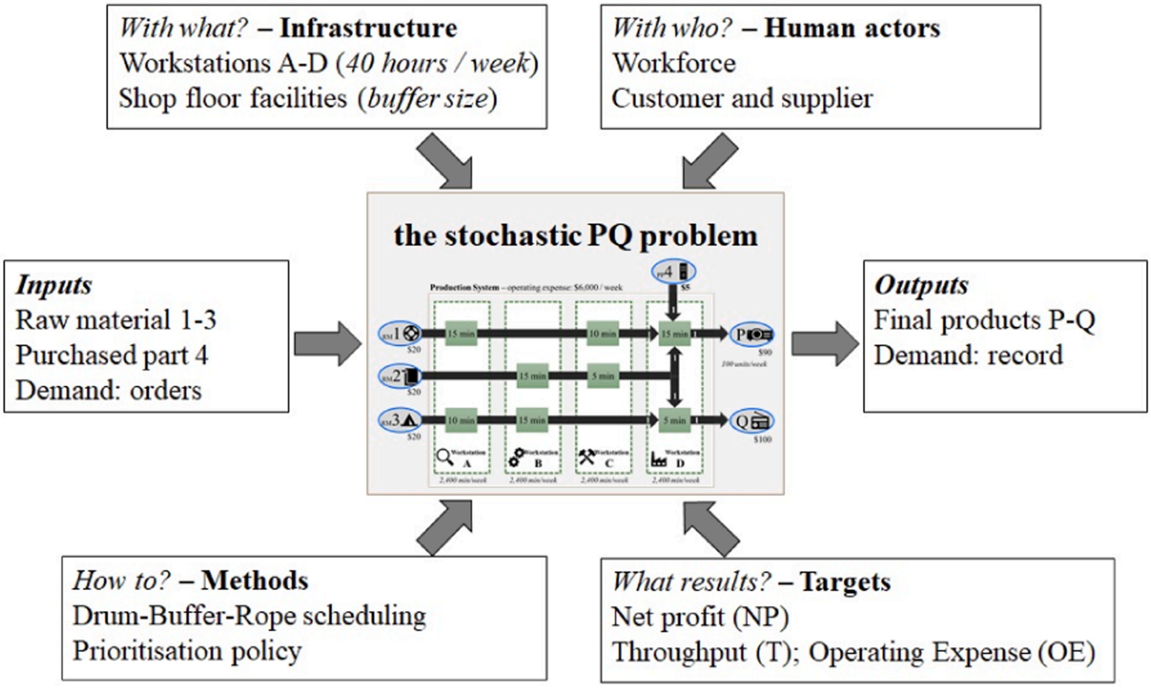

Fig. 3. Turtle diagram of the agent-based system.

execution semantics expected from the formal Petri net.[6] In this sense, we defined different breeds of agents to implement the places, transitions, interconnections, colours, and priority relations. We highlight that each of these agents is characterised by a collection of: (i) *attributes*, defining their characteristics and state (e.g. 'pool' establishes the difference for units of P and Q); and (ii) *methods*, describing how they perform tasks, make decisions, and communicate with each other and the outside world, which were previously formalised through Petri nets. In the following subsections, we explain how the agent-based system operates through schematic representations. The description is structured according to the main loops that form the model: sourcing, manufacturing, and shipping.

Before these explanations, we note some important structural aspects that were used in the implementation of the Petri net model in NetLogo. The graphics window was divided in six vertical lanes (regions of patches in NetLogo). The first lane (*incoming*) receives the materials from the supplier. There are four reception points: for the raw materials (RM1, RM2, RM3) and the purchased part (PP4). The last lane (*shipping*) contains the final products, which are available for satisfying customer demand, with two dispatching zones: for P and Q. The intermediate four lanes represent the workstations (A, B, C, D). Each lane includes not only the necessary production resources, but also all other agents (mainly, transitions and places) that describe the flows of materials and information around the workstation. In this sense, the products flow horizontally, from the west to the east, as in Fig. 1.

3.2.1. *Sourcing loop*

Fig. 4 shows the operation of the sourcing loop. By way of illustration, this loop is displayed for RM3. The 'sourcing' transition associated with RM3 is fired when there is a token in the 'truck' place that verifies the 'is-due?' condition.[7] This visible token represents the arrival of a truck to the shop floor, which occurs periodically. Also, tokens at the 'PO' place represent the purchase orders, which emerge through the pacemaker of the production system ('prepare' transition), placed at workstation D; see Section 3.2.2. Every time the production process starts assembling a finished good in workstation D, a purchase order is issued (as material components are back-flushed) for the required raw materials (and PP4, when applicable). This results in the arrival of tokens in the incoming lane. Finally, each token that represents a purchase order, through the 'sourcing' transition, generates tokens that include materials in the 'on-hand' place of workstation A, which is the point of consumption of RM3 in this workstation.

[6] We note that we used first-class function values in the NetLogo environment, including 'closures' and 'lambdas', to implement the operations of the models according to the formal Petri net. More information can be found at https://ccl.northwestern.edu/netlogo/docs/.

[7] The 'is-due?' condition is a mechanism that we developed to implement the desired operation of the formal Petri net in NetLogo. It may be interpreted as a 'predicate' of the tokens that is verified when the simulation clock reaches the specific event in the future event list. Tokens are always on the 'truck' place, but they are not visible (and they do not fire the 'source' transition) until this event is reached. Also, we note that the tokens carry information about the rule that defines the time between consecutive arrivals. The initial marking places one ('truck') token in the 'truck' place.

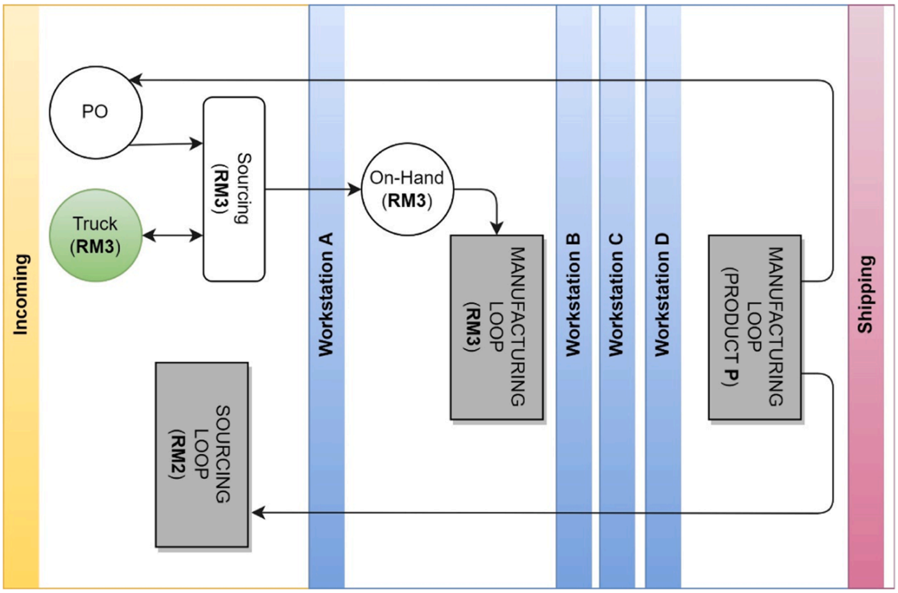

Fig. 4. Representation of the sourcing loop for RM3.

3.2.2. *Manufacturing loop*

We now focus on the manufacturing loop. In Fig. 5, we show the loop of product P at workstation D, as this is the most complex one in our system. The 'prepare' transition becomes enabled when: (i) there is a token at the '*Pacemaker*-WO' place, representing a work order; and (ii) there are enough intermediate products (RM1, RM2, and PP4, according to the bill of materials of P) in the 'on-hand' places, where tokens arrive from the previous workstations. As in the 'prepare' transition a back-flush of materials occurs, there are tokens with purchase orders flowing towards the 'PO' places in the incoming lane. We note that the 'reserved' place has a capacity of one token. If P (Q) is prioritised, there is an inhibitor arc towards the 'produce' transition of product Q (P), so that the same resource cannot be used to manufacture different products at the same time.[8] From the 'reserved' place, the token, together with the availability of the 'worker' resource in the 'idle' place, fires the 'produce' transition. This carries product P towards the 'WIP' (work-in-progress) place, where the token waits until the end of its cycle time. Later, the 'evacuate' transition fires. This moves product P downstream and leaves it in the 'on-hand' place of the shipping area. Finally, it replaces the 'worker' resource in the 'idle' place, so that a new production cycle can start.

3.2.3. *Shipping loop*

Fig. 6 shows the shipping loop of P, which occurs in the last lane. In the 'SO' (sale order) place, there is always a token with the next sale order (i.e. demand). The 'sales' transition is fired as soon as the 'is-due?' condition of this token is verified. The firing of this transition consumes as many tokens as indicated by the order quantity of the token in the 'SO' place, as long as there are enough black tokens available in the 'on-hand' place. Also, the firing of this transition adds a new token in the 'SO' place, which will fire the 'sales' transition in due course. Besides, tokens that represent work orders are sent to the pacemaker in workstation D. If, e.g., the system has sold 34 units of P, 34 tokens will be sent to the pacemaker. Finally, the sold units and the order satisfied are cloned in the 'sink-parts' and 'sink-orders' places, respectively, for statistical and monitoring purposes.

3.2.4. *Other implementation details*

Finally, we provide three additional considerations about how the agent-based simulator was developed in NetLogo 6.0.4. First, it is important to clarify that Figs. 4,5, and 6 do not represent formal Petri nets. Instead, they are implementations of a coloured Petri net with the functionalities of the NetLogo environment. These representations achieve the desired operation of the formal Petri net, using the main elements of these models (basically, transitions, places, and arcs), in NetLogo. In this sense, the figures provide information about both the operation of the formal Petri net model and its computer implementation. In this regard, we note that in the modelling stage we used the formal syntax and execution semantics of Petri nets proposed by Jensen and Kristensen [82]. By way of illustration, Appendix A shows the formal Petri net that models the operations of the sourcing system, allowing the reader to understand the procedure followed to convert the formal models to their NetLogo implementations.

[8] We note that the prioritisation of the work is a core issue of the PQ problem. This issue is critical at the bottleneck of the production system. Our agent-based system solves this requirement by combining the management of the cyclical list to decide which part is to pass through the bottleneck at each moment together with the implementation of a dynamic inhibitor to give priority to the appropriate part.

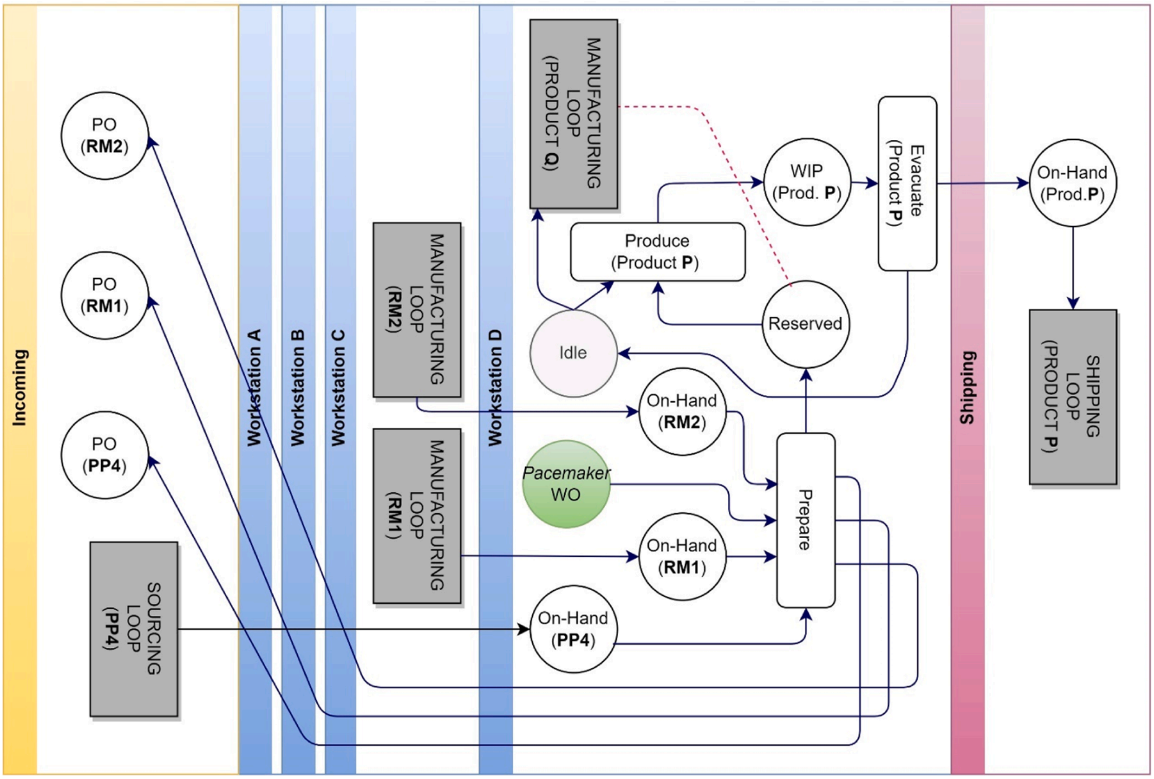

Fig. 5. Representation of the manufacturing loop for product P in workstation D.

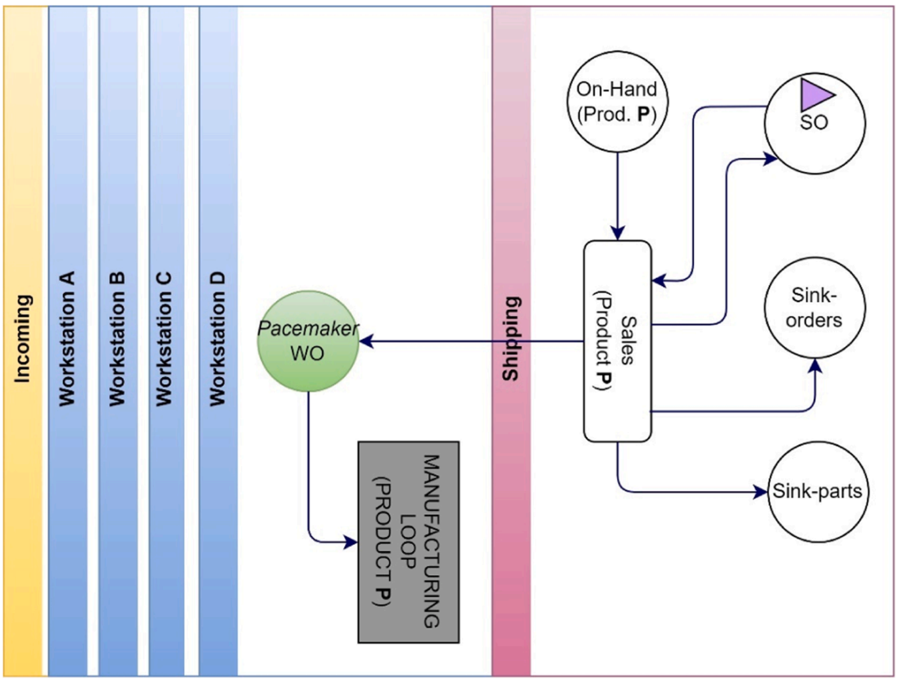

Fig. 6. Representation of the shipping loop for product P. .

Second, and in line with previous explanations, the production system has been programmed to operate according to a Drum-Buffer-Rope scheduling mechanism [36,37,83]. This focuses on utilising the entire capacity of the most limiting resource, i.e. the bottleneck, to maximise the system throughput. In the deterministic PQ problem, the bottleneck is placed at workstation B. However, in stochastic settings, it may move to other workstations. For this reason, we placed the *drum*, which acts as the pacemaker (it is regulated through the '*Pacemaker*-WO' place), at the assembly workstation D. This provides the control system with the ability to adapt to changes in the bottleneck position. The *rope* covers the release of products (including raw materials and intermediate products) from the incoming lane to the drum through the 'PO' places. This manages the flow of materials by subordinating the whole production system to the bottleneck. Finally, the *buffer* refers to the safety margin that protects the bottleneck with the aim of ensuring that the production system operates effectively and efficiently under demand uncertainty. We sized the buffer, which also determines the size of the rope that goes from 'SO' place to the 'Pacemaker-WO' place, to target a customer service level of 95%.

Third, we explain how we control the simulation time. As soon as one operation has been completed at a workstation, the next one must begin. This is of critical relevance at the bottleneck; otherwise, wasting bottleneck time results in a throughput decrease, which will eventually reduce the net profit and would distort our results. In agent-based systems, the simulation time can be managed through sequential 'ticks'. However, this is problematic in our application. Very small ticks are required to avoid losing time at the bottleneck, which would make simulation runs very long and/or may increase the experimental error. To solve this issue, we implemented a future event list —established by tokens that are on the arcs to transition with positive weights— that manages the clock in a discrete-event manner, see Costas et al. [84]. This allows us to obtain robust results in reasonable simulation times. Overall, this may be interpreted as a hybrid modelling approach (see [62]), similar to that followed by prior works (e.g. [85]), that aims to combine principles and advantages of the agent-based (for modelling complex logics) and discrete-event (for controlling the simulation time) models.

3.3. Verification and validation of the agent-based simulator

Verification requires making sure that the agent-based simulator operates as it was designed to. The simulation system needs to accurately match the conceptual design, i.e. with no programming errors, which includes checking the stability of the behaviour and the consistency of the results. Validation entails ensuring that the conceptual simulation model accurately represents the production system under study. Otherwise, the conclusions obtained from the study could not be extrapolated to the real world. In this sense, verification and validation are fundamental stages of modelling processes; see [55].

In our case, we followed four different techniques. First, we employed good programming practices that are common in robust software engineering approaches. Such practices include test-driven development, exception handling, failure mode analysis, and clean code. Second, we checked intermediate simulation outputs through tracing and statistical testing, both manually and automatically. For the latter, we created a specific breed of agents, labelled as 'police', which allowed us to detect system malfunctions and programming errors through cross-checking of the system invariants and the behavioural properties. Third, we analysed the dynamic animation of the simulator in some runs to check the operations performed by the system. Finally, we performed site acceptance tests (SATs) to compare the outputs of the agent-based simulation model against well-known results of the PQ problem (SAT #1) and the results of simpler mechanisms (including optimization models and spreadsheet simulation models) that model the deterministic PQ problem (SAT #2). Also, we used SATs to check that the simulator performs as intended when uncertainties are introduced (SAT #3). These tests are fully described in Appendix B.

In addition, Appendix C provides detail about the dashboard of the model-driven decision support system that we developed, which allows the reader to better understand how it was implemented in the NetLogo environment.

4. Numerical study

In this section, we investigate the impact of uncertainties in the volume and mix of customer demand on the economic performance of the PQ production system. In line with previous discussions, the economic performance is measured via the average net profit, $\overline{NP}$, while the shape parameters k_ρ, for the demand volume, and α_λ, for the demand mix, are used as proxies accounting for demand uncertainty.

We study the effects of both parameters separately. In each case, we explore five scenarios defined by different values of the shape parameter, which are detailed in the following subsections. They cover from a largely uncertain scenario, where the variability is very high, to a nearly deterministic scenario, with very low variability. Each scenario has been studied through five simulation runs to ensure high data availability for the subsequent statistical analysis. Each of the 50 ($2 \times 5 \times 5$) simulation runs has a time length of 75 weeks, where the first 25 weeks (a warm-up period) are not considered in order to minimise the effects of the initial state of the production system.

Finally, we note that we use a unit holding cost of $4 (that is, 4.44% and 4% of the selling prices of P and Q, respectively) in all cases. This applies to every additional material (raw material, purchased part, intermediate product, or final product) that is kept in the production system at the end of each week. In addition, we highlight that we implement for this analysis a strategy based on prioritising product P, which is well known to provide the optimal solution in the deterministic scenario. In this sense, we are exploring the impact of demand uncertainty on a (stochastic) production system that has been optimised in a deterministic scenario. In this fashion, we leave out of the scope of this paper the analysis of how to optimise the performance of the system in the stochastic scenario.

4.1. The impact of demand volume uncertainty

We use the following values of k_ρ to define the five scenarios of demand volume uncertainty, $k_\rho = \{2, 9, 34, 120, 300\}$. As we discussed in Section 2.2, θ_ρ has been appropriately adjusted in all cases to ensure that $\mathrm{E}[\rho] = 150$. As the coefficient of variation is $\mathrm{cv}[\rho] = 1/\sqrt{k_\rho}$, $k_\rho = 2$ results in $\mathrm{cv}[\rho] = 70.71\%$, representing a highly uncertain scenario, and $k_\rho = 300$ leads to $\mathrm{cv}[\rho] = 5.77\%$, i.e. a scenario where uncertainty is low.[9]

Figure D1 (in Appendix D) shows how the average net profit evolves over time in the first of the five simulation runs conducted in each scenario. From inspection of the different plots, we can observe that higher variabilities in the demand volume inevitably lead to higher variabilities in the economic performance of the system. This is the first —and reasonable— downside of demand volume uncertainty: it makes the net profit more uncertain.

Now we consider the mean results of the five replicates in each scenario with the aim of looking for the mathematical relationship between the average net profit, $\overline{NP}$, and the shape parameter, k_ρ. To this end, and after having checked the heteroscedasticity of the net profit (see Table E1 in Appendix E), we use a heteroskedastic fractional polynomial regression (see Table E3 in Appendix E).

Fig. 7 represents the regression model that we obtained together with the results of the simulation runs. Notice that for very high values of k_ρ, the stochastic PQ system performs very similarly to the deterministic one (i.e. $\overline{NP}$ =\$300. Inspection of this graph also reveals that net profit is relatively close to its optimal value (that of the deterministic setting) for $k_\rho = 120$, which results in a coefficient of variation of 9.13%. In the light of these results, we may conclude that the Drum-Buffer-Rope mechanism is able to reasonably well preserve the mean net profit in the production system even when there is a significant degree of variability in the volume of demand. However, as k_ρ decreases, uncertainty in the volume of demand dramatically decreases the average net profit of the PQ system.

To better understand the impact of variability in the volume of demand on the net profit, NP_t, we look at its two main components: the throughput, T_t, and the operating expense, OE_t; $\overline{NP} = \overline{T} - \overline{OE}$. Fig. 8 shows the box plots with the results of the three economic performance metrics that we obtained in the 25 simulations. The net profit graph provides a different view of the previous insight: when uncertainty grows (k_ρ decreases), the net profit tends to reduce. The other graphs explain why this occurs. We can see that, as k_ρ decreases, the throughput of the production system tends to decrease, while the operating expense tends to increase. Consequently, the net profit reduction occurs due to the combined effect of the throughput decrease and the operating expense increase. That is, not knowing the overall volume of customer demand provokes a reduction in the sales of the system as well as an increase in inventory costs. Notice that Fig. 8 also provides a clear perspective to see the heteroscedasticity in the data, as the variance is higher in the runs with lower values of k_ρ.

4.2. The impact of demand mix uncertainty

Now we conduct a similar study for the variability in the mix of demand. Here, the five scenarios were defined by $\alpha_\lambda = \{3, 7, 17, 40, 120\}$. In all of them, β_λ has been set such that $\mathrm{E}[\lambda] = 2/3$, as explained in Section 2.2. Given that the coefficient of variation of the mix ratio is $\mathrm{cv}[\lambda] = 1/\sqrt{2+3\alpha_\lambda}$, $\alpha_\lambda = 3$ results in $\mathrm{cv}[\lambda] = 30.15\%$ and $\alpha_\lambda = 120$ results in $\mathrm{cv}[\lambda] = 5.26\%$, representing two scenarios of a significantly different degree of uncertainty in the demand mix. Again, uncertainty decreases as the shape parameter, now α_λ, grows.

Figure D2 (in Appendix D) depicts the net profit in the first simulation run that was carried out in each scenario. The plots clearly show that mix uncertainty also creates variability in the net profit of the production system, which is the first downside provoked by this dimension of demand uncertainty. In addition, the plots suggest that the mean net profit decreases as α_λ reduces, which will be studied next. To analyse this relationship (between $\overline{NP}$ and α_λ), and after checking the heteroscedasticity of the net profit (shown in Table E2 in Appendix E), we also carried out a heteroskedastic fractional polynomial regression study (see Table E4 in Appendix E).

Fig. 9 represents the regression model obtained together with the results of the 25 simulation runs. Again, we can see that for high values of α_λ, the stochastic PQ system performs very similarly to the deterministic one (i.e. $\overline{NP} = \$300$), while as α_λ decreases, uncertainty dramatically damages economic performance. As for the analysis of demand volume, it is interesting to note that the PQ production system, controlled by the Drum-Buffer-Rope mechanism, is barely affected by demand mix uncertainty until a certain threshold. Note that, for example, for $\alpha_\lambda = 20$, where the coefficient of variation is 12.70%, the reduction in the net profit is relatively small.

Finally, Fig. 10 provides additional information on the five replicates in each scenario through a box plot of the average net profit, throughput, and operating expense. Here we can also observe that as the mix of demand becomes more uncertain, which is modelled through a decreasing α_λ, the average net profit reduces significantly. Interestingly, we can see that this occurs mainly due to the impact of a throughput decrease, although a small increase in the operating expense can also be perceived, especially for very low values of the shape parameter.

4.3. Discussion and implications: the trade-off between benefits and costs

Our analysis so far has provided evidence that demand uncertainty, both in terms of volume and mix, increases the volatility and

[9] Based on industrial data, Dejonckheere et al. [92] observed that the coefficients of variation of the demand faced by retailers typically range within the interval [15%, 50%]. In our study, we analyse several coefficients of variation within an even wider interval.

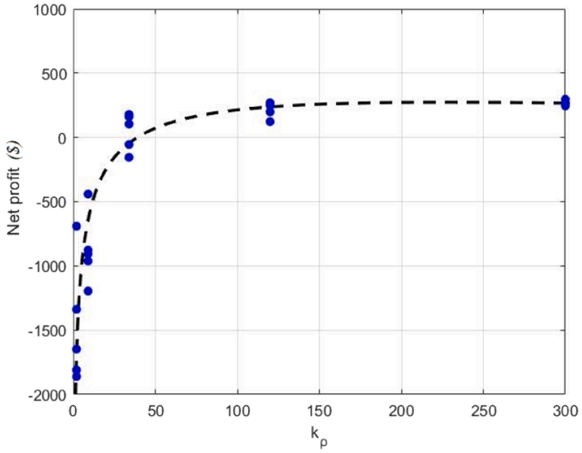

Fig. 7. Graphical representation of the regression model: Net profit as a function of demand volume uncertainty.

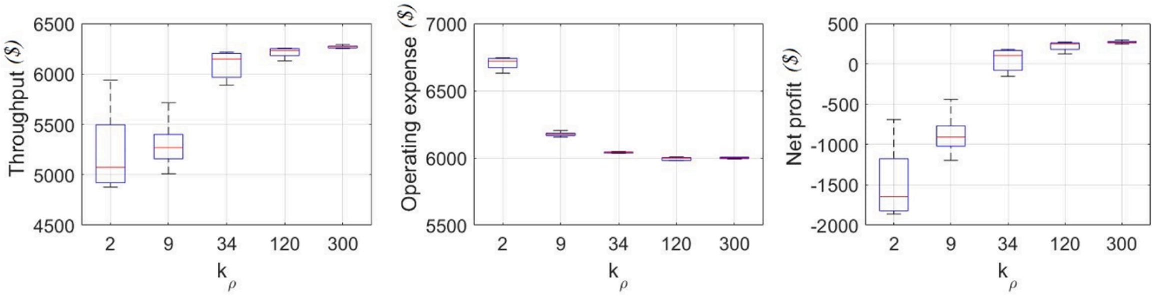

Fig. 8. Box plot representation of the throughput, operating expense, and net profit in the different scenarios (for demand volume uncertainty).

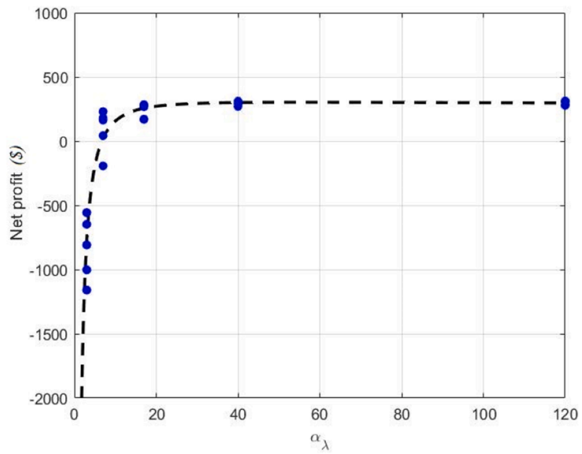

Fig. 9. Graphical representation of the regression model: Net profit as a function of demand mix uncertainty.

induces a decrease in the net profit of the production system, and it has allowed us to understand why this occurs. This highlights the importance of undertaking measures aimed at reducing demand uncertainty in manufacturing systems. First of all, in all types of businesses, this perspective places a premium on implementing forecasting methods that are well aligned with the nature (trend, seasonality) of customer demand in their specific context and appropriately adjusting their key parameters.

Also, in manufacturing industries where companies face the demand of lower echelons in the supply chain, it becomes fundamental to design and develop collaborative mechanisms that reduce the amplification of demand variability along the supply chain —that is, the well-known Bullwhip Effect. In this fashion, replenishment rules should not be individually (selfishly) established but need to take

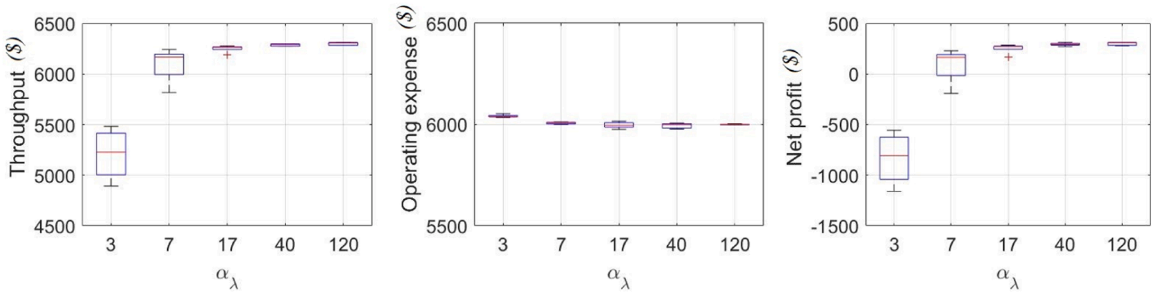

Fig. 10. Box plot representation of the throughput, operating expense, and net profit in the different scenarios (for demand mix uncertainty).

into consideration their implications in the wider supply chain.[10] Otherwise, as we have seen in this study, the manufacturer will suffer from a dramatically decreased economic performance, which would have negative repercussions on the rest of the supply chain, such as higher prices, uncertain product availability, and/or lower product qualities.

In those manufacturing systems that deal directly with the demand of the consumer or user of the product (i.e. the end customer), reducing demand variabilities may appear to be more difficult. However, this should not prevent managers from taking initiatives aimed at reducing them. Importantly, companies can use inventory pooling techniques —that is, serving separate markets using the same source— to mitigate demand uncertainty,[11] and thus significantly increase efficiency. Some marketing actions may also be useful for reducing uncertainty in demand, such as everyday low pricing —promising consumers the same (low) price without the use of promotions— or the offer of subscriptions. Indeed, subscriptions are becoming more and more popular, transforming many industries that are significantly different from those where they started to be used, such as the newspaper industry.[12]

In any of the previous cases, reducing demand uncertainties generally entails an economic cost and/or requires a financial investment, which may be shouldered by the organisation or shared by the different supply chain members. For example, increasing the degree of supply chain collaboration may require an investment in IT technologies and infrastructure to enable higher transparency. Similarly, not using promotion prices may have an opportunity cost for the organisation, while introducing a subscription model would reduce the gross margin per unit of product. All in all, this makes it necessary to consider the expected benefits derived from demand uncertainty mitigation, which were discussed in the previous subsections, in relative terms to the size of the required investment (or cost).

In the remainder of this subsection, we perform this trade-off analysis, given that it yields relevant implications for managers. To facilitate it, Fig. 11 first represents the mathematical relationship between the net profit and the coefficient of variation of both dimensions of demand uncertainty ($\mathrm{cv}[\rho]$ for the volume and $\mathrm{cv}[\lambda]$ for the mix), according to the regression models identified before and applying basing properties of the statistical distributions under consideration. Note that we use the same scale and limits in the y-axis in both to make the comparison easier, while the x-axis represents the intervals of $\mathrm{cv}[\rho]$ and $\mathrm{cv}[\lambda]$ where the regression model was developed.

Fig. 11 shows that, as discussed before, the production system is able to accommodate a certain degree of uncertainty in the volume and mix. Having noted that, we highlight that the system is more robust to variability in demand mix than to that in demand volume. To make the reduction in net profit, $\overline{NP}$, lower than 10% (in relation to the minimum coefficient of variation), the following conditions apply: $\mathrm{cv}[\rho] < 8.4\%$ for the demand volume, $\mathrm{cv}[\lambda] < 11.8\%$ for the mix. Similarly, to ensure that $\overline{NP} > 0$, $\mathrm{cv}[\rho] < 16.0\%$, while $\mathrm{cv}[\lambda] < 21.2\%$. However, the downward slope of the mix curve is stronger; notice that $\mathrm{cv}[\rho] = 30\%$ results in $\overline{NP} \approx -\$529$, while $\mathrm{cv}[\lambda] = 30\%$ leads to $\overline{NP} \approx -\$747$. In simple terms, this suggests that when both coefficients of variation are moderate, reducing uncertainty in the demand volume should be prioritised, as the system is more robust to variability in the mix ratio; while when they are both high, reducing uncertainty in the demand mix yields more benefits.

Now we focus on the trade-off analysis by considering the costs derived from taking initiatives aimed at reducing demand uncertainty. Specifically, we consider two different cost models: (i) a linear model in which costs are linearly related to the degree of reduction in the coefficient of variation, $C(\mathrm{cv}[\rho]) = \omega_l^v(\mathrm{cv}[\rho] - \mathrm{cv}[\rho_i])$ for the demand volume and $C(\mathrm{cv}[\lambda]) = \omega_l^m(\mathrm{cv}[\lambda] - \mathrm{cv}[\lambda_i])$ for the demand mix; and (ii) a quadratic model in which costs are proportional to the square of the reduction in the coefficient of variation, $C(\mathrm{cv}[\rho]) = \omega_q^v(\mathrm{cv}[\rho] - \mathrm{cv}[\rho_i])^2$ for the volume and $C(\mathrm{cv}[\lambda]) = \omega_q^m(\mathrm{cv}[\lambda] - \mathrm{cv}[\lambda_i])^2$ for the mix. In the equations, ω_i^j represents the unit costs, where i refers to the mathematical nature of the model ($i = l$ for linear, $i = q$ for quadratic), and j considers the dimension of demand uncertainty ($j = v$ for volume, $j = m$ for mix). In addition, $\mathrm{cv}[\rho_i]$ and $\mathrm{cv}[\lambda_i]$ refer to the initial state of variability in the system.

[10] From this perspective, the concept of 'smoothing replenishment rules' (see e.g. [93]) has become popular over the last decade to emphasize the benefits derived from avoiding the use of inventory policies that contribute to the (demand) variability amplification phenomenon in nowadays supply chains, which are often characterised by a high number of echelons and long lead times.

[11] When demand is stochastic, combining multiple markets allows for a reduction in aggregate demand uncertainty (see e.g. [83]).

[12] The automotive industry provides a good example where the subscription model is currently attracting many manufacturers. Since 2017, many brands have introduced such offers, including Volvo (Care by Volvo), BMW (Access by BMW), and Porsche (Porsche Passport). One of the key advantages of subscription models is that they reduce demand uncertainty, and thus generate a consistent revenue stream.

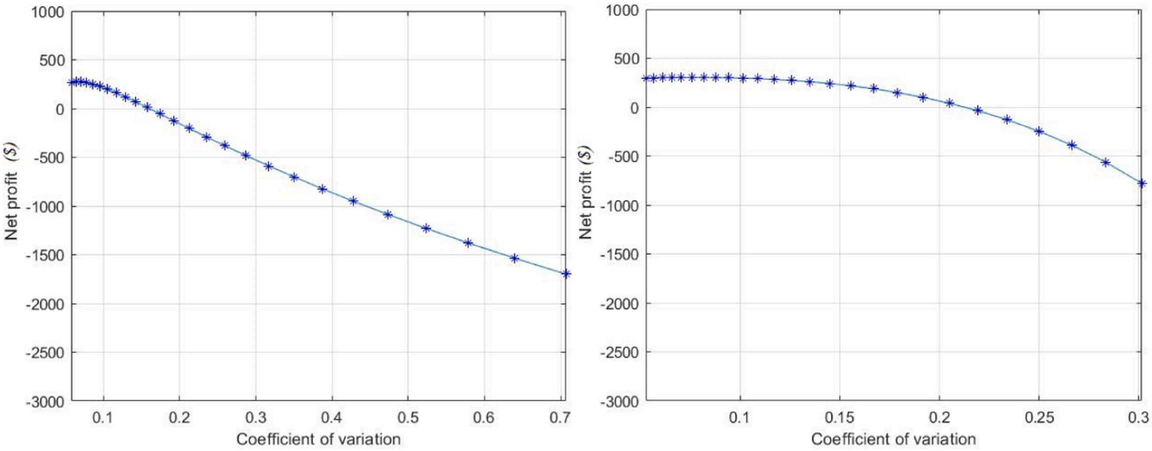

Fig. 11. Net profit as a function of the coefficient of variation of the demand volume (a) and mix ratio (b).

For the sake of simplicity, in the subsequent analysis, we will assume that the initial state corresponds to the upper limit of the interval under consideration, i.e. cv$[\rho_i]$ = 70.71%, cv$[\lambda_i]$ = 30.15%.

In the linear model, the cost of reducing variability is always the same; in the quadratic one, small reductions of demand variability can be achieved at a relatively low cost, while larger reductions are significantly more expensive. Considering the economic efforts

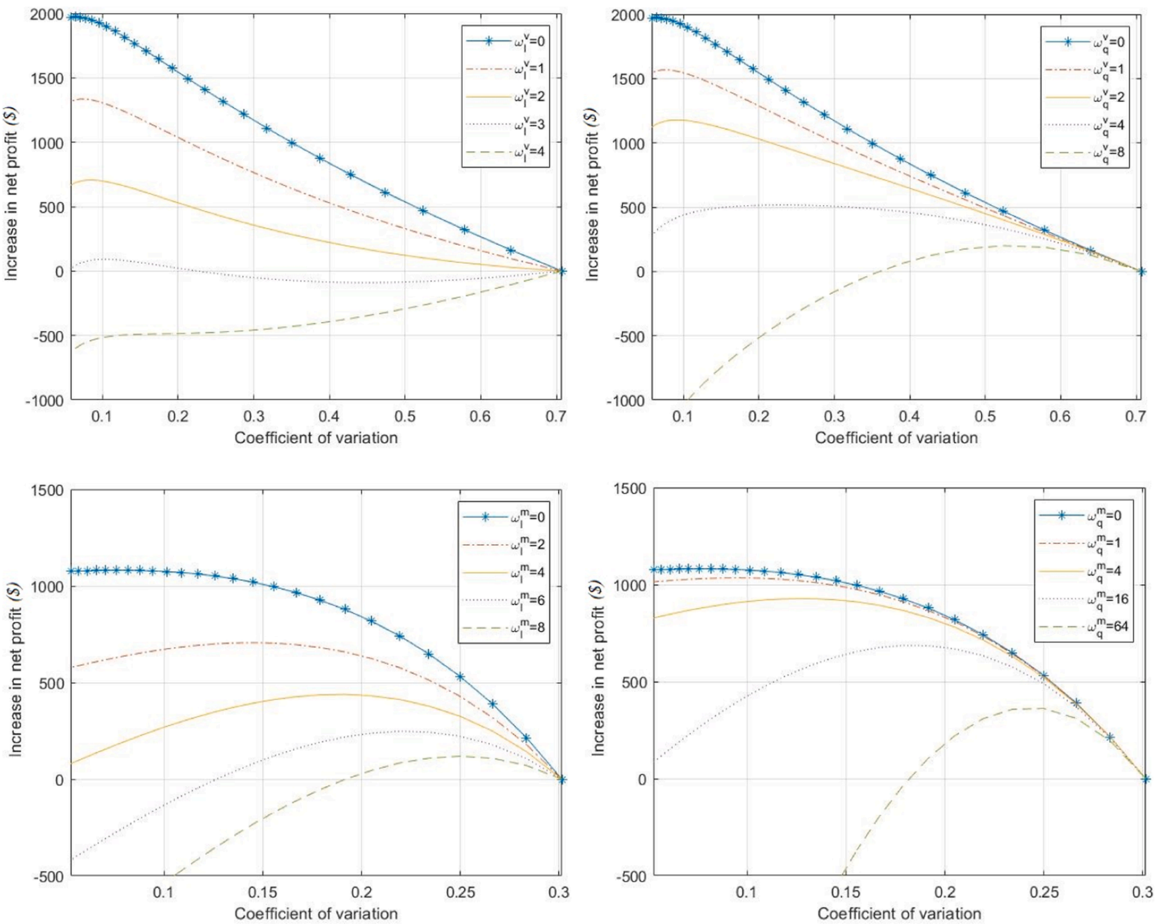

Fig. 12. Net profit increase induced by reducing demand volume uncertainty (top graphs) and demand mix uncertainty (bottom graphs) in the linear (left) and quadratic (right) cost models. [Note: ω_l^m and ω_q^m are expressed in thousands of dollars.].

made to reduce uncertainty through these models allows us to re-calculate the net profit, with the aim of gaining additional managerial insights. From this perspective, Fig. 12 plots the increase in the net profit (in relation to the initial state) provoked by reducing the coefficient of variation of the demand volume (top graphs) and mix (bottom graphs) under the linear (left) and the quadratic (right) cost models. Note that the graphs with $\omega_i^j = 0$ represent the baseline system where the costs of variability reduction are not considered.

First, we focus on the demand volume. For the linear model (top-left graph of Fig. 12), for high values of the unit cost ω_l^v (specifically, $\omega_l^v \geq 4$), the production system should not incur variability-reduction costs, as this would reduce the net profit. However, for lower values of the unit cost, $\omega_l^v \leq 3$, the system should undertake variability-reduction initiatives with the aim of reducing the coefficient of variation to very low values, i.e. $\text{cv}[\rho] < 10\%$. This duality —the optimal solution is either $\text{cv}[\rho] = \text{cv}[\rho_i]$ or $\text{cv}[\rho] < 10\%$, depending on the unit cost$---$ is interesting and can be explained by the linear-like relationship between $\overline{NP}$ and $\text{cv}[\rho]$ in Fig. 11, except for very low values of $\text{cv}[\rho]$. For this reason, this duality does not occur in the quadratic model where intermediate values are $\text{cv}[\rho]$ are generally the optimal decision; see the top-right graph of Fig. 12. Inspection of the curves shows that the coefficient of variation that maximises the net profit, $\text{cv}[\rho]^*$, increases as ω_q^v grows.

Now we consider uncertainty in the demand mix by looking at the bottom graphs of Fig. 12. Here we can also see that there is significant scope for improvement derived from implementing variability-reduction processes, even when they entail a high cost. Also, we can see that the twofold nature of the economically-optimal coefficient of variation, $\text{cv}[\lambda]^*$, does not occur here either in the linear or in the quadratic cost model. Indeed, in both models, $\text{cv}[\lambda]^*$ grows as ω_l^m/ω_q^m increases. Finally, it is interesting to look at these graphs in comparison with those for the demand volume. This allows us to observe that when the unit costs of variability reduction, ω_i^j, are low, reducing volume uncertainty should be prioritised. For example, the scope for improvement in the $\overline{NP}$ curve is significantly higher when $\omega_q^v = 1$ (top-right graph) than when $\omega_q^m = 1$ (bottom-right graph), and similar conclusions hold for the analysis of the linear cost models (top- and bottom-left graphs). However, when the reduction of demand variability is very costly (i.e. the unit costs ω_i^j are high), reducing mix uncertainty is often more profitable. Note that $\omega_q^m = 16$ results in a maximum increase in $\overline{NP}$ of \$686 (bottom-right graph), while $\omega_q^v = 8$ only allows for an economic improvement of \$199 (top-right graph). The same occurs in the linear system, where indeed variability-reduction initiatives become prohibitively expensive from $\omega_l^v = 4$ (top-left graph), but would be reasonable for $\omega_l^m = 8$ (bottom-left graph).

5. Conclusions and next steps

Optimisation-based approaches are prevalent in the literature on product-mix problems. These studies provide solution strategies that support organisational decision-making, but they often ignore the sources of uncertainty affecting the system and the related issues of production planning and inventory control. From this perspective, we suggest that simulation-based works are needed to complement optimisation studies. We claim that this approach may enable a better understanding of the effectiveness of solution strategies in real-world, uncertain contexts, which would facilitate the design of mechanisms for appropriately managing flexible production systems.

In this work, we considered two dimensions of demand uncertainty that typically emerge in product-mix settings: the total volume of demand across the family of products that share the same production resources and the mix of demand amongst said products. Integrating simulation and statistical techniques, we quantified the impact of demand uncertainties on the economic performance of the production system when it operates according to a Drum-Buffer-Rope mechanism. In this fashion, we provided regression equations that link the net profit to the relevant parameters in the statistical distributions that define the volume and mix of the demand.

Interestingly, we observed that the Drum-Buffer-Rope scheduling mechanism effectively accommodates a certain degree of variability. This is able to preserve the mean net profit up to a coefficient of variation of approximately 10%; specifically, around 8% for the demand volume and 12% for the demand mix, which allowed us to conclude that the production system is more robust to uncertainties in the mix of demand. As we move away from these thresholds, the net profit is strongly penalised. That is, our analysis clearly shows how the net profit decreases as uncertainty grows, thus compromising the results that can be obtained under the deterministic assumption and highlighting the need to explore the behaviour of such solutions in stochastic contexts. We found that volume uncertainty provokes a profit reduction due to the combined effect of the (decreased) throughput and the (increased) operating expense; however, mix uncertainty reduces the profit mainly as a result of a decrease in the throughput.

Having noted the potential benefits derived from demand uncertainty reduction, we explored the key trade-off between those benefits and the cost of initiatives aimed at reducing such uncertainty, including the implementation of mechanisms to enable information sharing in the supply chain and some marketing actions. By analysing two different cost models, we showed that companies should pursue an optimal level of demand variability, as removing all variability (when possible) nearly always becomes economically inefficient. As a rule, we concluded that, when variability reduction is affordable, mitigating volume uncertainty should be the priority; however, when variability reduction is prohibitively expensive, efforts should be concentrated on reducing mix uncertainty as much as possible.

We highlight that our study was conducted by modelling the production system via Petri nets and agent-based techniques, capturing the relevant characteristics of the stochastic product-mix problem under study. In this sense, we developed a model-driven decision support tool for our production system, which was described in great detail. Not only does this facilitate the replicability of our work, but it also offers practitioners a comprehensive framework to approach product-mix problems in practice via agent-based techniques. In this sense, our study may also encourage key decision-makers in production systems to develop powerful prototypes for dealing with complex real-world problems. This is particularly interesting in the implementation of I4.0 technologies, given that advanced modelling techniques would facilitate the instrumentation of advanced characteristics of I4.0 (such as self-optimization, self-control, networking capability, or decision-making ability) that are necessary to achieve higher levels of efficiency, flexibility and resilience in the emerging production systems.

Finally, we note that the literature that explores product-mix problems under uncertainty is still scarce due to their analytical and computational complexity. In this sense, a more solid understanding of the 'bridge' between product-mix decision making and financial performance in stochastic scenarios is required. There are two major next steps that we would like to pursue in this area. First, we plan to enhance the belief-desire-intention (BDI) model of the agents in our system; see Rao and Georgeff [86] and Gelaim et al. [87]. We aim to make them able to predict, evaluate options, and decide in order to implement in the production system the most appropriate counter-measure to variability. This would generate an adaptive behaviour that would elevate the value of the agent-based simulator and would strengthen its I4.0 capabilities.

Second, we will extend our model to include other important sources of uncertainty that are common in the real world but frequently ignored in research studies. Together with the external sources considered in this work, we plan to include internal uncertainties. This can be done by following Nakajima's categorisation of equipment-based losses [88], which is well-aligned with product-mix problems. This approach identifies three main factors, which may be interpreted as different types of losses: (i) availability loss; (ii) performance loss; and (iii) quality loss. Understanding their economic implications in flexible production systems that face product-mix problems is a research avenue worth exploring. To this end, it would be necessary to expand the model by modelling and implementing more advanced, stochastic Petri nets, with transitions that fire after probabilistic delays that are determined by random variables.

Declaration of competing interests

The authors wish to confirm that there are no known conflicts of interest associated with this research work.

Data availability

Data will be made available on request.

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Supplementary materials

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