MISO for NOMA in Small-Cell Scenarios

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***Abstract*—Non-Orthogonal Multiple Access (NOMA) is a promising technique for enhancing spectral efficiency in multiuser wireless communications. However, its application comes with certain limitations. For example, in a scenario involving two users, their channel conditions must be significantly different, which is not a common occurrence in small cell environments. In this paper, we investigate the use of beamforming in a Multiple-Input-Single-Output (MISO) system to address this constraint through channel shaping. We derive the problem formulation and apply an optimization algorithm to jointly find the power allocation and beamforming weights that maximize the geometric mean of the rates. Performance evaluations and a comparison with a Time-Division-Multiple-Access (TDMA) solution demonstrate the effectiveness of the proposed method in achieving high rates for both users while maintaining rate fairness.**

I. Introduction

The study of Non-Orthogonal Multiple Access (NOMA) has recently garnered significant interest due to its potential to enhance spectral efficiency in wireless communications. In fact, it has been identified as a key component in the future of 5G networks [1]. NOMA relies on the principles of Superposition Coding (SC) and Successive Interference Cancellation (SIC) to serve multiple users simultaneously within the same time, frequency, and code slot.

NOMA performance can be improved by incorporating spatial diversity in Multiple-Input Multiple-Output (MIMO) systems. The joint optimization of power allocation and beamforming is a complex problem that has been the subject of recent research [2], [3]. Another challenge related to NOMA is user grouping (or pairing for two users). The successful performance of SIC, and consequently NOMA, necessitates that users are grouped in such a way that their channel conditions are significantly different, i.e., one strong user is paired with one weak user [4]. This constraint poses a limitation on the applicability of NOMA, as there are scenarios, such as picocells or femtocells, where meeting this condition might not be possible.

In this paper, we investigate the application of beamforming in a two-user Multiple-Input-Single-Output (MISO) downlink NOMA system to address the limitations of Single-Input Single-Output (SISO) NOMA when both users have similar channel conditions. Although this scenario has been previously explored for NOMA in [5], [6], both studies assume that one user has priority and thus a higher Quality of Service (QoS) than the other. This assumption leads to solving the beamforming problem by maximizing focus for the primary user, with power allocation serving as a degree of freedom for system adjustment. In contrast, our proposed method considers the general case where users cannot be classified based on their QoS and rate fairness is a priority.

Notation: h, $\mathbf{h}$ and $\mathbf{H}$ denote a scalar, column vector and matrix, respectively. $(\cdot)^T$ and $(\cdot)^H$ denote transpose and conjugate transpose (or hermitian), respectively. $|\cdot|$ signifies the absolute value, $\|\cdot\|$ refers to the $\ell 2$-norm and $\mathbb{E}\{\cdot\}$ to the expected value. A three-dimensional position vector in cartesian coordinates is represented as $\vec{r}$. The abbreviation s.t. stands for *subject to*.

II. System Model and Optimization

A. System Model

We consider a MISO NOMA scenario, where a Base Station (BS) equipped with N antennas serves 2 users denoted as user 1 and user 2, respectively. To this end, the superposed signal of both users is transmitted by the N antennas multiplied by a complex weight factor. The received signal at each user y_i can be expressed as

$$\begin{cases} y_1 = \sqrt{p_1} g_1 x_1 + \sqrt{p_2} g_1 x_2 + n_1 \\ y_2 = \sqrt{p_2} g_2 x_2 + \sqrt{p_1} g_2 x_1 + n_2 \end{cases} \tag{1}$$

where p_i is the allocated power to the i-th user and x_i is its intended transmitted symbol, $\mathbb{E}\{|x_1|^2\} = \mathbb{E}\{|x_2|^2\} = 1$. The term n_i accounts for the additive white Gaussian noise with zero mean and variance $\sigma_{n_i}^2$, $n_i \sim \mathcal{CN}\left(0, \sigma_{n_i}^2\right)$. We denote g_i as the effective gain, which includes channel propagation losses and beamforming gain at the position of the i-th user

$$g_i = \left| \sum_{n=1}^{N} w_n h_{i,n} \right|^2 = \left| \mathbf{w}^T \cdot \mathbf{h}_i \right|^2 \tag{2}$$

where the vector $\mathbf{w}$ contains the weights, $w_n \in \mathbb{C}$, each applied to the n-th element of the array, and $\mathbf{h}_i$ is the vector containing the complex channel coefficients between each element of the array and the i-th user, $h_{i,n}$. For the considered scenario, the assertion that both users experience similar channel conditions implies that $\|\mathbf{h}_1\| \approx \|\mathbf{h}_2\|$.

User 1, which is the one performing SIC in our notation, will first decode the signal intended for user 2 and then subtract it from its received signal. The successful decoding of x_2 at user 1 its subjected to the condition $\mathbb{E}\{\sqrt{p_2} g_1 x_2\} >> \mathbb{E}\{\sqrt{p_1} g_1 x_1\} + \mathbb{E}\{n_1\}$, where the uncorrelation between signal and noise has been considered. In the interference-limited case of $\sqrt{p_1} g_1 >> \sigma_{n_1}^2$, this implies $p_2 >> p_1$. We define $p_2 = \gamma p_1$ and refer to γ as the power allocation factor. To ensure proper SIC performance, we assume a system constraint where the ratio of the powers p_2 and p_1 is greater than a minimum threshold $\gamma_{\min} > 1$. This minimum threshold depends on the chosen modulation and system realization [7], [8]. Additionally, the system is constrained by the power budget, given by, $p_{max} = p_1 + p_2$. This results in:

$$p_1 = \frac{p_{\max}}{1+\gamma} = \frac{p_2}{\gamma}\,. \tag{3}$$

Assuming perfect SIC, and the transmission of proper Gaussian signals, the rate achieved by each user corresponds to the channel capacity, and can be expressed as [2]

$$\begin{cases} R_1 = \log_2\left(1 + \dfrac{p_1 g_1}{\sigma_{n_1}^2}\right) \\ R_2 = \log_2\left(1 + \dfrac{p_2 g_2}{\sigma_{n_2}^2 + p_1 g_2}\right). \end{cases} \tag{4}$$

It is important to note that the constraint $\frac{p_2}{p_1} > \gamma_{\min} > 1$ compromises rate fairness, as user 2 is allocated a higher power than user 1, even when both users experience similar channel conditions. Nonetheless, adjusting the g_i terms offers an extra degree of freedom to balance the power disparity. By setting $g_1 > g_2$, the system can yield a higher focus at user 1's location, effectively compensating for the difference in power allocation.

B. Optimization Problem

In a scenario characterized by N, $\mathbf{h}_1$, $\mathbf{h}_2$, $\sigma_{n_1}^2$, $\sigma_{n_2}^2$, our objective is to find the optimal set of weights, $\mathbf{w}_{\text{opt}}$, and power allocation factor, γ_{opt}, that maximize a goal function. To enhance rate fairness, we choose to maximize the geometric mean of the rates, $R = \sqrt{R_1 R_2}$ [3], which is equivalent to maximize its product. The optimization constraints, $|\mathbf{w}|^2 \leq 1$ and $\gamma \geq \gamma_{\min}$, ensure that the total power is maintained and correct SIC performance is achieved, respectively. As a result, the optimization problem can be formulated as:

$$\begin{aligned} \max_{\gamma, \mathbf{w}} \quad & R = \sqrt{R_1 R_2} \\ \text{s.t.} \quad & \gamma \geq \gamma_{\min} \\ & \|\mathbf{w}\|^2 \leq 1 \end{aligned} \tag{5}$$

which results in a non-convex optimization problem with non-linear constraints. To address this issue, we utilize the Sequential Quadratic Programming (SQP) algorithm [9] as our solution approach.

III. Performance Evaluation

In this section, we present the simulation results for an antenna array consisting of 8×8 isotropic elements, $N = 64$, spaced at a distance of 0.63λ, and centered at the origin $\vec{r} = (0, 0, 0)$. The array is arranged on the XY plane. We considered the positions of users at $\vec{r}_1 = (48, 0, 96)\lambda$ and $\vec{r}_2 = (-120, 0, 216)\lambda$, respectively. The norms of the corresponding channel vectors, calculated assuming free-space conditions, are $\|\mathbf{h}_1\| = 5.9e^{-3}$ and $\|\mathbf{h}_2\| = 5.1e^{-3}$. By default, we set $\sigma_{n_1}^2 = \sigma_{n_2}^2 = \sigma_n^2$; a Signal-to-Noise-Ratio (SNR) of 80 dB, calculated as $10\log_{10}\left(\frac{p_{max}}{\sigma_n^2}\right)$; and $\gamma_{min} = 20$. To evaluate the performance of the proposed NOMA method, we compare it with the same beamforming algorithm (excluding power allocation) applied to TDMA. In that case, both users are served half of the time with maximum power p_{max}.

Fig. 1 displays the resulting beamforming gain, g_i, normalized with respect to the non-beamforming case, in which an isotropic element at the origin would transmit all the power. In the TDMA case, the optimization produces similar gain levels for both users, while in the NOMA case, user 1 receives a higher gain to compensate for the power allocation disparity.

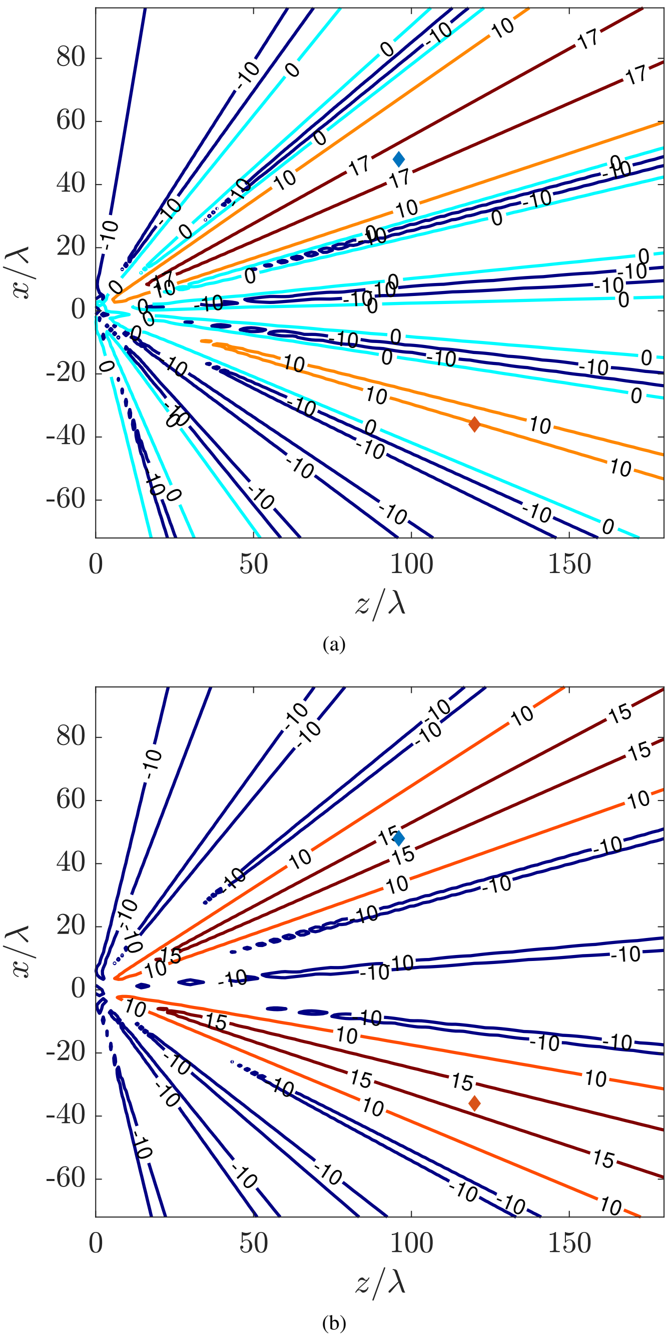

Fig. 1. Beamforming gain (dB) in the XZ plane normalized with respect the non-beamforming case. (a) NOMA, (b) TDMA. User 1 and user 2 are represented by blue and red diamonds, respectively.

Table I presents obtained values of user rates and effective gains, as well as the optimal power allocation factor for the NOMA case. Note that the proposed method results in higher rates than beamforming TDMA for both users. Additionally, despite the optimization algorithm yielding $\gamma_{opt} = 48.33$, which implies that user 2 is allocated $\frac{p_2}{p_1} = 48.33$ times more power under similar channel conditions, the obtained R_1 and R_2 are similar. The power allocation disparity is successfully compensated in the NOMA case through both beamforming, since $\frac{g_1}{g_2} = 8.1$, and SIC.

TABLE I
USER CAPACITIES, R_i (BPS/HZ); GOAL RATE, R (BPS/HZ); AND EFFECTIVE GAINS, g_i FOR BOTH NOMA AND TDMA.

	R	R_1	R_2	g_1	g_2	γ_{opt}
NOMA	5.74	6.04	5.46	$3.21e^{-5}$	$3.97e^{-6}$	48.33
TDMA	5.35	5.44	5.25	$1.91e^{-5}$	$1.45e^{-5}$	–

IV. CONCLUSION

In this paper, we have investigated the application of MISO in a two-user NOMA scenario, specifically focusing on cases where both users experience similar channel conditions, as is common in small cell environments. We show how our proposed beamforming strategy effectively addresses the rate fairness limitations typically associated with NOMA in such scenarios.

First, we derived the problem formulation and the optimization problem, aiming to maximize the rate product to prevent one user from being allocated most of the resources. The optimization leads to a non-convex problem with non-linear constraints, which we address using the SQP algorithm. Performance evaluations and comparisons with TDMA demonstrate that MISO NOMA can achieve higher rates for both users, even in scenarios that are unfavorable for SISO NOMA.

Future work includes examining a broader range of scenarios and use cases, and investigating more complex models for NOMA (e.g., considering the effects of imperfect SIC) and the antenna array. Additionally, efforts will be made to enhance the efficiency and robustness of the optimization algorithm.

ACKNOWLEDGEMENTS

This work was supported in part by the Ministerio de Ciencia e Innovación and the Spanish Research Agency (AEI) under the project ENHANCE-5G (PID2020-114172RB-C21 / AEI / 10.13039/501100011033), and in part by the Gobierno del Principado de Asturias/FEDER under Project GRUPIN-IDI-2021-000097 and "Severo Ochoa" Program Grant no. BP21-116.

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