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DYNAMIC RESPONSE OF LAMINATED GLASS ELEMENTS IN TIME DOMAIN

Manuel Aenlle-López[1], F. Pelayo[2], Natalia García-Fernández[3], Miguel. Muniz-Calvente[4], M.J. Lamela-Rey[5].

[1] *Professor, University of Oviedo, aenlle@uniovi.es*
[2] Associate Professor, University of Oviedo, fernandezpelayo@uniovi.es
[3] PhD Student, University of Oviedo, garciafnatalia@uniovi.es
[4] Associate Professor, University of Oviedo, munizcmiguel@uniovi.es
[5] Associate Professor, University of Oviedo, mjesuslr@uniovi.es

ABSTRACT

Laminated glass consists of two or more layers of monolithic glass and one or more interlayers of a polymeric material, the polyvinyl butyral (PVB) being the most used interlayer material. In the last years, the concept of effective thickness was proposed to simplify the calculation of these elements subject to static loadings, which consists of using a monolithic model with mechanical properties equal to those of the laminated element. However, when laminated glass is subject to dynamic loadings, the mechanical properties of the monolithic model have to be defined time and temperature dependent, which complicates the use of this technique in numerical models.

In this paper, expressions for obtaining the modal parameters of a laminated glass beam using a monolithic model are presented. On the other hand, a time domain effective stress thickness is defined and used to predict the dynamic response of laminated glass beams The proposed techniques are, firstly, validated comparing the analytical response of a laminated glass beam with the numerical results obtained with numerical model assembled in ABAQUS. Secondly, the dynamic responses are estimated and compared with experimental tests carried out on a laminated glass beam, where the modal parameters were estimated with operational modal analysis.

Keywords: Viscoelasticity, Dynamic Behaviour, Laminated Glass Elements.

1. INTRODUCTION

Laminated glass is a layered material that consists of two or more plies of monolithic glass and one or more polymeric interlayer material subject to high pressure and temperature in autoclave. All polymeric interlayers present a viscoelastic behavior, i.e. their mechanical properties are time and temperature dependent [1, 2]. In analytical and numerical models, glass mechanical behavior is usually modelled as linear-elastic whereas the PVB is commonly considered as linear-viscoelastic [3, 4, 5].

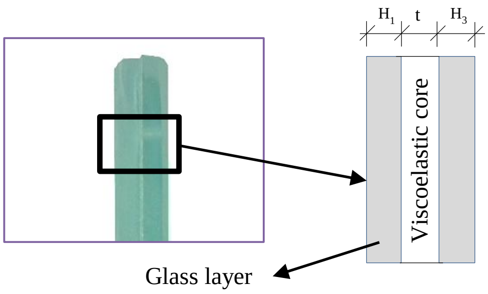


Figure 1: Example of laminated glass in sandwich configuration.

In order to obtain the dynamic behaviour of laminated glass element, a finite element analysis can be assembled. However, these methods are high time consuming because the viscoelasticity (time and temperature dependence) of the interlayers cannot be considered with modal superposition techniques.

In the last years, the concept of effective thickness has been proposed to estimate deflections and stresses in laminated glass beams using simplified monolithic models which thickness is time and temperature dependent [3, 4, 5]

In this work, expressions and techniques to predict the dynamic response of laminated glass beams are presented. The technique combines the modal parameters and a linear elastic monolithic model. The method is validated by analytical and numerical simulation as well as experimentally.

2. BASIC THEORY

2.1. Modal Parameters of laminated glass beams

The natural frequencies ω_{mon} of a monolithic beam with stiffness EI_{mon} and constant mass per unit length are given by:

$$\omega_{mon}^2 = k_I^4 \frac{EI_{mon}}{m_{mon}} \tag{1}$$

where the wavenumber k_I is constant for each mode. The stiffness EI_{mon} is expressed as:

$$EI_{mon} = (EbH_{TOT}^3)/12 \tag{2}$$

with b being the width of the beam, H_{TOT} the total thickness and E the Young's modulus.

The mass per unit length can be calculated with:

$$m_{mon} = \rho_{mon} H_{TOT} \tag{3}$$

where ρ_{mon} is the mass-density

The natural frequencies and loss factors of a laminated glass beam can be obtained from the expression [6-9]:

$$\omega^2(1 + j\eta) = \omega_{mon} \frac{\rho_{mon}}{\rho} \frac{E_{eff}^*(\omega, T)}{E} \tag{4}$$

Where the term $E^*_{eff}(\omega, T)$is an effective frequency domain stiffness [10]:

$$E^*_{eff}(\omega, T) = \frac{E}{1+Y}\left(1 + \frac{Y}{1 + \frac{EH_1tH_2k_I^2(\omega, T)}{G_t^*(\omega, T)(H_1+H_2)}}\right) \tag{5}$$

A description of the parameters in Eq. (5) are presented in Appendix.

With respect de mode shapes, it has been experimentally demonstrated [11] that there are not discrepancies between the mode shapes of monolithic beam and a laminated glass beam with same geometry and boundary conditions, i.e:

$$\psi \cong \psi_{mon} \tag{6}$$

where ψ and ψ_{mon} are mode shapes normalized to the largest component equal to unity.

2.2. Dynamic response in Time Domain

In structural dynamics, the responses of the system can be decomposed in terms of modal coordinates using the mode superposition method, i.e. [12]:

$$w(x,t) = \sum_{i=1}^{Nmodes} \phi_i(x) q_i(t) \tag{7}$$

Where $\phi_i(x)$ and $q_i(t, T)$ are the i-th mode shape and the i-th modal coordinate, respectively. For a laminated glass beam, the maximun stresses in the glass layers can be obtained with the following expressions [7, 13]:

$$\sigma_1(x,t,T) \cong \frac{H_1}{2} \cdot \sum_{i=1}^{Nmodes} E_{1\sigma i}(T) \cdot \phi_i''(x) \cdot q_i(t,T) \tag{8}$$

Whereas the stresses at the bottom of layer 3 are obtained from:

$$\sigma_2(x,t,T) \cong \frac{H_2}{2} \cdot \sum_{i=1}^{Nmodes} E_{1\sigma i}(T) \cdot \phi_i''(x) \cdot q_i(t,T) \tag{9}$$

Where $E_{1\sigma i}(T)$ and $E_{2\sigma i}(T)$ are constant time domain stress effective Young modulus [7], which are dependent on the geometry and the mechanical properties of the glass and the interlayer [14]. These time domain stress effective Young modulus can be obtained with the equations:

$$E_{1\sigma i}(T) = E_{1\sigma eff}(\omega_i, T) \tag{10}$$

$$E_{2\sigma i}(T) = E_{2\sigma eff}(\omega_i, T) \tag{11}$$

where ω_i is the natural frequency of the i-th mode, and $E_{1\sigma eff}(\omega_i, T)$ and $E_{2\sigma eff}(\omega_i, T)$ are frequency domain effective Young modulus whose expressions can be retreived from literature [9].

If the experimental modal parameters of the beam (natural frequencies, mode shapes and damping ratios) are known, i.e. by modal analysis, and the experimental response time histories $w(x,t)$ are measured at several points of the structure, the vector of experimental modal coordinates $\{q_x(t,T)\}$ can be estimated by:

$$\{q_x(t,T)\} = [\phi_x]^{-1}\{w_x(t,T)\} \tag{12}$$

Where subscript 'x' indicates experimental data, $[\phi_x(x)]^{-1}$represents the inverse matrix of the experimental mode shapes and $\{w_x(t,T)\}$ the vector of experimental responses. If Eq. (12) is substituted in Eqs. (8) and (9), the stresses at any point of the layer 1 can be obtained with the expression [13]:

$$\sigma_1(x,t,T) \cong \frac{H_1}{2} \cdot \sum_{i=1}^{Nmodes} E_{1\sigma i}(T) \cdot \phi''_{xpi} \cdot q_{xi}(t,T) \tag{13}$$

and at the bottom of layer 3 with:

$$\sigma_2(x,t,T) = \frac{H_2}{2} \cdot \sum_{i=1}^{Nmodes} E_{2\sigma i}(T) \cdot \phi''_{xpi} \cdot q_{xi}(t,T) \tag{14}$$

Where ϕ''_{xpi} are the experimental mode shapes expanded to the unmeasured DOF's using one of the techniques proposed in the literature [15].

3. EXAMPLES OF APPLICATION IN A LAMINATED GLASS BEAM

3.1. Modal parameter of a laminated glass beam

In order to validate the technique proposed in this paper, the modal parameters of a simply supported laminated glass beam, made of annealed glass layers, PVB core and with the following geometrical data: L = 1 m, H_1 = 4 mm, t = 0.76 mm, H_2 = 4 mm, b = 0.1 m, were predicted at 25^oC using Eq. (4) and validated with a numerical model assembled in ABAQUS.

As a first step, two numerical models were assembled using available mechanical properties for glass and PVB [14]. The details of the models used are:

- A simply supported monolithic glass model with thickness $H_{TOT} = H_1 + t + H_2 = 8.76\ mm$.. The beam was meshed using quadratic hexahedral elements (20 nodes per element) with an approximate size of 4 mm.

- A layered model with glass layers modelled as linear elastic and the PVB interlayer model as linear viscoelastic, was also meshed using 3 quadratic hexahedral elements through the beam thickness (one element for each material layer).

The natural frequencies ω_{mon}, corresponding to the first four bending modes of the monolithic model, were obtained solving the eigenvalue problem, and the results are presented in Table 1 for a temperature of 25ºC. The wavenumbers k_I were estimated from the monolithic model using Eq. (1).

The natural frequencies and the corresponding loss factors were estimated from the frequency response function (FRF), which was isolated around the peaks of resonance and taken to the time domain using the Inverse Discrete Fourier Transform (IDFT). The resonance frequency is obtained by determining the zero crossing times, and the damping by the logarithmic decrement of the corresponding free decay. The predicted natural frequencies and loss factors at 25ºC are shown in Tables 1, respectively.

Table 1. Modal parameters at 25º C.

Mode	Monolithic glass beam	Laminated glass beam			
	Nat. freq [Hz]	Nat. freq		Loss factor	
		Eq. (4)	FEM Visco	Eq. (4)	FEM Visco
1	21.31	21.60	21.62	0.0154	0.0157
2	85.24	85.86	84.95	0.0305	0.0306
3	191.72	186.49	186.81	0.0497	0.0488
4	340.48	323.81	324.50	0.0683	0.0715

From Table 1, it is concluded that the modal parameters have been predicted with a good accuracy. Although damping estimation in laminated glass panels using Eq. (13) are not accurate, in this particular case (simply supported beam) $k_R = 0$ for all the modes (which is the assumption considered for deriving Eq. (4)), which explains the large precision of the predictions.

3.2. Stresses estimation in a laminated glass beam under impact loading

3.2.1 Modal Analysis

A laminated glass beam was used in the experiments. The dimensions of the beam were 1000 mm x 100 x 6.38 mm, being the thicknesses of glass H_1 and H_2 three milimeters and the thickness of PVB interlayer t=0.38 mm. The total mass of the beam was 1.544 kg. The beam was clamped at both ends between rubbery bands in a glass standard impact frame [16] (see Figure 2).

The modal parameters of the beam were experimentally determined by operational modal analysis at 22 ºC. The responses of the beam were measured with 7 accelerometers with a sensitivity of 10 mV/g which were uniformly distributed along the beam and seven strain gages using a sampling frequency of 2132 Hz (see Figure 2), the test duration being approximately 2 minutes. The modal parameters were estimated in the frequency domain using the ARTEMIS Modal Pro software. The singular value decomposition of the experimental responses is presented in Figure 3. The first five experimental natural frequencies and the corresponding damping ratios are presented in Table 2. The mode shapes for the beam are those presented in Table 2.

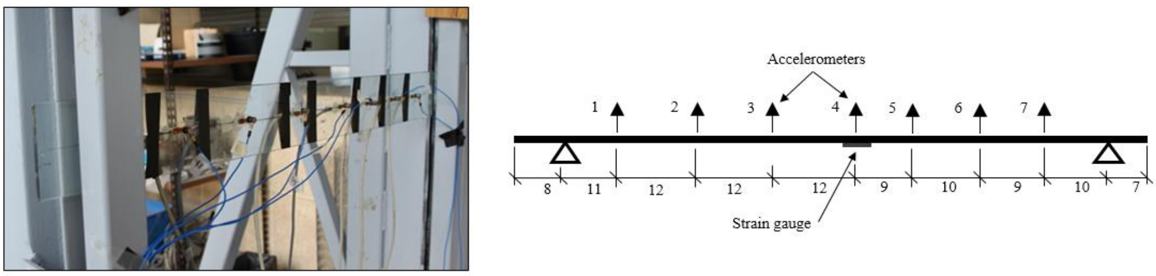


Figure 2. Experimental set-up for the laminated glass beam (distances in cm).

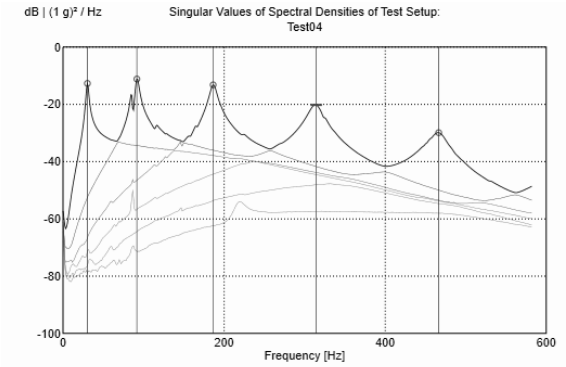
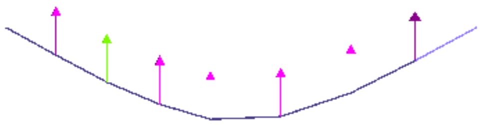
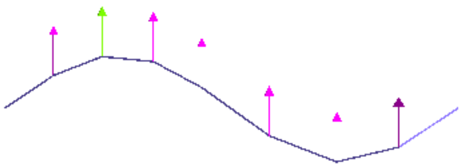
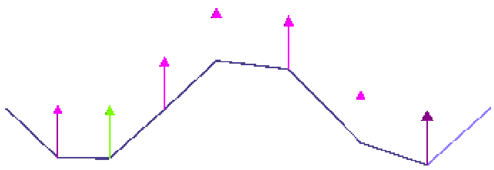
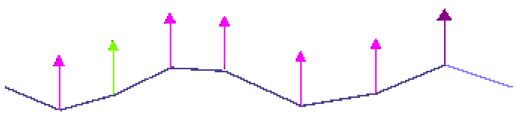
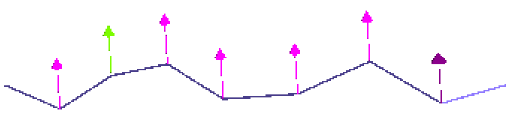


Figure 3. Singular value decomposition of the spectral densities for the OMA in the beam.

3.2.2 Impact tests

The beam was also subjected to an impact test using an impact hammer with a medium stiffness head and the additional mass of 75 grams. The response of the beam was measured with the same configuration used in modal analysis tests (Fig. 2).

Table 2. Modal parameters of the laminated glass beam.

Mode	Natural frequencies [Hz]	Exp. Damp. [%]	Experimental mode Shape
1	30.16	4.28	
2	92.70	3.23	
3	186.49	2.92	
4	313.41	2.89	
5	465.53	2.82	

3.2.3 Analytical Predictions

From the experimental responses and the experimental mode shapes, the modal coordinates were estimated using Eq. (12). Then, the experimental mode shapes were expanded using a monolithic 1D finite element model of the beam assembled in ABAQUS. In order to take into account the effect the rubbery at the supports, the finite element model was updated with elastic fixed supports.

The stresses at the midpoint of the bottom of layer H_2 , predicted with Eq. (14) is presented in Fig (4) together with those obtained with the strain gage at the same point. The effective Young modulus $E_{2\sigma i}(T)$ at temperature $T = 22^oC$ was calculated with Eq. (11) [7].
It can be observed that the calculated stresses are predicted with an error less than 6 % demonstrating that a reasonable good accuracy can be obtained with this technique.

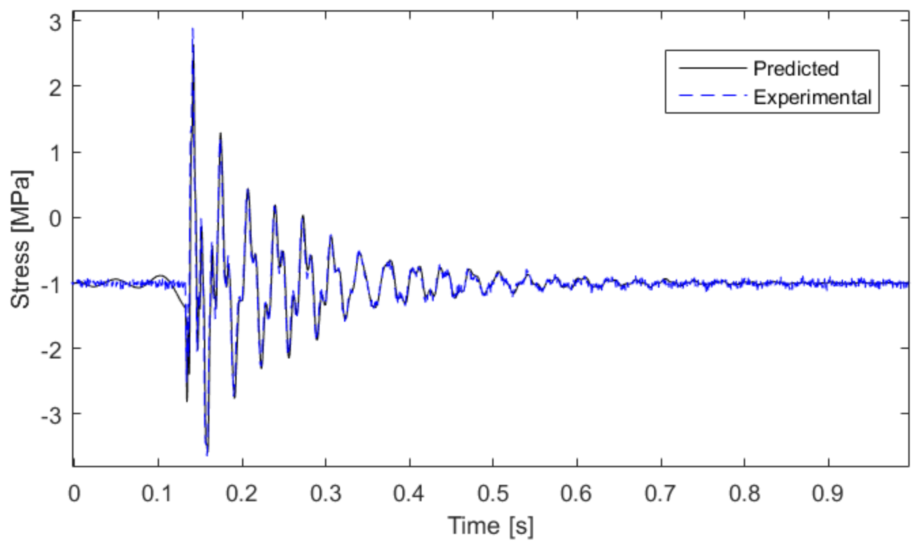


Figure 4. Predicted and experimental stresses for a laminated glass beam under soft impact loading (mid-point).

4. CONCLUSIONS

- In this paper, a methodology to predict the dynamic response of laminated glass elements using a linear-elastic monolithic model, has been proposed and validated.
- The method for obtaining the dynamic responses was also validated comparing the stresses estimated on a laminated glass subjected to an impact loading. The errors between the two models are less than 6%, which demonstrates that the technique can be used to predict with a good accuracy the dynamic response of laminated glass elements.

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Appendix

List of Parameters and Symbols

$$H_0 = t + \left(\frac{H_1 + H_2}{2}\right)$$

$$Y = \frac{H_0^2 E_1 H_1 E_2 H_2}{EI_T (E_1 H_1 + E_2 H_2)}$$

$$I_T = I_1 + I_2 = \frac{H_1^3 + H_2^3}{12}$$

$$I_1 = \frac{H_1^3}{12}$$

$$I_2 = \frac{H_2^3}{12}$$

E	Young's modulus of glass layers
ρ	Mass-density of the glass layers.
$G_t^*(\omega)$	Complex shear modulus for the polymeric interlayer
$k_I^2(\omega, T)$	Wavenumber of the beam