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Dualidad holográfica, teoría de defectos y agujeros negros
Holographic duality, theory of defects and black holes

Tesis doctoral presentada por Cristian Risco González
Dirigida por la Dra. Yolanda Lozano Gómez

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RESUMEN DEL CONTENIDO DE TESIS DOCTORAL

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Nombre: Cristian Risco González	DNI/Pasaporte/NIE:
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RESUMEN (en español)

Esta tesis está enfocada en el estudio de la correspondencia AdS/CFT. En particular, su meta principal consiste en proveer nuevas evidencias que apoyen los casos en dimensiones bajas: AdS_2/SCQM y AdS_3/CFT_2, los cuales no se comprenden completamente todavía.

Para alcanzar esta meta, construimos nuevas soluciones AdS_2 y AdS_3 de las supergravedades de Tipo II, ya que no se ha obtenido aún una clasificación completa. Esto se consiguió mediante la aplicación de un conjunto diverso de técnicas y conceptos: dualidades de cuerdas (por ejemplo, dualidad-T abeliana y no abeliana), estructura-G, soluciones de branas, etc. Tras esto, construimos las teorías de campos de \textit{quivers} que viven en la configuración de branas subyacente; las branas son objetos no perturbativos que aparecen en teoría de supercuerdas. Estas configuraciones de branas proveen una noción de dualidad entre la solución de supergravedad y la teoría de campos. En nuestro caso, se espera que esta última fluya en el IR a una teoría conforme de campos (CFT). El último paso consiste en poner a prueba esta hipótesis calculando un observable llamado carga central de la teoría de campos y ver si coincide con aquella de la solución de supergravedad en el IR.

Esta tesis está dividida en dos partes distintas. En la parte I introducimos los ingredientes y conceptos principales que será clave para entender la tesis. Están relacionados con temas tales como teoría de cuerdas, supergravedad y holografía. La parte II está dedicada a la presentación de los resultados originales de esta tesis.

RESUMEN (en Inglés)

This thesis is focused on the study of the AdS/CFT correspondence. In particular, its main goal is to provide new evidence supporting the low-dimensional AdS_2/SCQM and AdS_3/CFT_2 cases, which are not fully understood yet.

In order to achieve this goal, we built new AdS_2 and AdS_3 solutions to Type II supergravities, as a complete classification has not been obtained yet. This was done by applying a diverse set of techniques and concepts: string dualities (e.g. Abelian and non-Abelian T-duality), G-structure, brane solutions, etc. We then built quiver field theories living in the underlying brane set-up, branes being non-perturbative objects that appear in superstring theory. These brane set-ups provide a notion of duality between the supergravity solution and the field theory. In our case the latter one is expected to flow in

the IR to a conformal field theory (CFT). The final step is to trial this hypothesis by computing an observable called the central charge of the field theory and see if it coincides with that of the supergravity solution in the IR.

The thesis is divided in two distinct parts. In part I we introduce the main ingredients and concepts that are key to understand the thesis. They are related to such topics as string theory, supergravity and holography. Part II is devoted to presenting the original results of the thesis.

SR. PRESIDENTE DE LA COMISIÓN ACADÉMICA DEL PROGRAMA DE DOCTORADO EN _________________

A mis padres

"Life before death. Strength before weakness. Journey before destination."

—Brandon Sanderson, The Way of Kings

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Contents

II Original Work

Introduction

One of the most sought-after goals in modern theoretical physics is to come up with a simple and elegant theory that describes all physical phenomena, i.e. a Theory Of Everything. The most seriously considered candidate for a Theory Of Everything since the nineties is no other than string theory. However, the expectations and interest surrounding it have notably waned due to its multiple issues (lack of experimental evidence, an order of 10^{500} vacua and no way to tell the "real" one apart from the others...). Nevertheless, its importance as driving force for the development of theoretical physics can't be denied, as we review below. This fact along with its influence in our original work, which deals with supergravity solutions and the holographic duality, should be enough motivation to justify the following historical summary of the born and development of string theory and supergravity, as its low-energy limit. This introduction will also serve the purpose of presenting some of the key concepts that will be further discussed through the thesis.

String theory was born in the sixties in the context of strong interactions, when a quantum theory describing them was yet to be found. As a result of Veneziano's work on hadronic amplitudes [1], it was realised that the underlying theory contained an infinite number of massive particles in its spectrum, unlike any known QFT at the moment.

In order to better explain this spectrum Nambu [2] and Goto [3] developed in 1970 the model we now call the bosonic string. Around a year after Nambu's original paper, Ramond [4] and, independently, Neveu and Schwarz [5] proposed the superstring action. This new action incorporated fermions to the bosonic strings and displayed supersymmetry related to the interchanging of both kinds of fields. However, various theoretical obstacles appear when it comes to using one of these string theories to describe the strong nuclear force:

1. Consistency at the quantum level fixes the spacetime dimension D. More specifically, $D = 26$ for the bosonic string and $D = 10$ for the superstring. In both cases, we seem to obtain unrealistic spacetimes, far from our 4d universe.

2. The spectrum of particles is fixed. Apart from the infinite tower of massive string modes, it also contains a massless spin 2 particle and a tachyon (a particle with imaginary mass). The latter one present both theoretical and experimental problems as its presence violates causality and such behaviour lacks any empirical confirmation.

In the 70s, string theory was discarded as a theory for the strong interaction. The two main reasons that justify this are the new experimental high-energy results, which didn't exactly fit the Virasoro amplitudes, and the formulation of QCD, which was regarded as a superior candidate for the description of this fundamental force. However, this doesn't mean that string theory fell into oblivion. On the contrary, the interpretation of the aforementioned spin 2 particle as a graviton led to the conclusion that string theory was actually a theory of quantum gravity.

In 1976, Gliozzi, Scherk and Olive proposed the so-called GSO projection [6], which removed the tachyon from the spectrum and imposed spacetime supersymmetry. It is important to remark that this procedure applies to superstring theories, but not to the bosonic string due to its absence of supersymmetry. In the seventies, in parallel to the development of string theory, the notions of supersymmetry and its gauged version, supergravity, were also researched in detail.

In 1981 Polyakov used the path integral formulation to quantise string theory [7]. He used a linear action which was equivalent at the classical level to the non-linear one presented by Nambu and Goto [2, 3]. In the early years of that decade three different superstring theories were formulated. Two of them are theories of closed superstrings preserving $\mathcal{N} = 2$ supersymmetry in ten dimensions, known as Type IIA and Type IIB. Their main difference is that Type IIA presents two spinors of opposite chirality, i.e. $\mathcal{N} = (1, 1)$ supersymmetry, while Type IIB is a chiral theory with $\mathcal{N} = (2, 0)$ supersymmetry. The remaining one is an $\mathcal{N} = 1$ theory of open and closed strings which also presents gauge symmetry called Type I. In 1984, the proof by Green and Schwarz that the theories were anomaly-free [8] triggered what we now know as the first superstring revolution.

In 1985, two $\mathcal{N} = 1$ closed string theories where formulated by Gross, Harvey, Martinec and Rohm [9], which were called Heterotic strings. These string theories are built by considering that the left- and right-moving modes are completely decoupled in such a way that the former ones behave like $D = 26$ bosonic strings while the latter ones are treated as $D = 10$ superstrings. These theories also display gauge symmetry. However, the modular invariance of their one-loop partition functions restricts the possible gauge groups to either $\mathrm{SO}(32)$ or $\mathrm{E}_8 \times \mathrm{E}_8$, hence the two Heterotic theories. These two groups are also the only possible ones for which the conditions for anomaly cancellation are satisfied. Both Type I and Heterotic string theories represent a clear advantage when compared with the closed string theories. Type II string theories lacked a way of obtaining a non-Abelian group at the perturbative level, meaning that the $\mathrm{SU}(3)\times\mathrm{SU}(2)\times\mathrm{U}(1)$ gauge group that underlies the Standard Model could not be recovered. This is no longer a problem for the Type I and Heterotic string theories due to the intrinsic presence of gauge groups, being $\mathrm{E}_8 \times \mathrm{E}_8$ the one that better fulfils this purpose.

In order to avoid the remaining obstacle of superstring theory, namely the high spacetime dimension, we have to reduce from 10d to 4d. This is usually achieved via a Kaluza-Klein dimensional compactification. For this purpose, we consider that the spacetime manifold can be written as $\mathcal{M}_4 \times K_6$, where $\mathcal{M}_4$ is a 4d spacetime and K_6 is a 6d compact manifold. Under the assumption that K_6 is small enough, we can impose that the physical fields do not depend on the corresponding coordinates and so the 10d and the 4d theories are related. The idea of increasing the spacetime dimension in order to attain a clearer interpretation of a theory was originally introduced by Kaluza and Klein [10, 11].

There is an important issue we must address: although we are looking for a unique theory, it doesn't seem the case. On one hand, there are five perfectly consistent superstring theories, but only one of them should be the right one. What makes it special? And why is that the one that actually describes our universe? On the other hand, the way in which one compactifies is not fixed. On the contrary, the number of string vacua (the so-called string landscape) is often estimated to be of order 10^{500} in the literature.

Important breakthroughs in this regard where made in the eighties when the concept of duality became popular. T-duality was first noticed by Kikkawa and Yamasaki in the context of toroidal string compactifications. They proved in 1984 that a string theory with a circle direction of radius R was related to a different one with radius $1/R$ [12]. The study of this duality, along with the works of other scientists like Sen [13], Schwarz [14], Vafa [15] and Witten [16] tracing connections between the superstring theories, was key to motivate the conjecture that stated that all 10d superstring theories are different descriptions of the same theory. This was also the birth of M-theory, which underlies the ten-dimensional superstring theories and whose low-energy limit is 11d supergravity. These events rekindled the interest in string theory, sparking the so-called second superstring revolution (1994-1995).

The next big step in the development of string theory was holography. In 1997 Maldacena conjectured that various anti-de Sitter (AdS) string theory backgrounds were related to dual conformal field theories (CFT) living in the boundary of the AdS [17]. A key concept to understand this duality are *branes*, which are non-perturbative string theoretical objects that generalise the concept of string and its magnetic dual, the NS5-brane. In particular, *Dirichlet branes* or D-branes are of special interest in the context of holography. They were first introduced in 1989 by Dai, Leigh and Polchinski, when they noticed that T-duality interchanges Neumann and Dirichlet boundary conditions for open strings [18]. The locus of Dirichlet boundary conditions was shown to be the dynamical and non-perturbative object we now call D-brane, or Dp-brane when the spacial dimension p needs to be emphasised. In the context of holography, Maldacena first stated his conjecture for D3-branes [17], based on which an AdS_5 solution of Type IIB supergravity was proposed to be holographically dual to four-dimensional CFT, namely 4d $\mathcal{N} = 4$ Super Yang-Mills. This is particularly relevant because it provided a new path to study a four-dimensional theory, moreover in the strongly-coupled regime. The exploration and applications of this so-called AdS/CFT conjecture has constituted a very active line of research in the last couple of decades. It is quite prominent its application to the study of strongly-coupled quantum systems (the quark-gluon plasma in QCD, phase transitions, etc). The AdS/CFT correspondence has been also quite useful in the resolution of the black hole information paradox: AdS black holes (those living in a universe with a negative cosmological constant) are holographically dual to a configuration of particles on the conformal boundary of Anti-de Sitter space, which allows to compute the entropy of the black hole by counting the number of microstates of this other system on the boundary.

The AdS_3 and AdS_2 cases and their holographic duals are of special interest because of the role they play as horizons of black objects in supergravity [19]. In spite of all the steps taken towards understanding the lower-dimensional AdS holography, we still lack a complete and consistent string theory description. On the supergravity side, the classification of AdS_2 and AdS_3 solutions is still under development. For instance, one can check [20–52] for a non-exhaustive list of references. The underlying brane description of these solutions, key to the study of their CFT interpretation, becomes more complicated as the dimensionality of the internal space increases, due to the richer structure of the possible geometries and fluxes. An interesting interpretation of these low-dimensional AdS backgrounds is as holographic duals to defect CFTs living in higher-dimensional ones (see, for example, [23, 27, 34, 43, 46, 47, 53–59]). These defect theories are associated to a system of *defect branes* localised within a bound state of *background branes*, where the higher-dimensional CFT lives. A clue that this interpretation may be possible is when a low-dimensional AdS solution flows asymptotically locally to a higher dimensional one, containing extra fluxes. In such cases the *holographic free energy* (or *central charge*) diverges, which is interpreted as the need for UV completion into higher dimensions.

The AdS_2/CFT_1 case is particularly challenging. The boundary of AdS_2 is not connected so it is not quite clear where the dual CFT should live. There is also an issue with the interpretation of the central charge of the dual superconformal quantum mechanics (SCQM). Namely, as the trace of the energy-stress tensor of a conformal field theory must vanish, this implies that the energy of any SCQM equals zero and there are no finite energy excitations [60–64]. Thus, applying the AdS_2/CFT_1 correspondence in the microscopic study of black holes is not straightforward at all and some alternatives have being explored.

This thesis is divided in two distinct parts. In part I, we review the basic concepts regarding string theory, supergravity and holography. The intention behind this first part is to make the thesis more or less self-contained, providing the reader with the necessary tools to understand the rest of the thesis. We start in chapter 1 by introducing the main concepts related to string theory: the string actions and their boundary conditions, the dimensional reduction technique, supergravity as the low-dimensional limit of string theory and the string dualities. Then in chapter 2 we introduce supergravity as a gauged version of supersymmetry, review the simplest solutions of Type IIA/B and eleven-dimensional supergravity and conclude by exploring the notion of G-structure and its relation to supersymmetry. In chapter 3, we explore the concept of holography: we start with the original AdS_5/CFT_4 correspondence and then give an additional example of the AdS_3/CFT_2 case, which is particularly interesting for us, as it is related to our research. Finally, we review the Hanany-Witten brane set-ups and the quiver field theories one can construct in them in chapter 4.

We then present the original work in part II, where we explain the most relevant results that appear in [65–68]. In general terms, we have contributed not only to the classification of AdS_2 and AdS_3 solutions to Type IIA/B supergravities, but also to the study of the AdS/CFT correspondence for these cases. We remark the importance of the AdS geometries as horizons of black holes and strings. The AdS_2 and AdS_3 vacua are particularly interesting, as they are known to describe the horizons of extremal 4d and 5d black holes, respectively. Holography then provides a way of counting the number of microstates of these black objects and, therefore, of computing their Bekenstein-Hawking entropy. As for the dual field theories, we have been interested in giving a defect interpretation when possible. Defects consist on insertions of operators in higher-dimensional field theories.

For an outline of our original results, one can consult chapter 5. Chapter 6 is devoted to the $\mathrm{AdS}_3/\mathrm{CFT}_2$ correspondence. In particular, in section 6.1 we explore a new class of AdS_3 solutions to massive Type IIA supergravity. Sections 6.2-6.4 are dedicated to presenting the dual field theories of different subclasses within the aforementioned family of solutions. On the other hand, chapter 7 is dedicated to our results on $\mathrm{AdS}_2/\mathrm{SCQM}$ holography. Here we present the new AdS_2 solutions to Type IIA/B supergravity we found, as well as their field theory interpretation whenever it was possible and relevant.

Part I

Bibliographical Review

Chapter 1

String Theory

In this first chapter, we provide a general introduction to string theory as well as a motivation to why we should consider strings as the fundamental objects that underlie all the elementary particles. Thus, in section 1.1 we present the most explored string actions and outline their most relevant features. Then we introduce the notion that string theory at low energies becomes supergravity in section 1.3. Section 1.4 is dedicated to the relations connecting the various string theories, also known as dualities.

The chapter itself is bibliographical in nature and heavily based on [69–74] so those are the sources we recommend for further details on the topic at hand.

1.1. String dynamics

All through this section, we will consider a D-dimensional manifold of signature $(D-1,1)$ and denote its metric by g_{MN}. A classical point particle describes a worldline in spacetime when we let it evolve in time. Thus, the action describing the dynamics of the point particle is naturally proportional to the length of the worldline,

$$S_{\text{particle}} = m \int_{\tau_0}^{\tau_1} d\tau \sqrt{-g_{MN}(\tau, X^K(\tau))\dot{X}^M(\tau)\dot{X}^N(\tau)}\,, \tag{1.1}$$

where m is the mass of the particle, τ its proper time, $X^M(\tau)$ its spacetime coordinates and $\dot{X}^M(\tau)$ its proper velocity.

When we go one step further and substitute the particle with a one-dimensional string, we observe that it sweeps a two-dimensional surface in spacetime called *worldsheet*. We will denote it by Σ and its coordinates by $\sigma^i = (\tau, \sigma)$. Analogously to the particle case, the path chosen by the string should be extremal in the area of the worldsheet. Such dynamics are encapsulated by the Nambu-Goto action [2,3],

$$S_{NG} = T \int_\Sigma d^2\sigma \sqrt{|\det(g_{MN}(\tau, X^K(\sigma^k))\partial_i X^M(\sigma^l)\partial_j X^N(\sigma^m))|}\,, \tag{1.2}$$

where the string tension T makes the action dimensionless. For reasons that will be discussed later and to distinguish between different cases, we call this one-dimensional object a *bosonic* string. As the $X^M(\sigma^i)$ depend on the worldsheet coordinates, they can be interpreted as a set of D dynamical scalar fields living on it. The spacetime parametrised by these fields is usually referred to as *target space*.

We also have the Polyakov action, which is completely equivalent to (1.2), but it is easier to quantise as the velocity is not enclosed within a square root,

$$S_P = \frac{T}{2}\int_\Sigma d^2\sigma\sqrt{|h|}h^{ij}\partial_i X^M \partial_j X^N g_{MN}\,. \tag{1.3}$$

We have denoted by h_{ij} the worldsheet metric and h the determinant of said metric. The metric on the worldsheet h_{ij} is an independent but non-dynamical variable and, as we will see, it can be completely gauged away with the aid of the symmetries of (1.3). One of these symmetries is invariance under local two-dimensional reparametrisation, i.e. under

$$(\tau,\sigma) \to f(\tau,\sigma) = (f_1(\tau,\sigma), f_2(\tau,\sigma))\,, \tag{1.4}$$

where f is a diffeomorphism. The action (1.3) is also invariant under general transformations of the D-dimensional spacetime coordinates. It is also symmetric under Weyl transformations given by

$$h_{ij} \to e^{\Lambda(\sigma^i)} h_{ij}\,. \tag{1.5}$$

This invariance can be justified as follows. Let h_{ij} be the metric of an n-dimensional manifold, then under the transformation (1.5) we have that

$$\sqrt{-h} \to \sqrt{-e^{n\Lambda(\sigma^i)}h} = e^{\frac{n}{2}\Lambda(\sigma^i)}\sqrt{-h}\,, \quad h^{ij} \to e^{-\Lambda(\sigma^i)}h_{ij} \tag{1.6}$$

and, therefore

$$\sqrt{-h}h^{ij} \to \exp\Big\{\Big(\frac{n}{2}-1\Big)\Lambda(\sigma^i)\Big\}\sqrt{-h}\,, \tag{1.7}$$

where the exponential equals 1 if and only if $n=2$.

The stress-energy tensor is proportional to the variational derivative of (1.3) with respect to the metric,

$$T_{ij} = \frac{2}{T}\frac{1}{\sqrt{|h|}}\frac{\delta S}{\delta h^{ij}} = \partial_i X^M \partial_j X_M - \frac{1}{2}h_{ij}h^{kl}\partial_k X^M \partial_l X_M\,. \tag{1.8}$$

The infinitesimal version of the Weyl invariance implies that, if we consider a variation of the two-dimensional metric of the form

$$\delta h_{ij} = v h_{ij}\,, \tag{1.9}$$

then the variation of the action vanishes,

$$0 = \delta S_P = \int \frac{\delta S_P}{\delta h^{ij}}\delta h^{ij} = \int \frac{\delta S_P}{\delta h^{ij}} v h^{ij} \overset{(1.8)}{=} -\frac{T}{2}\int \sqrt{|h|}vT^i{}_i\,. \tag{1.10}$$

This implies that the stress-energy tensor is traceless, even without the requirement of the equations of motion. In the case we do consider a solution of said equations, the corresponding stress-energy tensor must be null as the equation of motion for the metric is $\delta S/\delta h^{ij} = 0$. Therefore, the vanishing of (1.8) tells us that

$$\partial_i X^M \partial_j X^N = \frac{1}{2} h_{ij} h^{kl} \partial_k X^M \partial_l X^N \tag{1.11}$$

and taking the square root of the determinant of both sides of this matrix equation, we have

$$\sqrt{|\det(\partial_i X^M \partial_j X^N)|} = \frac{1}{2}\sqrt{|h|} h^{kl} \partial_k X^M \partial_l X^N \,, \tag{1.12}$$

which proves the equivalence between the actions (1.2) and (1.3).

Now let us use the invariance under (1.4) and (1.5) to choose an appropriate gauge which simplifies the action (1.3). First, the reparametrisation invariance lets us consider the following local expression for the metric,

$$h_{ij} = \Omega(\sigma^i)\eta_{ij} \,. \tag{1.13}$$

Then we can also drop the $\Omega(\sigma^i)$ factor due to the Weyl invariance. For obvious reasons, this choice of the metric receives the name of *conformal gauge*. The resulting action is that of a free string,

$$S_P = \frac{T}{2}\int_\Sigma d^2\sigma\, \eta^{ij} \partial_i X^M \partial_j X_M \,. \tag{1.14}$$

In this gauge, the equations of motion for the string boil down to the following,

$$(\partial_\tau^2 - \partial_\sigma^2) X^M = 0 \,. \tag{1.15}$$

The general solution of the 2d wave equation can be written in terms of two arbitrary functions,

$$X^M(\tau,\sigma) = X_-^M(\tau-\sigma) + X_+^M(\tau+\sigma) \,, \tag{1.16}$$

where $X_-^M(\tau-\sigma)$ and $X_+^M(\tau+\sigma)$ are the right- and left-moving modes of the string, respectively. In order to determine these functions, we must impose boundary conditions on the endpoints of the string. In particular, we must differentiate between two topological inequivalent cases: open and closed strings.

The closed string is a loop and, as such, has no free ends. It must satisfy a periodicity condition for consistency,

$$X^M(\tau,\sigma+2\pi) = X^M(\tau,\sigma) \,. \tag{1.17}$$

The most general solution of (1.15) compatible with (1.17) is described by the following modes,

$$\begin{aligned} X_-^M(\tau-\sigma) =& \frac{x^M}{2} + \frac{p_-^M}{4\pi T}(\tau-\sigma) + \frac{i}{\sqrt{4\pi T}}\sum_{n\neq 0}\frac{\alpha_n^M}{n} e^{-in(\tau-\sigma)} \,, \\ X_+^M(\tau+\sigma) =& \frac{x^M}{2} + \frac{p_+^M}{4\pi T}(\tau+\sigma) + \frac{i}{\sqrt{4\pi T}}\sum_{n\neq 0}\frac{\tilde{\alpha}_n^M}{n} e^{-in(\tau+\sigma)} \,. \end{aligned} \tag{1.18}$$

We have that α_k^M and $\tilde{\alpha}_n^M$ are arbitrary Fourier coefficients which modulate the different oscillation modes. We observe that condition (1.17) imposes that

$$p_-^M = p_+^M \equiv p^M \,. \tag{1.19}$$

Thus, x^M and $p_\pm^M$ can be interpreted as the centre of mass of the string and its linear momentum. Besides, the reality of $X^M(\tau,\sigma)$ implies that the Fourier coefficients are constrained by the following conditions,

$$(\alpha_n^M)^* = \alpha_{-n}^M \quad \text{and} \quad (\tilde{\alpha}_n^M)^* = \tilde{\alpha}_{-n}^M \,. \tag{1.20}$$

Let us now consider the case of open strings. The Fourier expansion given by (1.16) and (1.18) is still valid, but now we take the space coordinate to satisfy $\sigma \in [0,\pi]$. Nevertheless, the periodicity constrain (1.17) is not present this time so we must look for another reasonable boundary condition. Let us consider the evolution of the string between an initial time τ_i and a final time τ_f. This can be achieved by varying (1.14) and taking $\delta X^M = 0$ at τ_i and τ_f,

$$\begin{aligned}\delta S_P =& T\int_{\tau_i}^{\tau_f} d\tau \int_0^\pi d\sigma\, \eta^{ij}\partial_i X^M \partial_j \delta X_M = \\ =& -T\int_{\tau_i}^{\tau_f} d\tau \int_0^\pi d\sigma\, \delta X^M \eta^{ij}\partial_i\partial_j X_M + T\int_{\tau_i}^{\tau_f} d\tau \int_0^\pi d\sigma\, \partial_i[\eta^{ij}\delta X^M \partial_j X_M] \,.\end{aligned} \tag{1.21}$$

The first term of the last line gives rise to (1.15), while the second one is a total derivative term and we want it to vanish. If we expand and integrate it, we arrive at the following,

$$\begin{aligned} &T\int_{\tau_i}^{\tau_f} d\tau \int_0^\pi d\sigma\, \partial_i[\eta^{ij}\delta X^M \partial_j X_M] = \\ =& -T\left[\int_0^\pi d\sigma\, X^M \partial_\tau \delta X_M\right]_{\tau_i}^{\tau_f} + T\left[\int_{\tau_i}^{\tau_f} d\tau\, \delta X^M \partial_\sigma X_M\right]_0^\pi \,.\end{aligned} \tag{1.22}$$

Of these two terms we obtained, the first is the typical one that appears when the equations of motion are computed via the principle of least action. It vanishes due to the fixing of the endpoints, $\delta X^M = 0$, at τ_i and τ_f. Nevertheless, the other term is not trivially equal to zero, but imposes a new constraint. Consequently, we will consider the following condition,

$$\delta X^M \partial_\sigma X_M = 0 \quad \text{at} \quad \sigma = 0\,,\ \pi \,. \tag{1.23}$$

There are two ways of meeting this equation:

- Neumann boundary conditions impose the vanishing of the normal derivative of the endpoints of the string,

  $$\partial_\sigma X_M = 0 \quad \text{at} \quad \sigma = 0\,,\ \pi \,. \tag{1.24}$$

  As δX^M is unconstrained under these boundary conditions, the endpoints of the string move freely.

- Dirichlet boundary conditions, on the other hand, fix the endpoints of the string,

$$\delta X^M = 0 \quad \text{at} \quad \sigma = 0\,, \pi \iff X^M = C^M \quad \text{at} \quad \sigma = 0\,,\ \pi \tag{1.25}$$

  for a certain constant vector C^M. The fact that C^M is the same for both ends of the string is a consequence of the Fourier expansions (1.16) and (1.18).

The Dirichlet boundary conditions may seem a bit strange at first glance as there is, in principle, no reason why the endpoints of the string should be fixed in space. What are they fixed to? In order to shed some light on this conundrum, let us impose Neumann boundary conditions to $p+1$ coordinates including the time direction X^0 and Dirichlet to the remaining $D-p-1$. This constrains the endpoints of the string to lie on a $(p+1)$-dimensional manifold in spacetime, which is called a *Dirichlet brane* or D-brane. When one wants to specify the number of tangent space dimensions p, we refer to them as Dp-branes instead. This new kind of objects turns out to be dynamical and highly relevant both in non-perturbative string theory and as supergravity solutions.

The Fourier expansion of the right- and left-moving modes is once again described by (1.18), but now we have different constraints. The Neumann boundary conditions (1.24) impose that

$$p_+^M = p_-^M \equiv p^M\,, \quad \alpha_n^M = \tilde{\alpha}_n^M\,, \tag{1.26}$$

while the Dirichlet boundary conditions (1.25) imply the following restrictions,

$$x^M = C^M\,, \quad p_+^M = -p_-^M\,, \quad \alpha_n^M = -\tilde{\alpha}_n^M\,. \tag{1.27}$$

For both boundary conditions, we only have a set of Fourier coefficients, for instance α_n^M, as the other set is fully determined by the boundary conditions. Thus, for the Neumann and Dirichlet boundary conditions the Fourier expansion in target space turns out to be, respectively,

$$\begin{aligned} X_N^M(\tau,\sigma) =& x^M + \frac{p^M}{2\pi T}\tau + \frac{i}{\sqrt{\pi T}}\sum_{n\neq 0}\frac{\alpha_n^M}{n}e^{-in\tau}\cos(n\sigma)\,, \\ X_D^M(\tau,\sigma) =& C^M - \frac{1}{\sqrt{\pi T}}\sum_{n\neq 0}\frac{\alpha_n^M}{n}e^{-in\tau}\sin(n\sigma)\,. \end{aligned} \tag{1.28}$$

The next step in this discussion would be to promote the classical string theory to a quantum one. By doing this, we could compute the string spectrum, scattering amplitudes, etc. However, we will skip those details as this thesis is supergravity-oriented, meaning that none of this more quantum-related aspects of string theory will be necessary to understand the original results presented in part II. Nonetheless, we must take into account that the bosonic string can only be consistently quantised in $D = 26$ dimensions. A quick justification of this fact is that Lorentz invariance requires all the string states to be massless and that only happens at that dimension. This is quite remarkable as the spacetime dimension is a free parameter of most theories (classical or quantum). It also raises the question of how this theory can be related to the 4d universe we live in.

We observe that the spectrum of the quantum bosonic string suffers of two critical problems. The first one is that it contains a tachyon, which renders the vacua unstable. The other one is that it lacks fermions, which should appear in any realistic description of our world. A way of hitting these two birds with the same stone is by considering a supersymmetric extension of the bosonic string theory. The simplest way in which this can be achieved is by adding free fermions as additional internal degrees of freedom that propagate along the string. We are left with the decision of whether these fermions are Dirac or Majorana. One interesting option consists on taking D Majorana fermions ψ^M transforming in the vectors representation of $\mathrm{SO}(D-1,1)$. Applying these considerations to the Polyakov action in the conformal gauge (1.14), one obtains a new action,

$$S = \frac{T}{2}\int_\Sigma d^2\sigma \left[\partial^i X^M \partial_i X_M - i\bar{\psi}^M \rho^i \partial_i \psi_M\right] , \tag{1.29}$$

where ρ^i are the two-dimensional Dirac matrices and $\bar{\psi} = \psi^\dagger \rho^0$. A convenient choice of basis is depicted below,

$$\rho^0 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} , \quad \rho^1 = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix} \tag{1.30}$$

and satisfy the anti commutation relation

$$\{\rho^i, \rho^j\} = -2\eta^{ij} . \tag{1.31}$$

In this basis, the spinors are decomposed in the left- and right-moving fermionic modes,

$$\psi^M = \begin{pmatrix} \psi_+^M \\ \psi_-^M \end{pmatrix} . \tag{1.32}$$

The Dirac matrices have being chosen to be imaginary so that the Dirac operator $i\rho^i\partial_i$ is real. Thus, it seems reasonable to impose that the $\psi_\pm^M$ are real-valued, i.e. the spinor is Majorana.

The action (1.29) is invariant under the following infinitesimal transformations,

$$\delta X^M = \bar{\epsilon}\psi^M , \quad \delta\psi^M = -i\rho^i \partial_i X^M \epsilon , \tag{1.33}$$

where ϵ is a constant Majorana spinor. As (1.33) interchanges the bosonic and fermionic coordinates, we say they are *supersymmetry* transformations. Because of this, the object underlying (1.29) receives the name of *superstring*. We observe that if ϵ is not a constant in (1.33), then the action is not left invariant, but its variation takes the following form,

$$\delta S = T\int_\Sigma d^2\sigma (\partial_i \bar{\epsilon}) J^i + \text{boundary term} , \tag{1.34}$$

where J^i is the Noether current associated to the local supersymmetry transformation, known as *supercurrent*,

$$J^i = \partial_j X^M \rho^j \rho^i \psi_M . \tag{1.35}$$

As we have written the action (1.29) in the conformal gauge, some restrictions must be satisfied. Apart from the conservation of the supercurrent, the stress-energy tensor must vanish. The symmetrised version of said tensor is given below,

$$\begin{aligned} T_{ij} =&\partial_i X^M \partial_j X_M - \frac{i}{2}\bar{\psi}^M \rho_i \partial_j \psi_M - \frac{i}{2}\bar{\psi}^M \rho_j \partial_i \psi_M - \frac{1}{2}\eta_{ij}\partial_k X^M \partial^k X_M + \\ &+ \frac{i}{2}\eta_{ij}\bar{\psi}^M \rho^k \partial_k \psi_M \,, \end{aligned} \tag{1.36}$$

which is traceless, as in the bosonic case.

As for the equations of motion, we get two decoupled PDEs. The one for the bosonic fields is still (1.15), while the fermionic one reads

$$\rho^i \partial_i \psi^M = 0 \tag{1.37}$$

or, for the components (1.32) in the chosen basis, we have

$$(\partial_\sigma \pm \partial_\tau)\psi^M_\mp = 0\,. \tag{1.38}$$

If we consider light cone coordinates

$$\sigma^\pm = \tau \pm \sigma \quad \text{and} \quad \partial_\pm = \partial_\tau \pm \partial_\sigma\,, \tag{1.39}$$

we can rewrite equations (1.15) and (1.38) in a much more enlightening way,

$$\partial_\pm(\partial_\mp X^M) = \partial_\pm \psi^M_\mp = 0\,. \tag{1.40}$$

We observe that $\partial_\pm X^M$ and $\psi^M_\pm$ are functions of $\sigma^\pm$ alone. Supersymmetry thus relates these two solutions of the same equation.

The boundary conditions for the fermionic modes can be obtained in an analogous way to the bosonic case. For the open string, they come once again from the vanishing of the surface term that appears when the action is varied. Knowing that $\sigma \in [0\,,\ \pi]$, we have the following

$$\bar{\psi}^M \rho^1 \delta\psi_M = 0 \quad \text{at} \quad \sigma = 0\,,\ \pi \tag{1.41}$$

or, in components,

$$\psi^M_+ \delta\psi_{+M} - \psi^M_- \delta\psi_{-M} = 0 \quad \text{at} \quad \sigma = 0\,,\ \pi\,. \tag{1.42}$$

This last condition is satisfied if $\psi^M_+ = \pm\psi^M_-$ and $\delta\psi^M_+ = \pm\delta\psi^M_-$ at each end. As the relative sign between ψ_- and ψ_+ is a matter of convention (not physics), we can set $\psi^M_+(\tau, 0) = \psi^M_-(\tau, 0)$. Nevertheless, the boundary condition at the other end now becomes relevant as a relative sign between both ends changes the solution. Thus, we have two different sets of boundary conditions, which are the ones below:

- We say that we are considering Ramond (R) boundary conditions when we impose $\psi^M_+(\tau, \pi) = \psi^M_-(\tau, \pi)$. The mode expansion for (1.38) takes the following form,

$$\psi^M_\pm(\tau, \sigma) = \frac{1}{\sqrt{2\pi T}} \sum_{n=-\infty}^{\infty} a^M_n e^{-in(\tau \pm \sigma)}\,. \tag{1.43}$$

- If we take $\psi_+^M(\tau,\pi) = -\psi_-^M(\tau,\pi)$, we say we are considering Neveu-Schwarz (NS) boundary conditions. Now the solution reads

$$\psi_\pm^M(\tau,\sigma) = \frac{1}{\sqrt{2\pi T}} \sum_{n=-\infty}^{\infty} \tilde{a}_n^M e^{-i(n+1/2)(\tau\pm\sigma)} \,. \tag{1.44}$$

Upon quantisation, the spinor solutions satisfying Ramond (Neveu-Schwarz) boundary conditions give rise to fermionic (bosonic) string modes in D-dimensional spacetime.

On the other hand, for closed strings, we can take $\sigma \in [0\,,\ 2\pi]$ and consider either periodic or antiperiodic boundary conditions:

- Periodic or Ramond (R) boundary conditions $\psi_\pm^M(\tau,0) = \psi_\pm^M(\tau,\pi)$ result in the following solution

$$\psi_\pm^M(\tau,\sigma) = \frac{1}{\sqrt{2\pi T}} \sum_{n=-\infty}^{\infty} b_n^M e^{-n(\tau\pm\sigma)} \,. \tag{1.45}$$

- Antiperiodic or Neveu-Schwarz (NS) boundary conditions $\psi_\pm^M(\tau,0) = -\psi_\pm^M(\tau,\pi)$ give rise to this other solution,

$$\psi_\pm^M(\tau,\sigma) = \frac{1}{\sqrt{2\pi T}} \sum_{n=-\infty}^{\infty} \tilde{b}_n^M e^{-i(n+1/2)(\tau\pm\sigma)} \,. \tag{1.46}$$

We observe that in the closed string case, the boundary condition satisfied by ψ_+^M is independent from that met by ψ_-^M and there are two options for both cases. Therefore, we end up with four combinations of boundary conditions, namely R-R, R-NS, NS-R, NS-NS. The first and last of these conditions correspond to bosonic string states, while the other ones manifest as fermionic.

For consistency at the quantum level, the spacetime dimension must be $D = 10$. This is closer to our 4d world than the $D = 26$ we have in the purely bosonic case, although there are still six extra dimensions. As we will further explain in the next section, the first step to relate ten-dimensional string theory with lower-dimensional ones is to use a dimensional reduction tool called the Kaluza-Klein compactification. Another advantage with respect to the bosonic case is that, although tachyons are still present in the spectrum, we can remove them using the so-called GSO projection [6]. This is a very specific truncation of the spectrum which provides 10d supersymmetry.

We conclude the section by noticing that worldsheet supersymmetry induces supersymmetry in the D-dimensional spacetime. In particular, open strings display $\mathcal{N} = 1$ supersymmetry in the D-dimensional sense and closed strings, $\mathcal{N} = 2$. The way in which the supersymmetry is realised gives rise to five (and no more) consistent superstring theories. The massless bosonic modes of these theories include the 10-dimensional metric g_{MN} and a scalar Φ called the dilaton. These fields are part of the so-called NSNS sector, as the associated fields come from closed strings satisfying double antiperiodic boundary conditions (1.46). In all superstring theories except for Type I, this sector also includes a gauge potential given by a 2-form B_2. Each theory also presents a specific extra set of massless bosonic fields. The particularities of each theory are discussed below,

- Type I is an $\mathcal{N} = 1$ theory of unoriented open strings. It also contains closed strings as they can be the result of the interaction of two open ones. The strings can be charged under a certain massless Yang-Mills field, whose group is fixed to be SO(32) for anomaly cancellation. The boundary conditions for open strings reduce the supersymmetry from $\mathcal{N} = 2$ to $\mathcal{N} = 1$, which corresponds to 16 preserved supercharges. Apart form the metric and dilaton, this theory contains a massless gauge potential in the adjoint representation of the SO(32) group as well as a 2-form gauge potential C_2.

- Type II string theories only contain oriented closed strings and present $\mathcal{N} = 2$ supersymmetry, thus preserving 32 supercharges. There are two Type II string theories, called Type IIA and Type IIB. The main difference between them is that, while the supercharges in Type IIA are of opposite chirality, those of IIB are of the same chirality. The extra massless bosonic excitations give rise to the RR sector, corresponding to double periodic boundary conditions (see (1.43) and (1.45)). This sector consists of a set of p-form gauge potentials C_p, which combines with the B_2 to produce gauge-invariant objects called RR fluxes F_q. In the case of Type IIA, $p = 1, 3$, while for Type IIB, $p = 0, 2, 4$. In the latter case, we have an extra condition $F_5 = \star F_5$, where $\star$ is the Hodge dual in 10-dimensional spacetime. We remark that there is a deformation of Type IIA, called *massive* Type IIA string theory, which includes a so-called *Romans mass* m, which is a constant F_0.

- The heterotic string theories only contain closed strings. They are built on the fact that left- and right-moving modes evolve independently for closed strings. Thus, it is considered that the left-moving sector is that of a purely bosonic string propagating in $D = 26$ dimensions, while the right-moving one is that of a superstring in $D = 10$. For consistency, the mismatch of 16 dimensions between the two sets of modes must be appropriately compactified on an even, self-dual lattice. There are only two of those, giving rise to two possible gauge groups, namely SO(32) and $\mathrm{E}_8 \times \mathrm{E}_8$. The gauge potentials associated to said groups are the only other massless bosonic modes present in these theories. As Type I, the heterotic string theories are also $\mathcal{N} = 1$ supersymmetric.

Although superstring theory is only consistent in 10 dimensions, an underlying 11d theory called M-theory has being a matter of debate for the past few decades. Although we lack a complete formulation of M-theory, it is defined by saying that it is the UV completion of 11d supergravity. The low-energy limit of these 10d and 11d theories will be explored in section 1.3, but let us review the dimensional reduction mechanism first.

1.2. Dimensional reduction

In this section we present the notion of Kaluza-Klein compactification in the context of string theory. We consider a spacetime of dimension $D = d+1$, which we denote $M^{1,d}$. In order to illustrate the idea behind the dimensional reduction, we present the example in which we can write $M^{1,d} = \mathcal{M}^{1,d-1} \times S^1_R$, where S^1_R is a circle of radius R. The D-dimensional coordinates are denoted by $X^M = (X^\mu, X^d)$, where $\mu = 0\,, \dots\,,\ d-1$ and X^d corresponds to the S^1_R and satisfies a periodic condition,

$$X^d + 2\pi R \simeq X^d\,. \tag{1.47}$$

We think of a closed string parametrised as $X^M(\tau,\sigma) = (X^\mu(\tau,\sigma), X^d(\tau,\sigma))$. We will consider, as in the previous section, that we are in the conformal gauge so the metric is η_{MN}. The mode expansion of the bosonic fields X^M is still given by (1.16) and (1.18) so we can write

$$X^M(\tau,\sigma) = x^M + \frac{p^M}{2\pi T}\tau + \frac{p^M_+ - p^M_-}{4\pi T}\sigma + \frac{i}{\sqrt{4\pi T}}\sum_{n\neq 0}\frac{e^{-in\tau}}{n}(\alpha^M_n e^{in\sigma} + \tilde{\alpha}^M_n e^{-in\sigma})\,, \tag{1.48}$$

where we have defined

$$p^M = \frac{p^M_+ + p^M_-}{2}\,. \tag{1.49}$$

Imposing the periodicity condition (1.17) on the X^μ coordinates leads us to $p^\mu_- = p^\mu_+$. However, this condition may not be met for the circle coordinate X^d as there may be winding modes, namely

$$X^d(\tau,\sigma+2\pi) = X^d(\tau,\sigma) + 2\pi w R \quad \text{with} \quad w\in\mathbb{Z}\,. \tag{1.50}$$

The winding number w is the number of times the string is wrapped around the circle and its sign depends on the orientation of the string. Thus, now the linear momenta of the two modes satisfy a more general condition,

$$\frac{p^d_+ - p^d_-}{4\pi T} = wR\,. \tag{1.51}$$

The classical string Hamiltonian in light cone coordinates is given by the following expression,

$$H = \frac{1}{2\pi T}p^i p_i + \frac{1}{2}\sum_{n=-\infty}^{\infty}\left(\alpha^i_{-n}\alpha_{ni} - \tilde{\alpha}^i_{-n}\tilde{\alpha}_{ni}\right)\,, \tag{1.52}$$

where $i = 2,\ 3,\ \dots,\ d$. Upon quantisation, the previous Hamiltonian must be modified in order to contain the following normal-ordered variables,

$$N_\perp = \sum_{n=1}^{\infty} :\alpha^i_{-n}\alpha_{ni}:\,, \quad \tilde{N}_\perp = \sum_{n=1}^{\infty} :\tilde{\alpha}^i_{-n}\tilde{\alpha}_{ni}:\,, \tag{1.53}$$

which satisfy the matching condition associated to the periodicity condition (1.51),

$$\tilde{N}_\perp - N_\perp = nw\,. \tag{1.54}$$

This along with (1.49) gives rise to the Hamiltonian below,

$$H = \frac{1}{2\pi T}\left[p^a p_a + \frac{\left(p_+^d - p_-^d\right)^2}{4} + \left(p^d\right)^2\right] + \left(N_\perp + \tilde{N}_\perp - 2\right)\,. \tag{1.55}$$

where $a = 2,\ 3,\ \ldots,\ d-1$. For the previous computation, we have implicitly taken light cone coordinates in the d-dimensional manifold,

$$X^\pm = \frac{1}{\sqrt{2}}\left(X^0 \pm X^1\right) \quad \text{and} \quad X^a \quad \text{with} \quad i = 2\,, \ldots\,, d-1\,. \tag{1.56}$$

In these coordinates, the spacetime metric is given by

$$\eta_{++} = \eta_{--} = 0\,, \quad \eta_{+-} = \eta_{-+} = -1\,, \quad \eta_{ij} = \delta_{ij} \tag{1.57}$$

and the mass of the d-dimensional string states can be computed as follows,

$$(M_d)^2 = -p^\mu p_\mu = 2p^+p^- - p^a p_a = \frac{\left(p_+^d - p_-^d\right)^2}{4} + \left(p^d\right)^2 + 2\pi T\left(N_\perp + \tilde{N}_\perp - 2\right)\,, \tag{1.58}$$

where we have used that

$$p^- = T\int_0^\pi d\sigma \partial_\tau X^- = 2\pi T\frac{H}{p^+}\,. \tag{1.59}$$

The mass of states with no excited oscillators, i.e. $N_\perp + \tilde{N}_\perp - 2 = 0$, is given by the following expression,

$$(M_{n,w})^2 = \left(\frac{n}{R}\right)^2 + \left(\frac{wR}{2\pi T}\right)^2\,, \tag{1.60}$$

where we have picked

$$p^d = \frac{n}{R}\,. \tag{1.61}$$

The reason why this is the most reasonable choice comes from the scalar field case. If we consider a massless scalar field Φ which lives on our D-dimensional spacetime, the periodicity of the X^d lets us write it as a Fourier series,

$$\Phi(X^M) = \sum_{n=-\infty}^{\infty} \phi(X^\mu) e^{i\frac{n}{R}X^d} \tag{1.62}$$

Also the n/R factor comes from dividing $2\pi n$ by the length of the circumference $2\pi R$ and it plays the role of p^d. The field must satisfy a massless Klein-Gordon equation,

$$0 = \partial_M \partial^M \Phi = \left(\partial_\mu \partial^\mu + \partial_d^2\right)\Phi\,. \tag{1.63}$$

Applying this equation to (1.62), we obtain an equation for the modes,

$$\left[\partial_\mu \partial^\mu - \left(\frac{n}{R}\right)^2\right]\phi_n = 0\,, \tag{1.64}$$

which is the Kaluza-Klein equation for a d-dimensional field of mass

$$M_n = \frac{n}{R}\,. \tag{1.65}$$

This infinite tower of massive states is a simplified version of what we had for a closed string (1.60). The only difference is that in this case we have no winding number.

We conclude this section by noticing that similar constructions can be done in the case where the circle is substituted by a higher-dimensional compact manifold. For instance, we could consider $M^{1,25} = \mathcal{M}^{1,3} \times K^{22}$ in the bosonic string case and $M^{1,9} = \mathcal{M}^{1,3} \times K^6$ in the superstring case for certain compact manifolds K^{22} and K^6 of the appropriate dimensions.

1.3. The low-energy limit

We are interested in exploring the low-energy particle limit of the string theories. In order to pursue this goal, we start by writing the most general action for a bosonic string with up to two worldsheet derivatives. This is the following non-linear sigma-model action,

$$\begin{aligned} S_D =& \frac{1}{4\pi\alpha'}\int_\Sigma d^2\sigma \left[\left(\sqrt{|h|}g_{MN}(X)h^{ij} + (B_2)_{MN}(X)\epsilon^{ij}\right)\partial_i X^M \partial_j X^N + \right.\\ &\left. + \frac{\alpha'}{2}\sqrt{|h|}\phi(X)\mathcal{R}^{(2)}(h)\right]\,, \end{aligned} \tag{1.66}$$

which describes the evolution of scalar fields X^M on a worldsheet parametrised by $\sigma^i = (\tau, \sigma)$ or, alternatively, the dynamics of a string embedded in the so-called D-dimensional *target space* described by the X^M coordinates. It contains the action (1.3), but also includes the 2-form potential B_2 and dilaton ϕ which appear naturally in superstring theories. The coefficient $\mathcal{R}^{(2)}(h)$ that multiplies the dilaton is the 2d Ricci scalar associated to h_{ij} and ϵ^{ij} is the 2d Levi-Civita symbol. One final remark, the string tension T is absent in the action because we have defined the so-called Regge slope parameter

$$\alpha' = \frac{1}{2\pi T} = l_s^2\,, \tag{1.67}$$

which has units of (length)^2 and thus introduces a fundamental length scale, namely the string scale $l_s \equiv \sqrt{\alpha'}$. If we see string theory as a 2d field theory living in the worldsheet, α' codifies the strength of the interactions of the fields X^M. Thus, an expansion in α' provides quantum corrections whenever they are required.

We observe that, although the first two terms in (1.66) are Weyl invariant, it is not the case for the coupling of the dilaton. However, there is a case where this coupling is Weyl invariant at the classical level. This happens when the dilaton is a constant

$$\phi = \phi_0 \,. \tag{1.68}$$

In this case, the last term in the action is purely topological because it turns out to be

$$\frac{1}{4\pi}\int_\Sigma d^2\sigma\sqrt{|h|}\frac{\phi_0}{2}\mathcal{R}^{(2)}(h) = \frac{\phi_0}{2}\left[\frac{1}{4\pi}\int_\Sigma d^2\sigma\sqrt{|h|}\mathcal{R}^{(2)}(h)\right] = \frac{\phi_0}{2}\chi(\Sigma)\,, \tag{1.69}$$

where $\chi(\Sigma)$ is the Euler characteristic of the worldsheet. This result seems to suggest that a genus expansion over the worldsheets may be useful. Indeed, equation (1.69) shows that the constant mode of the dilaton fixes the so-called *string coupling constant*,

$$g_s = e^{\phi_0}\,, \tag{1.70}$$

which encodes the strength of the string interactions and appears in the perturbative expansions of string theory. Moreover, as the dilatonic term of the action (1.66) is purely topological for a constant dilaton, this implies that an expansion in g_s is a genus expansion, where the different topologies of the worldsheets are considered. The usual choice for ϕ_0 is simply

$$\phi_0 = \lim_{X\to\infty}\phi(X)\,. \tag{1.71}$$

However, action (1.66) is not Weyl-invariant in general at the classical level. If we want to circumvent this issue at the quantum level, the coupling constants g_{MN}, B_{MN} and ϕ must be invariant under the Weyl transformation. In other words, their corresponding beta-functions must vanish. Among other things, this provides that the energy-momentum tensor becomes traceless. Without entering into detail, it is possible to compute said beta-functions as a perturbative series in α', their lowest-order being as below [75],

$$\begin{aligned}\beta(g)_{MN} =& R_{MN} + 2\nabla_M\partial_N\phi - \frac{1}{4}(H_3)_M{}^{PQ}(H_3)_{NPQ} + \mathcal{O}(\alpha')\,,\\ \beta(B)_{MN} =& \frac{e^{2\phi}}{2}\nabla^L\left(e^{-2\phi}(H_3)_{LMN}\right) + \mathcal{O}(\alpha')\,,\\ \beta(\phi) =& \frac{D-26}{6} + \frac{\alpha'}{2}\left(-R + 4(\partial\phi)^2 - 4\nabla^2\phi + \frac{H_3^2}{12}\right) + \mathcal{O}(\alpha'^2)\,,\end{aligned} \tag{1.72}$$

where R_{MN} and R are the D-dimensional Ricci tensor and scalar, respectively, and we have defined

$$H_3 = dB_2\,. \tag{1.73}$$

We have also used the following compact notations

$$H_3^2 = H^{KMN}H_{KMN}\,, \quad (\partial\phi)^2 = \partial_M\phi\partial^M\phi\,. \tag{1.74}$$

The difference in order between $\beta(\phi)$ and the other two beta-functions comes from the fact that the dilatonic term in (1.66) is multiplied by α', therefore, different orders are required in order for the trace of the energy-momentum tensor to be zero. The vanishing of the considered terms of these beta-functions yields a set of PDEs which constrain the massless string states. Surprisingly, these are a deformation of the Einstein equations that includes the string theoretical fields Φ and B_2. This is highly relevant, as it shows that (super)gravity arises in the low-energy limit of (super)string theory. Concretely, (1.72) are the Euler-Lagrange equations of the D-dimensional Einstein-Hilbert action below,

$$S = \frac{1}{2\kappa_0^2} \int d^D x \sqrt{|g|} e^{-2\phi} \left(-\frac{(D-26)}{3\alpha'} + R + 4(\partial\phi)^2 - \frac{H_3^2}{12} \right) . \tag{1.75}$$

The first term in the action equals zero at the critical dimension $D = 26$, which reinforces the idea that (1.75) is the low-energy limit ($\alpha' \to 0$) of bosonic string theory. The κ_0 constant is not fixed by the equations of motion and it scales as $\kappa_0 \sim l_s^{24} = \alpha'^{12}$. In order to relate κ_0 with the *D-dimensional gravitational coupling constant* κ_D, we redefine the dilaton so it has a vanishing expectation value,

$$e^{\Phi} \equiv \frac{e^{\phi}}{g_s} . \tag{1.76}$$

The action now reads

$$S = \frac{1}{2\kappa_D^2} \int d^D x \sqrt{|g|} e^{-2\phi} \left(-\frac{(D-26)}{3\alpha'} + R + 4(\partial\Phi)^2 - \frac{H_3^2}{12} \right) , \tag{1.77}$$

where the D-dimensional gravitational coupling constant is defined as follows,

$$\frac{1}{2\kappa_D^2} \equiv \frac{2\pi}{g_s^2 (2\pi l_s)^{D-2}} . \tag{1.78}$$

The previous treatment can be repeated for the five superstring theories consistent at $D = 10$, which gives rise to five different supergravity theories in their low-energy limits. The supergravity theories preserve the same amount of supersymmetry as their string theory counterparts. Moreover, these supergravity theories share a common sector, as in the full superstring case. Their actions can thus be split in three parts,

$$S_{\text{SUGRA}} = S_1 + S_2 + S_{\text{fermi}} . \tag{1.79}$$

The bosonic sector is described by $S_1 + S_2$, where S_1 is common to the five supergravity theories and S_2 is particular to each one. The term S_2 is added to the action in order to ensure anomaly cancellation. There are exactly five ways in which this term can be written, corresponding to the five anomaly-free supergravity theories in ten dimensions. Besides, S_{fermi} describes the fermionic superpartners of the aforementioned bosons. We will only present the first two parts of (1.79). The first part of the action describes the fields common to all five supergravity theories and reads

$$S_1 = \frac{1}{2\kappa_{10}^2} \int d^{10} x \sqrt{|g|} e^{-2\Phi} \left(R + 4(\partial\Phi)^2 \right) . \tag{1.80}$$

The action S_2 describes the dynamics of the massless bosonic fields which are different for each theory. We explain each case below:

- Type I. It contains an extra 1-form gauge potential $\mathcal{A}^I$ in the adjoint representation of the SO(32) group and a 2-form potential C_2. Thus, we have the following action,

$$S_2(I) = -\frac{1}{2\kappa_{10}^2}\int d^{10}x\sqrt{|g|}\left(\frac{F_3^2}{12}+\frac{1}{4}e^{-\Phi}\mathcal{F}^I\mathcal{F}_I\right), \tag{1.81}$$

where we have defined the field strengths

$$\mathcal{F} = d\mathcal{A}+\mathcal{A}\wedge\mathcal{A}, \quad F_3 = dC_2+\frac{1}{2}\,\mathrm{Tr}(\mathcal{A}\wedge\mathcal{A}). \tag{1.82}$$

- Type IIA. This theory contains an RR sector consisting of two gauge potentials $\{C_1, C_3\}$ and the associated action reads [76]

$$\begin{aligned} S_2(IIA) = -\frac{1}{4\kappa_{10}^2}\int d^{10}x\Bigg[&\sqrt{|g|}\left(e^{-2\Phi}\frac{H_3^2}{6}+\frac{F_2^2}{2}+\frac{F_4^2}{4!}\right)+ \\ &+dC_3\wedge dC_3\wedge B_2\Bigg], \end{aligned} \tag{1.83}$$

where we have defined

$$H_3 = dB_2, \quad F_2 = dC_1, \quad F_4 = dC_3 - H_3\wedge C_1. \tag{1.84}$$

A deformation of this theory can be performed by adding a constant scalar m, which plays the role of an RR 0-form flux F_0. This new scalar receives the name of *Romans mass* [77]. In this case (1.83) must be modified in order to include the new scalar [76],

$$\begin{aligned} S_2(\text{massive } IIA) = -\frac{1}{4\kappa_{10}^2}\int d^{10}x\Bigg[&\sqrt{|g|}\left(e^{-2\Phi}\frac{H_3^2}{6}+m^2+\frac{F_2^2}{2}+\frac{F_4^2}{4!}\right)+ \\ &+dC_3\wedge dC_3\wedge B_2+\frac{m}{3}dC_3\wedge(B_2)^3+\frac{m^2}{20}(B_2)^5\Bigg], \end{aligned} \tag{1.85}$$

where the topological term comes from a eleven-dimensional Chern-Simons term,

$$\begin{aligned} &-\frac{1}{2}\int_{M_{11}} F_4\wedge F_4\wedge H_3 = \ldots = \\ &= -\frac{1}{2}\int_{M_{10}}\left(dC_3\wedge dC_3\wedge B_2+\frac{m}{3}dC_3\wedge(B_2)^3+\frac{m^2}{20}(B_2)^5\right). \end{aligned} \tag{1.86}$$

The exponents of the B_2 indicate repeated wedge product by itself and we have redefined the fluxes in order to include m,

$$F_2 = dC_1+mB_2, \quad F_4 = dC_3 - H_3\wedge C_1+\frac{m}{2}B_2\wedge B_2. \tag{1.87}$$

Sometimes the democratic formulation of the theory is considered, which means that the RR fluxes F_6, F_8 and F_{10} are treated on the same footing as the lower-ranked ones. Analogously, one can also define C_5, C_7 and C_9. In this formulation, the Bianchi identities yield all the equations of motion except for the Einstein one. The higher-ranked p-forms are related to the lower-ranked ones as follows,

$$F_p = (-1)^{\lfloor \frac{p}{2} \rfloor} \star F_{10-p}\,, \tag{1.88}$$

where $\lfloor x \rfloor$ stands for the floor function, i.e. the greatest integer smaller than or equal to x, and $\star$ is the ten-dimensional Hodge star operator. In general, we will use the following notation for a D-dimensional Hodge dual,

$$(\star F_p)_{M_1 \ldots M_{D-p}} = \frac{\sqrt{|g|}}{p!} \epsilon_{M_1 \ldots M_D} F^{M_{D-p+1} \ldots M_D}\,, \tag{1.89}$$

where $\epsilon_{01 \ldots (D-1)} = 1$. In our conventions, we consider that

$$\star \star F_p = s(-1)^{p(D-p)} F_p\,, \tag{1.90}$$

where $s = (-1)^n$ for a spacetime signature of (p, n).

- Type IIB. In this case, we have three RR gauge potentials $\{C_0, C_2, C_4\}$ whose dynamics are codified by the following action [76],

$$\begin{aligned} S_2(IIB) = &- \frac{1}{4\kappa_{10}^2} \int d^{10}x \Bigg[\frac{\sqrt{|g|}}{2} \left(e^{-2\Phi} \frac{H_3^2}{3!} + F_1^2 + \frac{F_3^2}{3!} + \frac{F_5^2}{5!} \right) + \\ &+ dC_4 \wedge H_3 \wedge dC_2 \Bigg]\,, \end{aligned} \tag{1.91}$$

where the fluxes are defined in a similar way as in the previous case,

$$H_3 = dB_2\,, \quad F_1 = dC_0\,, \quad F_3 = dC_2 - C_0 H_3\,, \quad F_5 = dC_4 - H_3 \wedge C_2\,. \tag{1.92}$$

However, the action (1.91) is not enough to specify the theory, but an extra condition must be imposed,

$$F_5 = \star F_5\,. \tag{1.93}$$

This theory can also be described in the democratic formulation. Whenever the higher-ranked RR fluxes F_7 and F_9 are required, they are related to the lower-ranked ones as in (1.88). One can also consider the C_6 and C_8 gauge potentials when they are needed.

- Heterotic string. As in the case of Type I, we have gauge potentials corresponding to the appropriate group, namely SO(32) or $\mathrm{E}_8 \times \mathrm{E}_8$ depending on the theory. The specific piece of the action for these theories reads

$$S_2(H) = -\frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{|g|} e^{-2\Phi} \left(\frac{H_3^2}{12} + \frac{1}{4} \mathcal{F}^I \mathcal{F}_I \right)\,, \tag{1.94}$$

where $\mathcal{F}$ is defined as in (1.82) and we also have

$$H_3 = dB_2 + \frac{1}{2}\,\mathrm{Tr}(\mathcal{A} \wedge \mathcal{A})\,. \tag{1.95}$$

Before we conclude this section, 11d supergravity deserves a special mention because, as we explained in section 1.1, it is the low-energy limit of M-theory. The eleven-dimensional case is quite elegant as the field content consists simply on a metric g_{MN} and a three-form potential A_3. The bosonic part of the corresponding action is given below [78],

$$S_{11d}(\mathrm{SUGRA}) = -\frac{1}{2\kappa_{11}^2}\int d^{11}x \left[\sqrt{|g|}\left(R - \frac{G_4^2}{48}\right) + \frac{1}{6}A_3 \wedge G_4 \wedge G_4\right]\,, \tag{1.96}$$

where κ_{11} is the eleven-dimensional gravitational coupling constant and as usual

$$G_4 = dA_3\,. \tag{1.97}$$

This is the only possible supergravity theory in eleven dimensions and is connected to Type IIA via dimensional reduction. It displays $\mathcal{N} = 1$ supersymmetry, meaning it preserves 32 supercharges.

1.4. String dualities

It is a key feature of the string theories that they are related under certain transformations called *dualities*. We observe that the duality associated to the inversion of the coupling constant (S-duality) is already present in quantum field theories like QED in the presence of magnetic charges. However, string theory displays a new kind of duality called T-duality, which is purely a consequence of strings having length (as opposed to point particles).

As we will see in the next sections, all these relations between 10d and 11d string theories can be summarised in a simple and visual manner, as done in figure 1.1

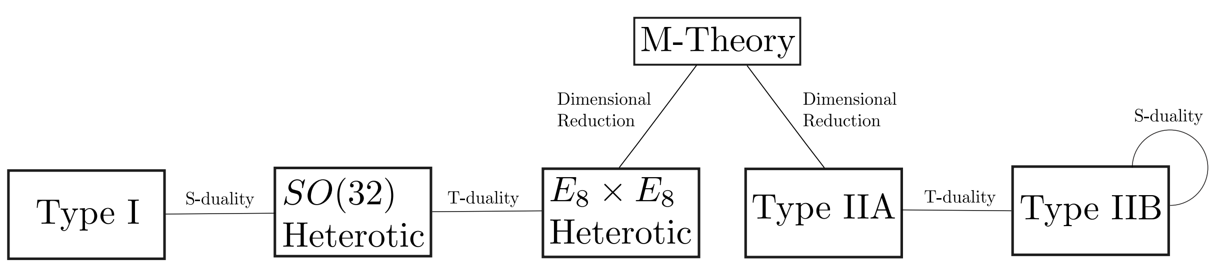


Figure 1.1: All the relations between string theories.

1.4.1. T-duality

T-duality can be applied when the theory is weakly coupled, i.e. at the perturbative level with respect to g_s. The basic idea comes from a construction that we have already presented, indeed. As we saw in section 1.2, if we consider a closed string theory living in a spacetime which has a circle direction of which all fields are independent, then the ground states consist on an infinite tower with masses given by (1.60). Thus, we conclude that this spectrum is invariant under the following transformation of the radius

$$R \leftrightarrow \tilde{R} = \frac{\alpha'}{R} \,. \tag{1.98}$$

This interchange relates a theory where the circle is small (stringy effects are relevant) to another one with a big circle (stringy effects are irrelevant). In other words, T-duality connects a theory where α' is large to another one where it is small. The momentum and winding numbers are also interchanged as below

$$(n, w) \leftrightarrow (\tilde{n}, \tilde{w}) = (w, n) \,. \tag{1.99}$$

The below discussion on T-duality follows in a certain way [79], so it can be consulted if the reader is interested in the topic.

Abelian T-duality

Buscher first formulated Abelian T-duality for the non-linear sigma-model described by (1.66) in [80,81]. However, for the purposes of this thesis, we are more interested in the way it was developed by Rocek and Verlinde [82], as it provides a simpler link to its non-Abelian extension. We start by assuming that the model presents an Abelian isometry, i.e. all the fields are independent on one of the coordinates θ of the D-dimensional spacetime. We thus split the coordinates into $X^M = (X^\mu, \theta)$, where $\mu = 0, 1, \ldots D-2$. The main idea is to gauge the Abelian isometry $\delta\theta = \epsilon$. This is done by introducing a gauge potential A_i on the worldsheet (with $\delta A_i = -\partial_i \epsilon$) and substituting the partial derivatives of the isometry coordinate by covariant ones,

$$\partial_i \theta \to D_i \theta = \partial_i \theta + A_i \,. \tag{1.100}$$

These considerations are key to our derivation as we will obtain the T-dual theory upon integrating the gauge potentials and fixing the gauge. We also add a Lagrange multiplier to the action through a term proportional to A_i. The resulting action can be interpreted as that of a $(D+1)$-dimensional non-linear sigma-model and reads

$$\begin{aligned} S_{D+1} =& \frac{1}{4\pi\alpha'} \int_\Sigma d^2\sigma \Big[\sqrt{|h|} h^{ij} \left(g_{\mu\nu} \partial_i X^\mu \partial_j X^\nu + 2 g_{\mu\theta} \partial_i X^\mu D_j \theta + g_{\theta\theta} D_i \theta D_j \theta \right) + \\ &+ \epsilon^{ij} \Big((B_2)_{\mu\nu} \partial_i X^\mu \partial_j X^\nu + 2 (B_2)_{\mu\theta} \partial_i X^\mu D_j \theta + 2\tilde{\theta} \partial_i A_j \Big) + \frac{\alpha'}{2} \sqrt{|h|} \Phi \mathcal{R}^{(2)} \Big] \,. \end{aligned} \tag{1.101}$$

By construction, this action depends on $\tilde{\theta}$, but not on its derivatives. This can be exploited as it implies that $\tilde{\theta}$ is a Noether charge, its associated equation being simply

$$\epsilon^{ij}\partial_i A_j = \partial_\tau A_\sigma - \partial_\sigma A_\tau = 0\,. \tag{1.102}$$

In topologically trivial worldsheets the previous condition forces A to be an exact form, i.e. it can be written as

$$A = d\omega \tag{1.103}$$

for a certain function of the worldsheet $\omega = \omega(\tau,\sigma)$. In this case, we recover the original theory upon fixing $\omega = 0$. On the other hand, we could compute the equations of motion for A_i,

$$\sqrt{|h|}h^{ij}\left(g_{\mu\theta}\partial_j X^\mu + g_{\theta\theta}A_j\right) + \epsilon^{ij}((B_2)_{\theta\mu}\partial_j X^\mu + \partial_j\tilde{\theta}) = 0\,. \tag{1.104}$$

It is trivial to solve for A_i and obtain the following,

$$A_i = -\frac{1}{g_{\theta\theta}}\left[\frac{\epsilon_i{}^j}{\sqrt{|h|}}\left((B_2)_{\theta\mu}\partial_j X^\mu + \partial_j\tilde{\theta}\right) + g_{\mu\theta}\partial_i X^\mu\right]. \tag{1.105}$$

If we substitute this solution in (1.101), assume that the boundary term associated to $\partial_i(\epsilon^{ij}\tilde{\theta}A_j)$ vanishes and set $\theta = 0$, we obtain a new D-dimensional sigma-model,

$$\begin{aligned}\tilde{S}_D =& \frac{1}{4\pi\alpha'}\int_\Sigma d^2\sigma\left[\sqrt{|h|}h^{ij}\left(\tilde{g}_{\mu\nu}\partial_i X^\mu\partial_j X^\nu + 2\tilde{g}_{\mu\tilde{\theta}}\partial_i X^\mu\partial_j\tilde{\theta} + \tilde{g}_{\tilde{\theta}\tilde{\theta}}\partial_i\tilde{\theta}\partial_j\tilde{\theta}\right) + \right.\\ &\left. +\epsilon^{ij}\left((\tilde{B}_2)_{\mu\nu}\partial_i X^\mu\partial_j X^\nu + 2(\tilde{B}_2)_{\mu\tilde{\theta}}\partial_i X^\mu\partial_j\tilde{\theta}\right) + \frac{\alpha'}{2}\sqrt{|h|}\Phi\mathcal{R}^{(2)}\right].\end{aligned} \tag{1.106}$$

The new metric and 2-form potential are related to the old ones by the so-called Buscher rules given below,

$$\begin{aligned}\tilde{g}_{\mu\nu} =& g_{\mu\nu} - \frac{g_{\theta\mu}g_{\theta\nu} - (B_2)_{\theta\mu}(B_2)_{\theta\nu}}{g_{\theta\theta}}\,, \quad \tilde{g}_{\tilde{\theta}\mu} = \frac{(B_2)_{\theta\mu}}{g_{\theta\theta}}\,, \quad \tilde{g}_{\tilde{\theta}\tilde{\theta}} = \frac{1}{g_{\theta\theta}}\,,\\ (\tilde{B}_2)_{\mu\nu} =& (B_2)_{\mu\nu} - \frac{g_{\theta\mu}(B_2)_{\theta\nu} - g_{\theta\nu}(B_2)_{\theta\mu}}{g_{\theta\theta}}\,, \quad (\tilde{B}_2)_{\tilde{\theta}\mu} = \frac{g_{\theta\mu}}{g_{\theta\theta}}\,.\end{aligned} \tag{1.107}$$

A more delicate computation involving the path integral shows that the previous transformation modifies the measure,

$$\sqrt{|g|}e^{-2\Phi} \to \sqrt{|\tilde{g}|}e^{-2\tilde{\Phi}} = \frac{\sqrt{|g|}}{g_{\theta\theta}}e^{-2\tilde{\Phi}}\,. \tag{1.108}$$

This can be compensated by an appropriate shift in the dilaton,

$$\tilde{\Phi} = \Phi - \frac{1}{2}\log(g_{\theta\theta})\,. \tag{1.109}$$

It can be proven that the fields given by (1.107) and (1.109) satisfy the conformal invariance conditions at first order in α' that we saw in (1.72).

We observe that $\tilde{\theta}$ is an isometry direction of the new theory, as the associated metric depends on the fields of the original action (1.66), where $\tilde{\theta}$ did not appear. We also highlight the fact that the geometry of the new theory is completely different from that of the old one. For instance, both non-diagonal terms in the metric and a 2-form gauge potential can be generated via T-duality.

However, Rocek and Verlinde proved that the T-dual theories are indeed the same one as conformal field theories [82]. The discussion begins by considering the $(D+1)$-dimensional action (1.101). This model displays two U(1) isometries and, therefore, the isometry group can be written as $\mathrm{U}(1)_L \times \mathrm{U}(1)_R$. One can gauge (1.101) with respect to the vector or axial subgroup, i.e. $\mathrm{U}(1)_V = \mathrm{U}(1)_L + \mathrm{U}(1)_R$ or $\mathrm{U}(1)_A = \mathrm{U}(1)_L - \mathrm{U}(1)_R$, respectively. The result is an action which is locally invariant under the subgroup that was considered for the gauging. If we then integrate the gauge field, the actions (1.66) and (1.106) are obtained for the vector and axial gaugings, respectively. This procedure of gauging and integrating the resulting gauge field is usually referred to as a coset construction, which presents some interesting properties. For instance, if one of the actions (1.101), (1.66) or (1.106) is conformal, then the other two also are. This is due to the fact that the vanishing of the beta functions at all orders in α' (which is the condition for a conformal field theory) provides the same set of restrictions for each of these three actions. Also T-duality can be shown to act on the $(D+1)$-dimensional action as a symmetry interchanging vector and axial fields with one another. We conclude the equivalence of the original and T-dual D-dimensional conformal field theories at all orders in α'.

We notice that the previous derivation is only valid for trivial worldsheets. A way to see this is by noticing that, in order to show the equivalence between the actions (1.101) and (1.66), we used that conditions (1.102) and (1.103) are equivalent, which is only true if the worldsheet is simply connected. However, this can be generalised to arbitrary topologies, at least in the Abelian case.

Let us consider the case where the sigma model presents no dilaton and let M and $\tilde{M}$ be the manifolds describing the original and dual spacetimes. We observe that the Buscher rules (1.107)-(1.109) were given in a special set of coordinates which included the isometry direction θ. However, in order to study the topological properties of the dual theory $(\tilde{M}, \tilde{g}, \tilde{B}_2)$, it is necessary to derive these rules in a general set of coordinates, which enables the use of different charts for M and $\tilde{M}$. In order to perform the analysis in an arbitrary set of coordinates, we consider the following D-dimensional sigma-model in the conformal gauge without dilaton,

$$\begin{aligned}S_D =&\frac{1}{4\pi\alpha'}\int_\Sigma d^2\sigma\,(g_{MN}+(B_2)_{MN})\,\partial_+X^M\partial_-X^N = \\ =&\frac{1}{4\pi\alpha'}\int_\Sigma d^2\sigma\left(g_{MN}\partial_iX^M\partial^iX^N+(B_2)_{MN}dX^M\wedge dX^N\right)\,,\end{aligned} \tag{1.110}$$

where $\partial_\pm$ are given by (1.39). The second term of the last line of (1.110) is a so-called *Wess-Zumino term*. In the case where the worldsheet is a closed Riemann surface of genus g, which we denote by Σ_g, the Wess-Zumino term can be written as the integral of an element of the set of harmonic three-forms on M, i.e. there is a certain $H_3 \in H^3(M, \mathbb{R})$.

In particular, if Σ_g^0 is the region of space enclosed by Σ_g, then the Wess-Zumino term reads

$$\Gamma = \frac{1}{4\pi\alpha'}\int_{\Sigma_g^0} H_3 \quad \text{with} \quad \partial\Sigma_g^0 = \Sigma_g\,. \tag{1.111}$$

We assume that H_3 can be defined so that the ambiguity in Γ, due to the freedom of choice of Σ_g^0, is $2\pi n$ for some integer n so that the theory is well-defined at the quantum level. Let k^M be the Killing vector of the 10d metric g_{MN} associated to the isometry we started with, meaning that

$$\mathcal{L}_k g_{MN} = \nabla_M k_N + \nabla_N k_M = 0\,. \tag{1.112}$$

For the action to be invariant under $\delta X^M = \epsilon k^M$, the following requirement has to be met,

$$\delta_k\Gamma = \frac{1}{4\pi\alpha'}\int_{\Sigma_g^0}\epsilon\mathcal{L}_k H_3 = \frac{1}{4\pi\alpha'}\int_{\Sigma_g^0}\epsilon(di_k + i_k d)H_3 = \frac{\epsilon}{4\pi\alpha'}\int_{\Sigma_g} i_k H_3 = 0\,, \tag{1.113}$$

where we have used the notation

$$(i_k H_3)_{MN} \equiv k^K (H_3)_{KMN} \tag{1.114}$$

and the generalised Stokes theorem in order to substitute

$$\int_{\Sigma_g^0} di_k H_3 = \int_{\Sigma_g} i_k H_3\,, \tag{1.115}$$

as well as the fact that $dH_3 = 0$. We observe that (1.113) vanishes if $i_k H_3 = -dv$ for some one-form v. This implies that

$$\mathcal{L}_k B_2 = d\omega\,, \quad \omega = i_k B_2 - v\,, \tag{1.116}$$

where we have used that locally $H_3 = dB_2$. According to Noether's theorem, we have a conservation law associated to this symmetry,

$$\partial_+ J_k^- + \partial_- J_k^+ = 0\,, \tag{1.117}$$

where

$$J_k^\pm = (k \mp i_k B_2 \pm \omega)_M \partial_\pm X^M = (k \mp v)_M \partial_\pm X^M \equiv (k \mp v)\cdot\partial_\pm X\,. \tag{1.118}$$

As we did in the trivial worldsheet case, we now gauge the isometry. This is accomplished by introducing the gauge field $A_\pm$ satisfying

$$\delta_\epsilon A_\pm = -\partial_\pm\epsilon\,, \quad \delta_\epsilon X^M = \epsilon k^M(X)\,, \tag{1.119}$$

where ϵ is a function on the worldsheet. We observe that now the variation of the action (1.110) is given by

$$\delta_\epsilon S_D = \frac{1}{4\pi\alpha'}\int d^2\sigma (J_k^- \partial_+\epsilon + J_k^+\partial_-\epsilon)\,, \tag{1.120}$$

which can be cancelled out by adding the term

$$S'_D = \frac{1}{4\pi\alpha'}\int d^2\sigma (A_+ J_k^- + A_- J_k^+)\,. \tag{1.121}$$

However S'_D is not gauge invariant, as $J_k^\pm$ change under gauge transformations. If we include the term

$$S''_D = \frac{1}{4\pi\alpha'}\int d^2\sigma k^2 A_+ A_-\,, \tag{1.122}$$

then the total variation is given by

$$\delta_\epsilon(S_D + S'_D + S''_D) = \frac{1}{4\pi\alpha'}\int d^2\sigma [A_+\partial_-(\epsilon k\cdot v) - A_-\partial_+(\epsilon k\cdot v)]\,. \tag{1.123}$$

This anomalous variation cannot be cancelled out unless extra fields are added to the action. The simplest way to achieve this is by defining a real scalar field χ over the worldsheet which contributes to the action through the following term,

$$S_\chi = \frac{1}{4\pi\alpha'}\int d^2\sigma (A_+\partial_-\chi - A_-\partial_+\chi) \tag{1.124}$$

and changes under the gauge transformation as

$$\delta_\epsilon\chi = -\epsilon k\cdot v\,. \tag{1.125}$$

Interpreting this new scalar field as a spacetime coordinate, we end up with the following $(D+1)$-dimensional action,

$$\begin{aligned} S_{D+1} =& \frac{1}{4\pi\alpha'}\int_\Sigma d^2\sigma \left[(g_{MN} + (B_2)_{MN})\partial_+X^M\partial_-X^N + (J_k^+ - \partial_+\chi)A_- + \right.\\ &\left. + (J_k^- + \partial_-\chi)A_+ + k^2 A_+A_-\right]\,. \end{aligned} \tag{1.126}$$

We observe that, if the genus of the worldsheet is $g \geq 1$ and the gauge orbits are compact, then multivalued gauge transformations may appear,

$$\oint_\gamma d\epsilon = 2\pi\sqrt{\alpha'}n(\gamma) \quad \text{with} \quad n(\gamma)\in\mathbb{Z}\,, \tag{1.127}$$

where γ is a cycle of non-trivial homology in Σ_g. As we are only considering Abelian isometries, we can restrict our study to the $g = 1$ case. Thus, the variation of (1.126) is given by

$$\begin{aligned} \delta_\epsilon S_{D+1} =& \frac{1}{4\pi\alpha'}\int_\Sigma d^2\sigma\,(\partial_+\chi\partial_-\epsilon - \partial_-\chi\partial_+\epsilon) = \frac{1}{8\pi\alpha'}\int_{\mathbb{T}^2} d\chi\wedge d\epsilon = \\ =& \frac{1}{8\pi\alpha'}\left(\oint_a d\chi\oint_b d\epsilon - \oint_a d\epsilon\oint_b d\chi\right)\,, \end{aligned} \tag{1.128}$$

where a and b are the generator of the homology group of the torus $\mathbb{T}^2$. As ϵ has a period of $2\pi n(\gamma)$, in order for S_{D+1} to be a multiple of 2π, we must impose

$$\oint_\gamma d\chi = 8\pi\sqrt{\alpha'}m(\gamma) \quad \text{with} \quad m(\gamma) \in \mathbb{Z}\,, \tag{1.129}$$

If the isometry is non-compact then the variation of (1.126) vanishes so the periods of χ may take any real value.

The original theory is recovered by setting the Lagrange multipliers in action (1.126) to zero. They are closed forms and consequently can be decomposed as the sum of an exact component plus a harmonic one in worldsheets with non-trivial topologies. Their contribution to S_{D+1} is given by

$$S_\chi = -\frac{1}{4\pi\alpha'}\int (d\chi_0 + \chi_h)\wedge A\,, \tag{1.130}$$

where the closed 1-form has being written as $d\chi_0 + \chi_h$, χ_0 is a scalar, χ_h is a harmonic 1-form and $A = A_+ d\sigma^+ + A_- d\sigma^-$. We can write this action as

$$S_\chi = \frac{1}{4\pi\alpha'}\int \chi_0 \wedge dA - \frac{1}{4\pi\alpha'}\left(\oint_a \chi_h \oint_b A - \oint_a A \oint_b \chi_h\right)\,, \tag{1.131}$$

where we have integrated by parts the exact part and used the Riemann bilinear identity in the harmonic one. Taking a look to this action, it is clear that the equations of motion for χ_0 imposes $dA = 0$, which implies that A is exact in trivial worldsheets. For arbitrary topologies, we observe that the equation of motion for χ_h imposes

$$\oint_a A = \oint_b A = 0\,. \tag{1.132}$$

The Hodge decomposition theorem lets us write

$$A = d\omega + (n_a\phi^a + n_b\phi^b)\,, \tag{1.133}$$

where ϕ^a and ϕ^b constitute the basis of harmonic 1-forms on the torus satisfying

$$\oint_i \phi^j = \delta_{ij} \quad \text{with} \quad i,j \in \{a,b\}\,. \tag{1.134}$$

This normalization lets us easily identify

$$n_i = \oint_i A \quad \text{with} \quad i = a, b\,. \tag{1.135}$$

We conclude that (1.132) sets the harmonic part of A to zero so $A = d\omega$, which is the pure gauge solution. Fixing $\omega = 0$ the original theory (1.110) is recovered.

By construction the action (1.126) is invariant under changes of coordinates, meaning that we can infer geometrical properties from it. It can be shown that the spacetime of the T-dual theory $\tilde{M}$ is given by the product of the quotient of the original spacetime M by the orbit of the gauge group and the space parametrised by χ. If the isometry is compact, this implies that

$$\tilde{M} = (M/S^1)``\times" S^1_\chi \,, \tag{1.136}$$

as χ describes a circle. Depending on the case "$\times$" can stand for either a direct or twisted product. We highlight that the structure of $\pi_1(M)$ plays no role in the periodicity properties of χ, as all that matters is whether the isometry with respect to which we T-dualise is compact or not. For toroidal compactifications $\pi_1(\mathbb{T}^n) = \mathbb{Z}^n$, meaning that there are winding modes describing how closed strings are wrapped, apart from the momentum ones. In that scenario, T-duality can be interpreted as a symmetry that exchanges both kinds of modes.

In order to obtain the T-dual theory, one has to integrate the gauge fields and fix the gauge. Fixing the gauge in this formalism is equivalent to picking a coordinate system. After integrating the gauge fields, the following rules arise,

$$\begin{aligned} \tilde{g}_{\chi\chi} =& \frac{1}{k^2}\,, \quad \tilde{g}_{\chi\mu} = \frac{v_\mu}{k^2}\,, \quad \tilde{g}_{\mu\nu} = g_{\mu\nu} - \frac{k_\mu k_\nu - v_\mu v_\nu}{k^2}\,, \\ (\tilde{B}_2)_{\chi\mu} =& \frac{k_\mu}{k^2}\,, \quad (\tilde{B}_2)_{\mu\nu} = (B_2)_{\mu\nu} - \frac{k_\mu v_\nu - v_\mu k_\nu}{k^2}\,, \quad \tilde{\Phi} = \Phi - \frac{1}{2}\log\big(k^2\big)\,, \end{aligned} \tag{1.137}$$

where the T-dual set of coordinates is $\{\tilde{x}^M\} = \{x^\mu, \chi\}$ and we have reincorporated the dilaton to the original action. We can go back to the coordinate system that contains the isometry direction θ as coordinate and then fix $\theta = 0$. This yields the Buscher rules (1.107)-(1.109), as in that case $k^2 = g_{\theta\theta}$, $v_\mu = (B_2)_{\theta\mu}$ and $k_\mu = g_{\theta\mu}$.

Taking into account the fermions, one observes that T-duality relates the Type II string theories to one another and also the Heterotic ones. The application of this duality in the context of Type II supergravity is one of the most fruitful solution generating techniques that was used in our original work. However, the previous derivation does not provide a relation between the RR fields of the T-dual theories. The original way in which these relations where derived consisted in truncating the Type IIA and Type IIB supergravities to nine dimensions. As there is a single supergravity theory living in nine dimensions, this procedure yielded a mapping relating the RR fluxes of the Type IIA and Type IIB supergravities. However, S.F. Hassan devised an alternative method that provides a simpler extension to the non-Abelian case [83,84]. It starts by writing the original metric in terms of vielbeins,

$$g_{MN} = \eta_{ab} e^a{}_M e^b{}_N \,, \tag{1.138}$$

where a and b are Lorentz indices and we are taking some set of flat coordinates $\{y^a\}$. It is known that there are two vielbeins $(e^a{}_M)_\pm$ compatible with the T-dual solution. The T-duality of the vielbeins are described by the transformations below,

$$(\tilde{e}^M_a)_\pm = (Q^M{}_N)_\pm e^N{}_b \,, \tag{1.139}$$

where the matrices $Q_\pm$ are defined as

$$Q_\pm = \begin{pmatrix} \mp g_{\theta\theta} & \mp(g \mp B_2)_{\theta\nu} \\ 0 & \mathbb{I}_9 \end{pmatrix}, \quad Q_\pm^{-1} = \begin{pmatrix} \mp g_{\theta\theta}^{-1} & -(g \mp B_2)_{\theta\nu} \\ 0 & \mathbb{I}_9 \end{pmatrix} \tag{1.140}$$

and we denote by $\mathbb{I}_n$ the n-dimensional identity matrix. We now observe that the $(e^a{}_M)_\pm$ vielbeins describe the same theory and, therefore must be related by a Lorentz transformation,

$$(e^a{}_M)_+ = \Lambda^a{}_b \left(e^b{}_M\right)_- \quad \text{for} \quad \Lambda = e^{-1} Q_-^{-1} Q_+ e\,. \tag{1.141}$$

Using (1.140), we can write the Lorentz matrix more explicitly,

$$\Lambda^a{}_b = \delta^a{}_b - 2\frac{e^a{}_\theta e_{\theta b}}{g_{\theta\theta}} \quad \text{with} \quad \det(\Lambda) = -1\,. \tag{1.142}$$

Now let us shift our gaze to the transformation of spinors under T-duality. Let $\Gamma^M = e^M{}_a \Gamma^a$ be the 10d Dirac matrices living in the curved spacetime of the original theory and Γ^a their flat spacetime counterparts. Similarly as before, after T-duality, there are two possible sets of gamma matrices depending on the choice of the new vielbein,

$$\tilde{\Gamma}^M_\pm = (\tilde{e}^M_a)_\pm \Gamma^a\,. \tag{1.143}$$

Analogously to the bosonic case, these two sets are related by a matrix Ω as below,

$$\tilde{\Gamma}^M_+ = \Omega^{-1} \tilde{\Gamma}^M_- \Omega \quad \text{with} \quad \Omega^{-1}\Gamma^a\Omega = \Lambda^a{}_b \Gamma^b\,. \tag{1.144}$$

It is clear that Ω is the spinorial representation of the Lorentz transformation (1.141). We can deduce the concrete form of Ω as follows. Let us rewrite (1.141) as below,

$$\Lambda^a{}_b = \delta^a{}_b - 2\omega^a{}_b \quad \text{with} \quad \omega^a{}_b = \frac{e^a{}_\theta e_{\theta b}}{g_{\theta\theta}}\,. \tag{1.145}$$

It is clear that $\omega^a{}_b \omega^b{}_c = \omega^a{}_c$, meaning that $\omega^a{}_b$ is a projection operator of rank 1. We observe that $\omega = \omega^a{}_b (\partial/\partial X^a) dX^b$ projects the vector $\Gamma = \Gamma^a(\partial/\partial X^a)$ along $K = \sqrt{g_{\theta\theta}^{-1}} e^a{}_\theta (\partial/\partial X^a)$, which is the unitary vector that generates the isometry. In turn, the projected component of Γ is given by the scalar product

$$\langle K, \omega\Gamma\rangle = \left(\sqrt{g_{\theta\theta}^{-1}} e^a{}_\theta\right)\left(\frac{e^a{}_\theta e_{\theta b}}{g_{\theta\theta}}\Gamma^b\right) = \sqrt{g_{\theta\theta}^{-1}}\Gamma_\theta\,, \tag{1.146}$$

where Γ_θ is the curved space gamma matrix. Taking a look to (1.145), we observe that $\Lambda^a{}_b$ changes the sign of (1.146), while the other components of Γ remain unchanged. This implies that the spinorial representation of the Lorentz transformation must be given by

$$\Omega = \Gamma_{11}\langle K, \omega\Gamma\rangle = \sqrt{g_{\theta\theta}^{-1}}\Gamma_{11}\Gamma_\theta\,, \tag{1.147}$$

where Γ_{11} is the product of all the Γ^a. If we now take

$$e^\theta{}_a = \sqrt{g_{\theta\theta}}\delta^9{}_a\,, \tag{1.148}$$

for certain flat spacetime coordinate y^9, we can write

$$\Omega = \Omega^{-1} = \Gamma_{11}\Gamma_9\,, \tag{1.149}$$

as $\Gamma_{11}^2 = \mathbb{I}_{10}$ in Minkowski spacetime and Γ_{11} commutes with the other gamma matrices.

Now, the RR fluxes of Type IIA/B supergravity can be combined into a polyform,

$$P = \begin{cases} \frac{e^\Phi}{2}\sum_{n=0}^4 \not{F}_{2n+1} & \text{for Type IIA}\,, \\ \frac{e^\Phi}{2}\sum_{n=0}^5 \not{F}_{2n} & \text{for Type IIB}\,, \end{cases} \tag{1.150}$$

where we have considered the democratic formulation of Type II supergravity and so we have to take into account all possible RR fluxes. The slashed fluxes are given by the below formulae,

$$\not{F}_n = \frac{1}{n!}F_{M_1\dots M_n}\Gamma^{M_1\dots M_n}\,, \quad (\not{F}_n)_{M_1\dots M_r} = \frac{1}{(n-r)!}F_{M_1\dots M_r M_{r+1}\cdots M_n}\Gamma^{M_{r+1}\cdots M_n}\,. \tag{1.151}$$

The polyform is a bispinor and thus transforms as follows under T-duality,

$$\tilde{P} = \Omega^{-1}P\,. \tag{1.152}$$

From (1.107) and (1.148), it is clear that

$$e^9 = \sqrt{g_{\theta\theta}}\,d\theta\,, \quad \tilde{e}^9 = \frac{d\tilde{\theta}}{\sqrt{g_{\theta\theta}}}\,. \tag{1.153}$$

Thus, if we write the original fluxes as

$$F_p = G_p^{(0)} + G_{p-1}\wedge e^9\,, \tag{1.154}$$

then the dual RR fluxes read

$$\tilde{F}_p = \sqrt{g_{\theta\theta}}\left(\tilde{G}_p^{(0)} + \tilde{G}_{p-1}\wedge\tilde{e}^9\right) = \sqrt{g_{\theta\theta}}G_p + G_{p-1}^{(0)}\wedge d\tilde{\theta}\,, \tag{1.155}$$

where

$$\tilde{G}_p^{(0)} = G_p\,, \quad \tilde{G}_{p-1} = G_{p-1}^{(0)}\,. \tag{1.156}$$

We observe that the rules in (1.156) imply that odd-ranked fluxes are transformed into even-ranked ones. This was expected as T-duality relates Type IIA and Type IIB supergravities to one another.

These considerations lead us to the rule of thumb that is usually applied when computing the Abelian T-dual of RR fluxes: *The RR n-form flux associated to a Type II theory that presents a U(1) isometry along θ can be written as*

$$F_n = \frac{(F_n)_{\mu_1\ldots\mu_n}}{n!} dx^{\mu_1} \wedge \ldots \wedge dx^{\mu_n} + \frac{(F_n)_{\mu_1\ldots\mu_{n-1}}}{(n-1)!} dx^{\mu_1} \wedge \ldots \wedge dx^{\mu_{n-1}} \wedge d\theta \tag{1.157}$$

with $\mu_i = 0, \ldots, 8$. Upon T-dualising along θ, F_n will contribute, when possible, through two different kinds of components, namely

$$\begin{aligned} \tilde{F}_{n-1} =& \frac{(F_n)_{\mu_1\ldots\mu_{n-1}}}{(n-1)!} dx^{\mu_1} \wedge \ldots \wedge dx^{\mu_{n-1}} \,, \\ \tilde{F}_{n+1} =& \frac{(F_n)_{\mu_1\ldots\mu_n}}{n!} dx^{\mu_1} \wedge \ldots \wedge dx^{\mu_n} \wedge d\tilde{\theta} \,. \end{aligned} \tag{1.158}$$

To conclude this dissertation, we observe that Abelian T-duality is only possible when the theory is invariant under translations along the θ direction. This is usually realised in string theory and supergravity by considering a compact direction in the internal manifold. As we explained in section 1.2, a solution cannot depend on the coordinate over which we compactify. Thus, this compact direction is usually taken to be a circle encoding an Abelian U(1) symmetry. The next natural step is to wonder whether or not this can be generalised into a non-Abelian group of isometries, realised by a higher-dimensional manifold.

Non-Abelian T-duality

For a reference for non-Abelian T-duality (NATD) with respect to a general isometry group G, one can check [85]. The main idea is to rewrite the action (1.66) in a way that emphasises the isometries,

$$\begin{aligned} S =& \frac{1}{4\pi\alpha'} \int_\Sigma d^2\sigma \, [Q_{\mu\nu} \partial_+ X^\mu \partial_- X^\nu + Q_{\mu n} \partial_+ X^\mu \partial_- X^n + \\ & + Q_{n\mu} \partial_+ X^n \partial_- X^\mu + Q_{mn} \partial_+ X^m \partial_- X^n + \frac{\alpha'}{2} \sqrt{|h|} \Phi \mathcal{R}^{(2)} \Big] \,, \end{aligned} \tag{1.159}$$

where we have taken light cone coordinates, as defined in (1.39). We have also defined $Q_{MN} = g_{MN} + (B_2)_{MN}$ with $X^M = (X^\mu, X^n)$ and assumed that there are N isometries so $\mu = 0, 1, \ldots D - N - 1$ and $n = D - N, \ldots, D - 1$. The matrix Q_{MN} in general depends on the X^n and said coordinates transform under the isometries as

$$X^n \to g^n{}_m X^m \quad \text{for} \quad g^n{}_m \in G \,. \tag{1.160}$$

Following what we did in the Abelian case, we are interested in gauging (1.160) in order to obtain a new action upon integrating the gauge potentials and fixing the gauge. For this purpose, the regular partial derivatives must be replaced by covariant ones in (1.159),

$$\partial_\pm X^n \to D_\pm X^n = \partial_\pm X^n + i(A_\pm)^n{}_m X^m \,, \tag{1.161}$$

where $A_\pm = A^a_\pm T_a$ is a gauge potential with $\{T_a\}_{a=1}^N$ a basis of an N-dimensional representation of the Lie algebra associated to G. Its transformation under the action of the group is given by

$$A_\pm \to g(A_\pm - i\partial_\pm)g^{-1}\,. \tag{1.162}$$

Apart from substituting the regular derivatives with the covariant ones, we add the following term to the action

$$-i\int d^2\sigma\,\mathrm{Tr}(vF_\pm)\,, \tag{1.163}$$

where $v = v^a T_a$ with Lagrange multipliers v^a and $F_\pm$ is the field strength defined below,

$$F_\pm = \partial_+ A_- - \partial_- A_+ - [A_+, A_-]\,. \tag{1.164}$$

We observe that the trace in (1.163) is simply the Killing form of the Lie algebra of G, as both v and F are in the adjoint representation of said algebra. Now we must choose a gauge. If we want to integrate on v, its equation of motion reads

$$\mathrm{Tr}(T_a F_\pm) = 0 \quad \text{for} \quad a = 1, 2, \ldots, N\,. \tag{1.165}$$

For a semisimple group, this implies that $F_\pm = 0$ and, therefore, A must be pure gauge and we can set $A_\pm = 0$, recovering the original action (1.106). However, if the group is not semisimple, the Killing form may be degenerate and $F_\pm$ may not be zero. The v field must be assumed to be in the dual basis of the T_a. Upon the previous considerations the action (1.159) becomes

$$\begin{aligned} S_{\text{gauge}} =&\frac{1}{4\pi\alpha'}\int_\Sigma d^2\sigma\,[Q_{\mu\nu}\partial_+X^\mu\partial_-X^\nu + Q_{\mu n}\partial_+X^\mu D_-X^n + Q_{n\mu}D_+X^n\partial_-X^\mu + \\ &+Q_{mn}D_+X^mD_-X^n - i\,\mathrm{Tr}(vF_\pm) + \frac{\alpha'}{2}\sqrt{|h|}\Phi\mathcal{R}^{(2)}\bigg]\,, \end{aligned} \tag{1.166}$$

In order to obtain the non-Abelian T-dual, we expand this action so that the $A_\pm$ appear explicitly an integrate by parts in order to eliminate its derivatives,

$$S_{\text{gauge}} =S[X] + \frac{1}{4\pi\alpha'}\int_\Sigma d^2\sigma\left[h_{+a}A^a_- + h_{-a}A^a_+ - A^a_+M_{ab}A^b_-\right]\,, \tag{1.167}$$

where $S[X]$ is the original action (1.159) and

$$\begin{aligned} h_{\pm a} =&i(Q_{\mu n}\partial_\pm X^\mu + Q_{mn}\partial_\pm X^m)(T_a)^n{}_kX^k \pm iT_R\partial_\pm v_a\,, \\ M_{ab} =&Q_{mn}(T_a)^m{}_k(T_b)^n{}_lX^kX^l + T_Rv^cf_{abc}\,. \end{aligned} \tag{1.168}$$

We have used that $[T_a, T_b] = if_{ab}{}^cT_c$ and also $\mathrm{Tr}(T_aT_b) = T_R\delta_{ab}$ in the semisimple case. If G is not semisimple, we take $\mathrm{Tr}(T_aT'_b) = T_R\delta_{ab}$ where $\{T'_a\}_{a=1}^N$ is the dual basis. The equations of motion for $A_\pm$ are

$$h_{+a} - A^b_+M_{ba} = 0\,, \quad h_{-a} - M_{ab}A^b_- = 0\,, \tag{1.169}$$

which fixes the gauge potentials as

$$A_+^a = h_{+b}(M^{-1})^{ba}\,, \quad A_-^a = (M^{-1})^{ab}h_{-b}\,. \tag{1.170}$$

If we substitute them in (1.167), then we obtain the non-Abelian T-dual action we were looking for,

$$\tilde{S} = S[X] + \frac{1}{4\pi\alpha'}\int_\Sigma d^2\sigma \left[h_{+a}(M^{-1})^{ab}h_{-b} + \frac{\alpha'}{2}\sqrt{|h|}\Delta\Phi\mathcal{R}^{(2)}\right]\,. \tag{1.171}$$

We observe that an explicit expression for the metric and NSNS 2-form potential cannot be written down, as they depend on the isometry group. On the other hand, the new dilaton is given by

$$\tilde{\Phi} = \Phi + \Delta\Phi = \Phi - \frac{1}{2}\log(\det M)\,, \tag{1.172}$$

which comes once again from the shift in the measure of the path integral. One needs to fix a gauge in order to eliminate the original X^m coordinates. However, this cannot be done in general as it again depends on the concrete isometry group. Once this is done, the v^a play the role of a new set of coordinates instead of the X^m.

Although the non-Abelian T-duality seems like a straightforward generalization of the Rocek-Verlinde formulation of the Abelian case, it is not so, as we encounter many difficulties in the former one to the point that it is not clear that non-Abelian T-duality is an exact symmetry of the theory. First although the non-Abelian NSNS sector we presented satisfies the equations (1.72), we lack a complete proof of them satisfying the supergravity equations at all orders in α'. We also have that the isometry group is completely destroyed by the non-Abelian T-duality (at least globally) and, therefore, this transformation is not an involution. One more inconvenient that does not appear in the Abelian case is that, while we derived the non-Abelian T-duality for a spherical worldsheet, we do not know how to generalise it to arbitrary topologies. The main obstacle to this generalisation is that, while in the Abelian case the isometry group of the dual theory is the representation ring of the original one, this no longer holds in the non-Abelian case, as the representation ring of a non-Abelian group is not even a group. This implies that we lose all global information of the non-Abelian T-dualised geometry.

Non-Abelian T-duality for SU(2)

Let us now consider the case where the isometry group of a Type II supergravity solution contains an SU(2) so we can T-dualise with respect to $G =$SU(2). We remark that SU(2) is a simple group and, therefore semisimple, which was an important fact in the general discussion. As some of our original solutions were obtained by doing exactly this, it is convenient for us to go into the details. The main source followed for the below derivation was [86]. For convenience, we will assume that the spacetime is described by a manifold containing a group submanifold that realises the SU(2) symmetry. The non-Abelian T-duality actually takes places in this submanifold. We start by considering the

Pauli matrices,

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\,, \quad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\,, \quad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\,. \tag{1.173}$$

The infinitesimal generators for SU(2) are then taken to be

$$t^i \equiv \frac{\sigma^i}{\sqrt{2}} \tag{1.174}$$

and satisfy

$$\mathrm{Tr}\big(t^i t^j\big) = \delta^{ij}\,, \quad [t^i, t^j] = i f^{ij}{}_k t^k = i\sqrt{2}\epsilon_{ijk} t^k\,. \tag{1.175}$$

In general, the left-invariant Maurer-Cartan one-forms of a Lie group G are defined for each group element $g \in G$ as below,

$$L^i = -i\,\mathrm{Tr}\big(t^i g^{-1} dg\big)\,, \quad \text{which satisfy} \quad dL^i = \frac{1}{2} f^i{}_{jk} L^j \wedge L^k\,. \tag{1.176}$$

On the other hand, the Euler parametrisation gives us a way of writing any element of a Lie group as the exponential of a vector in the associated Lie algebra. Thus, in the case at hand, we can write any $g \in$SU(2) as follows,

$$g = e^{\frac{i}{2}\phi\sigma_3} e^{\frac{i}{2}\theta\sigma_2} e^{\frac{i}{2}\psi\sigma_1} \text{ with } 0 \leq \theta \leq \pi\,,\ 0 \leq \phi \leq 2\pi\,,\ 0 \leq \psi \leq 2\pi\,. \tag{1.177}$$

In this parametrisation the left-invariants forms can be written explicitly,

$$\begin{aligned} L_1 =& \frac{1}{\sqrt{2}}(-\sin(\psi)\,d\theta + \cos(\psi)\sin(\theta)\,d\phi)\,, \\ L_2 =& \frac{1}{\sqrt{2}}(\cos(\psi)\,d\theta + \sin(\psi)\sin(\theta)\,d\phi)\,, \\ L_3 =& \frac{1}{\sqrt{2}}(\cos(\theta)\,d\phi + d\psi)\,. \end{aligned} \tag{1.178}$$

In these terms, we consider an SU(2)-symmetric Type II supergravity solution whose NSNS sector displays the below form,

$$\begin{aligned} ds^2 =& G_{\mu\nu}(x) dx^\mu dx^\nu + 2G_{\mu i}(x) dx^\mu L^i + g_{ij}(x) L^i L^j\,, \\ B_2 =& B_{\mu\nu}(x) dx^\mu \wedge dx^\nu + 2B_{\mu i}(x) dx^\mu \wedge L^i + \frac{1}{2} b_{ij}(x) L^i \wedge L^j\,, \\ \Phi =& \Phi(x)\,. \end{aligned} \tag{1.179}$$

where $\mu = 0, \ldots, 6$. We will need to use the vielbeins for the transformation of the RR sector, so we fix the notation as follows,

$$e^A = e^A_\mu dx^\mu\,, \quad e^a = k^a{}_j L^j + \lambda^a_\mu dx^\mu\,, \tag{1.180}$$

In our case, the original action (1.159) takes the following form

$$\begin{aligned} S =& \frac{1}{4\pi\alpha'}\int_\Sigma d^2\sigma \left[Q_{\mu\nu}\partial_+X^\mu\partial_-X^\nu + Q_{\mu i}\partial_+X^\mu L^i_- + \right. \\ & \left. + Q_{i\mu}L^i_+\partial_-X^\mu + E_{ij}L^i_+L^j_- + \frac{\alpha'}{2}\sqrt{|h|}\Phi\mathcal{R}^{(2)}\right], \end{aligned} \tag{1.181}$$

where $L^i_\pm = -i\,\mathrm{Tr}(t^i g^{-1}\partial_\pm g)$ and we have conveniently defined

$$Q_{\mu\nu} = G_{\mu\nu} + B_{\mu\nu}\,, \quad Q_{\mu i} = G_{\mu i} + B_{\mu i}\,, \quad Q_{i\nu} = G_{i\nu} + B_{i\nu}\,, \quad E_{ij} = g_{ij} + b_{ij}\,. \tag{1.182}$$

We observe that the action (1.181) is invariant under the global transformation

$$g \to h^{-1}g\,, \tag{1.183}$$

where $g, h \in$SU(2) and h does not depend on the worldsheet coordinates. Now we can apply the general proceeding by gauging the SU(2) symmetry and performing the following changes,

$$\partial_\pm g \to D_\pm g = \partial_\pm g - A_\pm g\,, \quad L^i_\pm \to L^i_\pm + iD^{ji}A^j_\pm\,, \tag{1.184}$$

where we have defined the matrix

$$D^{ij} = \mathrm{Tr}\big(t^i g t^j g^{-1}\big) \tag{1.185}$$

Then we add the Lagrange multiplier term (1.163), which takes the following form after integrating by parts,

$$-i\,\mathrm{Tr}(vF_\pm) = \mathrm{Tr}(i\partial_+vA_- - i\partial_-vA_+ - A_+fA_-) \quad \text{with} \quad f_{ij} = f_{ij}{}^k v_k\,. \tag{1.186}$$

The total action is invariant under

$$g \to h^{-1}g\,, \quad v \to h^{-1}vh\,, \quad A_\pm \to h^{-1}A_\pm h - h^{-1}\partial_\pm h\,, \tag{1.187}$$

where now $h = h(\sigma^+, \sigma^-) \in$SU(2).

The next step is to fix a gauge in order to simplify the next computations. The most reasonable choice of gauge is $\theta = \phi = \psi = 0$, which implies that $g = \mathbb{I}$. In this gauge the matrix $D = \mathbb{I}_3$ and the $L^i_\pm$ are set to 0. We end up with an action of the form (1.167), where $S[X]$ is no other than (1.181). As for the $h_\pm$ fields and the M matrix, they are given by the expression below,

$$h_{\pm i} = iQ_{\mu i}\partial_\pm X^\mu \pm i\partial_\pm v_i\,, \quad M = E + f\,. \tag{1.188}$$

The integration of the fields, given in general by (1.170), in our case are

$$A^i_+ = i(Q_{\mu j}\partial_+X^\mu + i\partial_+v_j)(M^{-1})^{ji}\,, \quad A^i_- = i(M^{-1})^{ij}(Q_{\mu j}\partial_-X^\mu - i\partial_-v_j)\,. \tag{1.189}$$

Finally, the dual action is given by

$$\begin{aligned}\tilde{S} = S[X] + \frac{1}{4\pi\alpha'}\int_\Sigma d^2\sigma \big[&-(Q_{\mu i}\partial_+X^\mu + i\partial_+v_i)(M^{-1})^{ij}(Q_{\mu j}\partial_-X^\mu - i\partial_-v_j)+\\ &+\frac{\alpha'}{2}\sqrt{|h|}\Delta\Phi\mathcal{R}^{(2)}\big] .\end{aligned} \tag{1.190}$$

From this expression, it is easy to infer the NSNS sector of the new theory,

$$\begin{aligned}\tilde{Q}_{\mu\nu} &= Q_{\mu\nu} - Q_{\mu i}(M^{-1})^{ij}Q_{\mu j} , \quad \tilde{E}_{ij} = M^{-1}_{ij} ,\\ \tilde{Q}_{\mu i} &= Q_{\mu j}M^{-1}_{ji} , \quad \tilde{Q}_{i\mu} = -M^{-1}_{ij}Q_{j\mu} .\end{aligned} \tag{1.191}$$

As for the dilaton, it is again given by (1.172).

We still need to address the transformation of the RR sector. The dual fluxes are given once again by equation (1.152), but now we have

$$\Omega^{-1} = (A_0\Gamma^1\Gamma^2\Gamma^3 + A_a\Gamma^a)\Gamma_{11} \quad \text{with} \quad a = 1,2,3 . \tag{1.192}$$

The coefficients in (1.192) are given by

$$A_0 = \frac{1}{\sqrt{1+\zeta^2}} , \quad A_a = \frac{\zeta^a}{\sqrt{1+\zeta^2}} \tag{1.193}$$

with

$$\zeta^a = \frac{k^a{}_i}{\det k}y^i \quad \text{with} \quad y_i = \epsilon_{ijk}b_{jk} + v_i \tag{1.194}$$

and k is the matrix that appears in (1.180). In order to obtain the new RR fluxes, we write the original ones as

$$F_p = G^{(0)}_p + G^a_{p-1}\wedge e^a + \frac{1}{2}G^{ab}_{p-2}\wedge e^a\wedge e^b + G^{(3)}_{p-3}\wedge e^1\wedge e^2\wedge e^3 , \tag{1.195}$$

where $G^{(0)}_p$, G^a_{p-1}, G^{ab}_{p-2} and $G^{(3)}_{p-3}$ do not have legs along the three isometry directions. Consequently, we have

$$\not{F}_p = \not{G}^{(0)}_p\mathbb{I}_{10} + \not{G}^a_{p-1}\Gamma^a + \frac{1}{2}\not{G}^{ab}_{p-2}\Gamma^{ab} + \not{G}^{(3)}_{p-3}\Gamma^{123} , \tag{1.196}$$

where the $\not{G}_q$ are defined by contracting with the Γ^A matrices corresponding to the seven-dimensional spectator spacetime. Upon using identities of gamma matrices and simplifying, the transformation of (1.196) can be proven to be

$$\not{F}_p\Omega^{-1} = \tilde{\not{F}}_{p-3} + \tilde{\not{F}}_{p-1} + \tilde{\not{F}}_{p+1} + \tilde{\not{F}}_{p+3} , \tag{1.197}$$

where

$$\begin{aligned}
\tilde{\not{F}}_{p-3} =& - A_0 \not{G}^{(3)}_{p-3} \,, \\
\tilde{\not{F}}_{p-1} =& A_a \not{G}^{a}_{p-1} - \frac{A_0}{2} \not{G}^{ab}_{p-2} \epsilon^{abc} \Gamma^c - A_a \not{G}^{ab}_{p-2} \Gamma^b + \frac{A_a}{2} \not{G}^{(3)}_{p-3} \epsilon^{abc} \Gamma^{bc} \,, \\
\tilde{\not{F}}_{p+1} =& A_a \not{G}^{(0)}_{p} \Gamma^a + \frac{A_0}{2} \not{G}^{a}_{p-1} \epsilon^{abc} \Gamma^{bc} - A_a \not{G}^{b}_{p-1} \Gamma^{ab} + \frac{A_a}{2} \not{G}^{bc}_{p-3} \epsilon^{abc} \Gamma^{123} \,, \\
\tilde{\not{F}}_{p+3} =& A_0 \not{G}^{(0)}_{p} \Gamma^{123} \,.
\end{aligned} \tag{1.198}$$

The new RR fluxes can be read off directly from this,

$$\tilde{F}_p = \tilde{G}^{(0)}_p + \tilde{G}^{a}_{p-1} \wedge \tilde{e}^a + \frac{1}{2} \tilde{G}^{ab}_{p-2} \wedge \tilde{e}^a \wedge \tilde{e}^b + \tilde{G}^{(3)}_{p-3} \wedge \tilde{e}^1 \wedge \tilde{e}^2 \wedge \tilde{e}^3 \,, \tag{1.199}$$

where

$$\begin{aligned}
\tilde{G}^{(0)}_p &= e^{\Phi - \tilde{\Phi}} \Big(-A_0 G^{(3)}_p + A_a G^a_p \Big) \\
\tilde{G}^{a}_{p-1} &= e^{\Phi - \tilde{\Phi}} \Big(-\frac{A_0}{2} \epsilon^{abc} G^{bc}_{p-1} + A_b G^{ab}_{p-1} + A_a G^{(0)}_{p-1} \Big) \\
\tilde{G}^{ab}_{p-2} &= e^{\Phi - \tilde{\Phi}} \Big[\epsilon^{abc} \Big(A_c G^{(3)}_{p-2} + A_0 G^{c}_{p-2} \Big) - \big(A_a G^{b}_{p-2} - A_b G^{a}_{p-2} \big) \Big] \\
\tilde{G}^{(3)}_{p-3} &= e^{\Phi - \tilde{\Phi}} \Big(\frac{A_a}{2} \epsilon^{abc} G^{bc}_{p-3} + A_0 G^{(0)}_{p-3} \Big) \,.
\end{aligned} \tag{1.200}$$

This transformation maps odd-ranked forms into even-ranked ones and vice-versa, supporting the fact that T-duality transforms Type IIA into Type IIB and the other way round also in the non-Abelian case with respect to SU(2).

The Wess-Zumino and DBI actions

Let us now take a look to what happens with open strings. As we saw in section 1.1 below equation (1.25), the coordinates of the ends open strings that evolve according to the Nambu-Goto action (1.2) satisfy either Neumann or Dirichlet boundary conditions. These open strings may have their ends fixed to so-called D-branes, which are the manifolds described by the Dirichlet boundary conditions. Nevertheless, free strings are possible in the bosonic string model.

Open strings can also appear in the Type II string theories, but they must always end on D-branes. As we will justify below, the presence of these open strings render the D-branes dynamical objects, which are described by worldvolume actions. An effective version of such an action is given below [87],

$$S_{eff} = S_{\mathrm{WZ}} + S_{\mathrm{DBI}} \,. \tag{1.201}$$

We have that the first term is a Wess-Zumino action generalising that of (1.110) and it reads

$$S_{\mathrm{WZ}}(\mathrm{D}p) = T_p \int_{M_{p+1}} C \wedge e^{B_2 + 2\pi\alpha' F} \,, \tag{1.202}$$

where T_p is the tension of the Dp-brane, M_{p+1} its worldvolume, C is a formal sum of the RR gauge potentials defined as

$$C = \begin{cases} \sum_{i=0}^{4} C_{2p+1} & \text{in Type IIA}\,, \\ \sum_{i=0}^{4} C_{2p} & \text{in Type IIB} \end{cases} \tag{1.203}$$

and

$$F = dA \tag{1.204}$$

with A the U(1) Born-Infeld gauge potential living in the brane. This action can be easily generalised to the case where F is a U(N) gauge field by taking the trace of $e^{B_2+2\pi\alpha' F}$ in (1.202), but we will not need it for the purposes of this thesis. We observe that, in the particular case in which we are only considering a Dp-brane, the Wess-Zumino term reads

$$S_{\mathrm{WZ}}(\mathrm{D}p) = T_p \int_{M_{p+1}} C_{p+1}\,. \tag{1.205}$$

This simply means that the Dp-brane is charged under C_{p+1} and, in this case, T_p plays the role of its charge density. In general, a Dp-brane can be coupled to all the present RR gauge potentials, which is reflected by (1.202).

The second term in (1.201) is the *Dirac-Born-Infeld (DBI) action*, which is displayed below

$$S_{\mathrm{DBI}}(\mathrm{D}p) = -\frac{1}{(2\pi)^p \alpha'^{(p+1)/2} g_s} \int d^{p+1}x e^{-\Phi} \sqrt{-\det(\mathcal{P}[g+B_2] + 2\pi\alpha' F)_{\mu\nu}}\,, \tag{1.206}$$

where x^μ denotes the worldvolume directions of the considered Dp-brane and

$$\mathcal{P}[\Omega]_{\mu\nu} = \Omega_{MN}\partial_\mu y^M \partial_\nu y^N \tag{1.207}$$

the pull-back over it. This term can be obtained via T-duality of the following effective action,

$$S_{eff} = -\frac{1}{(2\pi)^9 \alpha'^5 g_s} \int dx^{10} e^{-\Phi} \sqrt{\det(g + B_2 + 2\pi\alpha' F)_{MN}}\,, \tag{1.208}$$

which describes the aforementioned open strings charged under the Born-Infeld gauge field F in the low-energy regime [88]. We take a coordinate system $\{x^M\} = \{x^\mu, \theta\}$, where θ parametrises a circle of length $2\pi\sqrt{\alpha'}$. We also assume that all the coordinates of the gauge potential A_M are independent of θ. Additionally, we assume that A_θ is proportional to the identity matrix so we can identify $A_\theta = -y$ for some function $y = y(x^\mu)$. Thus, we have

$$F_{\theta\mu} = \partial_\theta A_\mu - \partial_\mu A_\theta = -\partial_\mu A_\theta \equiv \partial_\mu y\,. \tag{1.209}$$

For an arbitrary 10d square matrix M_{MN}, we have the following formula,

$$\det(M_{MN}) = M_{\theta\theta} \det\left(M_{\mu\nu} - \frac{M_{\mu\theta}M_{\theta\nu}}{M_{\theta\theta}} \right). \tag{1.210}$$

Applying it to our determinant, we have

$$\begin{aligned}\det(g + B_2 + F)_{MN} =& g_{\theta\theta} \det\Big[(\tilde{g} + \tilde{B}_2)_{\mu\nu} + F_{\mu\nu} + \tilde{g}_{\tilde{\theta}\tilde{\theta}}\partial_\mu y \partial_\nu y + \partial_\mu y(\tilde{g} + \tilde{B}_2)_{\tilde{\theta}\nu} + \\ &+\partial_\nu y(\tilde{g} - \tilde{B}_2)_{\tilde{\theta}\mu}\Big] ,\end{aligned} \tag{1.211}$$

where $\tilde{g}$ and $\tilde{B}_2$ are the dual fields as given by (1.107). Taking this into account, one can easily see that (1.208) can be rewritten as

$$S_{eff} = -\frac{1}{(2\pi)^8 \alpha'^{9/2} g_s} \int dx^9 e^{-\tilde{\Phi}} \sqrt{\det\Big(\mathcal{P}[\tilde{g} + \tilde{B}_2] + 2\pi\alpha' F\Big)_{\mu\nu}} \equiv S_{\text{DBI}}(\text{D8}) , \tag{1.212}$$

where we have used that

$$\int d\theta = 2\pi\sqrt{\alpha'} \quad \text{and} \quad \tilde{\Phi} = \Phi - \frac{1}{2}\det(g_{\theta\theta}) . \tag{1.213}$$

We observe that (1.207) is valid for a brane with a general worldvolume given by the equation below,

$$\tilde{x}^M = y^M(x^\mu) \quad \text{with} \quad \{\tilde{x}^M\} = \{x^\mu, \tilde{\theta}\} . \tag{1.214}$$

However, in our case we have considered $y^{\tilde{\theta}}(x^\mu) = y(x^\mu)$ and $y^\mu(x^\nu) = x^\mu$.

This analysis leads us to a very interesting interpretation of open strings as D9-branes, as (1.208) looks like the DBI action for a ten-dimensional worldvolume. Moreover, one can iterate this process to obtain the DBI action for all the Dp-branes, which is precisely (1.206).

1.4.2. S-duality

The second duality we must consider is *S-duality*. It is a duality that relates the strong coupling limit of a theory with the weak-coupling limit of another or the same one. This can be specified by the following relation of their string coupling constants,

$$g_s \leftrightarrow \tilde{g}_s = \frac{1}{g_s} . \tag{1.215}$$

This means that the duality is non-perturbative in g_s and a general derivation is not known. One interesting remark is that the role played by α' remains unchanged by this transformation, ergo an order by order expansion in said parameter could be written down.

One important example of S-duality is the relation between Heterotic SO(32) and Type I string theories. Both of them are very similar, as can be appreciated at the supergravity level by comparing (1.81) and (1.94). The similarities are even more obvious if we write these actions in the so-called *Einstein frame*, as opposed to the *string frame* that we used so far. They are related by the following expression for the metrics,

$$g^E_{MN} = e^{-\frac{4}{D-2}\phi} g^S_{MN} . \tag{1.216}$$

In this frame, the bosonic part of the actions of the aforementioned theories read

$$
\begin{aligned}
S^E(I) =& \frac{1}{2\kappa_{10}^2}\int d^{10}x\sqrt{|g|}\left[R^E - \frac{1}{2}(\partial\Phi)^2 - \frac{e^{\Phi}}{12}F_3^2 - \frac{1}{4}e^{\frac{\Phi}{2}}F_2^I F_{2I}\right] ,\\
S^E(H) =& \frac{1}{2\kappa_{10}^2}\int d^{10}x\sqrt{|g|}\left[R^E - \frac{1}{2}(\partial\Phi)^2 - \frac{e^{-\Phi}}{12}H_3^2 - \frac{1}{4}e^{-\frac{\Phi}{2}}F_2^I F_{2I}\right] .
\end{aligned}
\tag{1.217}
$$

It is clear that one theory is mapped into the other by the following transformations

$$
\Phi \leftrightarrow -\Phi\,, \quad F_3 \leftrightarrow H_3\,. \tag{1.218}
$$

Taking this and (1.216) into account, one concludes that, in the string frame, the actions of Type I and heterotic SO(32) supergravities are related by

$$
\Phi \leftrightarrow -\Phi\,, \quad F_3 \leftrightarrow H_3\,, \quad g_{MN} \leftrightarrow e^{-\Phi}g_{MN} \tag{1.219}
$$

Another important example is that of Type IIB. As it can be seen in (1.80) and (1.91), the role played by H_3 and F_3 in the overall action is quite similar. A more detailed analysis shows that the $\mathrm{SL}(2,\mathbb{R})$ group can act intermixing these two fields in a concrete manner that leaves the action invariant. Let us start by considering one such rotation

$$
R = \begin{pmatrix} \cos\xi & -\sin\xi \\ \sin\xi & \cos\xi \end{pmatrix} . \tag{1.220}
$$

If we act with this transformation on the "seed" fields, denoted by $F_{(n),s}$, Φ_s and $ds^2_{10,s}$, we obtain new ones with the aid of the following relations

$$
\begin{aligned}
\tau =& \frac{\cos\xi\,\tau_s - \sin\xi}{\sin\xi\,\tau_s + \cos\xi}\,, \quad F_{(5)} = F_{(5),s}\,, \quad ds^2_{10} = |\cos\xi + \sin\xi\,\tau|\,ds^2_{10,s}\,,\\
\begin{pmatrix} \hat{F}_{(3)} \\ H_{(3)} \end{pmatrix} =& \begin{pmatrix} \cos\xi & -\sin\xi \\ \sin\xi & \cos\xi \end{pmatrix}\begin{pmatrix} F_{(3),s} \\ H_{(3),s} \end{pmatrix} , \quad F_{(3)} = \hat{F}_{(3)} - C_{(0)}H_{(3)}\,,
\end{aligned}
\tag{1.221}
$$

where $\tau = C_{(0)} + ie^{-\Phi}$ is known as the axio-dilaton. We observe that either a dilaton or $C_{(0)}$ may be generated by these relations, even in cases where either of the seed ones were zero, provided that $\tau_s \neq 0$. It must be also noticed that these transformations depend on the parameter ξ; this parameter is real in principle, but quantisation of charges fixes it at either $\xi = 0$ or $\xi = \pi/2$.

Beyond the supergravity limit, there is also heavy evidence that S-duality must hold. Some non-perturbative checks has been made, usually considering BPS protected operators, i.e. those preserving some, but not all, of the supersymmetries. For instance, the branes, as BPS states, have been used to collect evidence which supports this duality.

As a simple test, one can study the S-duality that relates a D1-brane in Type I string theory to an F1-brane of SO(32) heterotic string theory by comparing their tensions and seeing if they agree [89]. The tension of a D1-brane in Type I string theory is given by

$$
T^{\mathrm{I}}_{\mathrm{D1}} = \frac{1}{g_s^{\mathrm{I}} 2\pi (l_s^{\mathrm{I}})^2}\,, \tag{1.222}
$$

where the I superscript emphasises that these quantities belong to Type I string theory. Besides, the tension of an F1-brane of SO(32) heterotic string theory reads

$$T^{\mathrm{H}}_{\mathrm{F1}} = \frac{1}{2\pi (l^{\mathrm{H}}_s)^2} \,, \tag{1.223}$$

where the H superscript corresponds to SO(32) heterotic string theory. Now it is easy to see that (1.222) and (1.223) are actually the same. As we saw in (1.219), the dilaton of both theories is related by $\Phi \leftrightarrow -\Phi$. We recall that, prior to the normalisation considered in (1.76), the string action was written in terms of ϕ instead of Φ. In that language, it is clear that the dilaton must transform as $\phi \leftrightarrow -\phi$ under S-duality. As the string coupling is the vev of e^{ϕ}, we conclude that

$$g^{\mathrm{I}}_s = \frac{1}{g^{\mathrm{H}}_s} \,. \tag{1.224}$$

On the other hand, the rescaling of the metric in (1.219) implies that, although the string length is the same in both theories, it is measured in different ways. Consequently, we observe that

$$l^{\mathrm{I}}_s = l^{\mathrm{H}}_s \sqrt{g^{\mathrm{h}}_s} \,. \tag{1.225}$$

Finally, applying (1.224) and (1.225) to (1.222), one obtains (1.223). These supports the idea that these two kinds branes are just two descriptions of the same object, which is the cornerstone of all dualities.

1.4.3. M-theory and U-duality

Although it is not strictly a duality, it is important to know that M-theory arises as the strongly coupled limit of Type IIA string theory. If we start with a solution to eleven-dimensional supergravity which presents a circle direction χ, on which all the fields are independent, it can be truncated to obtain a Type IIA supergravity solution. The formulae that relates these two solutions is given by the expression below,

$$\begin{aligned} ds^2_{11} =& e^{-2\Phi/3} ds^2_{10} + e^{4\Phi/3} \left(d\chi + C_{(1)} \right)^2 \,, \\ G_{(4)} =& F_{(4)} + H_{(3)} \wedge \left(d\chi + C_{(1)} \right) . \end{aligned} \tag{1.226}$$

Furthermore, these equations relate the actions of 11d and Type IIA supergravities.

Let us now consider an eleven-dimensional supergravity solution with two circular directions forming a torus, on which the fields are independent. The Type IIA backgrounds that result after dimensionally reducing along each of the circles are related via the so-called *U-duality*. It consists on a chain of T-S-T dualities linking both Type IIA supergravity solutions as displayed in Figure 1.2.

Before moving on to the next chapter, let us briefly recall the most relevant concepts that have been discussed in the current one. We have started by presenting the string action and the boundary conditions for open and closed strings in section 1.1. Then the concept of dimensional reduction has being described and illustrated with a simple

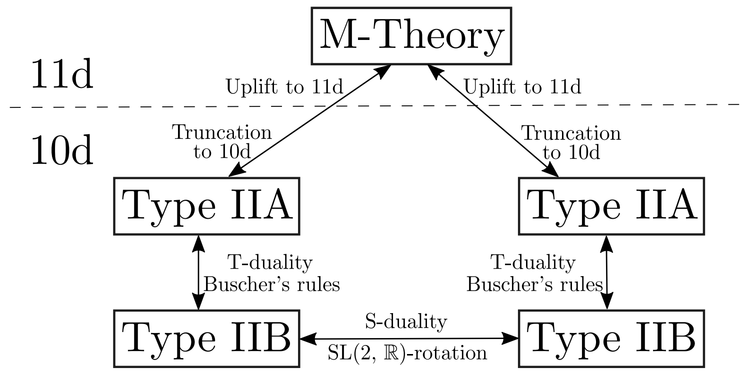


Figure 1.2: U-duality as a chain of T-S-T dualities coming from two Type IIA solutions with the same 11d origin.

example in section 1.2. In section 1.3 we saw that a gravity theory is always recovered as the low-energy limit of a string theory and, in particular, the five superstring theories reduce to five different supergravity theories. Finally, section 1.4 was dedicated to the review of the dualities connecting the different supergravity theories. We first explored T-duality, how it arises naturally as the interchanging of the momentum and winding numbers of string states, how the presence of an isometry (Abelian or not) in a Type IIA/B supergravity solution can be exploited to derive a new background in Type IIB/A string theory and how the DBI action of D-branes can be derived by applying this duality. We also introduced S-duality as a strong-weak duality and displayed the formulae for its realisation in Type IIB supergravity. We concluded this section by speaking of U-duality as a chain of T-S-T dualities that connect Type IIA supergravity solutions that share the same origin in M-theory.

Chapter 2

Supergravity

In this chapter, we want to explore supergravity from another perspective. These theories originally arose, not from string theory, but from the study of supersymmetry as a gauge theory. With that in mind, we provide a brief introduction to the notions of supersymmetry and superspace and how they can be gauged to obtain supergravity in section 2.1. We then introduce some basic notions and explore some simple solutions of Type II and 11d supergravities in sections 2.2 and 2.3, respectively. This will provide some context when we present our original solutions in the second part of the thesis. Finally, section 2.4 is devoted to introducing the notion of G-structure and how it is related to supersymmetry.

If the reader wishes to study this topics in more detail, we recommend to take a look at the references we used, namely [73, 90, 91].

2.1. Gauging supersymmetry

We begin by considering the D-dimensional Minkowski spacetime $\mathbb{R}^{1,D-1}$ with metric η_{MN} characterised by a signature $(-,+,+,\ldots,+)$. Its isometry group is the Poincaré group and its infinitesimal transformations form the Poincaré algebra. The generators of said algebra are P_M and M_{MN}, related to translations and Lorentz transformations, respectively. We recall that their commutation relations are the ones below,

$$\begin{aligned} [M_{MN}, M_{PQ}] =& \eta_{NP} M_{MQ} - \eta_{MP} M_{NQ} + \eta_{MQ} M_{NP} - \eta_{NQ} M_{MP}\,, \\ [M_{PQ}, P_M] =& P_P \eta_{QM} - P_Q \eta_{PM}\,, \quad [P_M, P_N] = 0\,. \end{aligned} \tag{2.1}$$

We are interested in extending this Lie algebra to include supersymmetry in an intrinsic way. This can be achieved by extending the spacetime (Minkowski in our case) into a *superspacetime* by adding a set of anticommuting coordinates $\{\psi^M\}$ to the regular "bosonic" ones $\{x^M\}$. This mirrors what we did in section 1.1 when we built the superstring action from the bosonic one. Supersymmetry then arises naturally as the invariance under the interchange of both kinds of coordinates. This gives rise to a set of anti-Hermitian fermionic generators of these new symmetries Q_α, where α is the spacetime spinor index, and receive the name of *supercharges*. If more than one set of fermionic coordinates is present, there is more than one collections of supercharges. These are then labelled as Q_α^I with $I = 1, 2, \ldots, \mathcal{N}$ the index that runs over the different sets of fermionic coordinates. Moreover, the symmetry that transforms the different supercharges into one another receives the name of *R-symmetry*. The new (anti)commutation relations depend on the spacetime dimension D and on the number of supersymmetries $\mathcal{N}$. For instance, when $D = 4$ and $\mathcal{N} = 1$, we have the below relations,

$$\{Q_\alpha, \bar{Q}_{\dot{\beta}}\} = -\frac{1}{2} P_M (\gamma^M)_{\alpha\dot{\beta}}\,, \quad [Q_\alpha, P_M] = 0\,, \quad [M_{MN}, Q_\alpha] = -\frac{1}{2} (\gamma_{MN})_\alpha{}^\beta Q_\beta\,, \tag{2.2}$$

where γ^M are the gamma matrices in four dimensions and

$$\gamma_{M_1 \ldots M_n} = \gamma_{[M_1} \gamma_{M_2} \cdots \gamma_{M_n]}\,. \tag{2.3}$$

This new algebra is called the super-Poincaré algebra. Supergravity arises when we gauge it by considering local supersymmetry transformations in the same way that General Relativity appears when the regular Poincaré algebra is gauged. In more concrete terms, this means that the constant spinor ϵ parametrising the transformations in the global case depends on the bosonic coordinates when we speak about supergravity. Supergravity theories are non-linear and they contain a gauge or gravity multiplet and optionally matter multiplets. The gauge multiplet consists in the graviton, given by the vielbein $e_M^a(x)$ in the second-order formalism, and a set of $\mathcal{N}$ spin 3/2 gravitino fields $\psi_M^I(x)$ with $I = 1, 2, \ldots, \mathcal{N}$. The concrete theory depends, as before, both on the dimension and the amount of supersymmetry. However, there is a part of the supergravity action that is shared by the $\mathcal{N} = 1$ case in all dimensions. It consists on the Hilbert action (written here in the second order formalism) plus a Lorentz and diffeomorphism invariant extension of the Rarita-Schwinger action for the gravitino,

$$S = S_2 + S_{3/2} \tag{2.4}$$

with

$$S_2 = \frac{1}{2\kappa_{10}^2} \int d^D x e e^{aM} e^{bM} R_{MNab}(\omega)\,, \quad S_{3/2} = -\frac{1}{2\kappa_{10}^2} \int d^D x e \overline{\psi}_M \Gamma^{MNP} D_N \psi_P\,, \tag{2.5}$$

where e is the determinant of the vielbein, Γ^M are the D-dimensional gamma matrices and the Γ with more than one index are defined as in (2.3). The gravitino covariant derivative reads

$$D_M \psi_N \equiv \partial_M \psi_N + \frac{1}{4} \omega_{Mab} \gamma^{ab} \psi_N \tag{2.6}$$

and $\omega_{Mab}(e)$ is the torsion-free spin connection below,

$$\omega_M{}^{ab}(e) = 2 e^{N[a} \partial_{[M} e_{N]}{}^{b]} - e^{N[a} e^{b]\sigma} e_{Mc} \partial_N e_\sigma{}^c\,. \tag{2.7}$$

Action (2.4) is invariant under the following local supersymmetry transformation,

$$\delta_\epsilon e_M^a = \frac{1}{2} \bar{\epsilon} \gamma^a \psi_M\,, \quad \delta_\epsilon \psi_M = D_M \epsilon = \partial_M \epsilon + \frac{1}{4} \omega_{Mab} \gamma^{ab} \epsilon\,. \tag{2.8}$$

There is a matter that must be addressed now: the supersymmetry solutions are never invariant under all the possible supersymmetry transformations. Those solutions that preserve some supersymmetry are called *supersymmetric* or *BPS states*. The question of how much supersymmetry does a particular solution preserve is a crucial one. Let us consider generic bosonic and fermionic fields B and F. The infinitesimal form of the supersymmetry transformation that interchanges them is given below,

$$\delta_\epsilon B \sim \epsilon F \,, \quad \delta_\epsilon F \sim \partial\epsilon + B\epsilon \,. \tag{2.9}$$

As it is usually done in supergravity, we consider purely bosonic solutions, which correspond to classical solutions. This is because in nature we only observe macroscopic bosonic fields, while fermions behave as particles at the classical level. A solution is said to be supersymmetric when (2.9) vanishes for some parameter $\epsilon(x)$. As we are considering $F = 0$, the condition for the bosons is always met, while that for the fermions reads

$$\delta_\kappa F \sim \partial\epsilon + B\epsilon = 0 \,. \tag{2.10}$$

These can be interpreted as an infinitesimal reparametrisation of the superspace. This heavily resembles the general relativistic case, in which Killing vectors are those whose Lie derivative leaves the metric invariant. For this reason, (2.10) is called the *Killing spinor equation* and its solution is the product of the infinitesimal anticommuting scalar ϵ and the finite commuting spinor κ, i.e. the *Killing spinor*. These spinors are the supersymmetry generators and, therefore, the dimension of the solution space of (2.10) is the number of preserved supersymmetries.

Each supergravity theory has a different Killing spinor equation. For instance, in Type II supergravity theories, the fermionic fields consist of two gravitini ψ^I_M and two dilatini λ^I with $I = 1, 2$ [92]. Thus, the Killing equation in these theories corresponds to the vanishing of their infinitesimal variation with respect to said spinor,

$$\delta_\kappa \psi_M = 0 \,, \quad \delta_\kappa \lambda = 0 \,. \tag{2.11}$$

More explicitly, these equations can be written in terms of a supersymmetry parameter ϵ,

$$\begin{aligned} \delta_\kappa \psi_M =& \nabla_M \epsilon + \frac{1}{4} \not{H}_M \mathcal{P} \epsilon + \frac{e^\Phi}{16} \sum_n \not{F}_n \Gamma_M \mathcal{P}_n \epsilon = 0 \,, \\ \delta_\kappa \lambda =& \left(\not{\partial}\Phi + \frac{1}{2} \not{H} \mathcal{P} \right) \epsilon + \frac{e^\Phi}{8} \sum_n (-1)^n (5-n) \not{F}_n \mathcal{P}_n \epsilon = 0 \,. \end{aligned} \tag{2.12}$$

We have used that $M = 0, \ldots, 9$. In addition, $n = 0, 2, 4, 6, 8$ for Type IIA and $n = 1, 3, 5, 7, 9$ for Type IIB. Besides, ψ_M, λ and ϵ are Majorana-Weyl bispinors of opposite (same) chirality in Type IIA (Type IIB). As for $\mathcal{P}$ and $\mathcal{P}_n$, they are 2×2 matrices as depicted below,

$$\mathcal{P} = \begin{cases} \Gamma_{11} & \text{in Type IIA} \,, \\ -\sigma^3 & \text{in Type IIB} \,, \end{cases} \quad \mathcal{P}_n = \begin{cases} \Gamma_{11}^{(n/2)} \sigma^1 & \text{in Type IIA} \,, \\ \begin{cases} \sigma^1 & \text{for } \frac{n+1}{2} \text{ even} \\ i\sigma^2 & \text{for } \frac{n+1}{2} \text{ odd} \end{cases} & \text{in Type IIB} \,. \end{cases} \tag{2.13}$$

The slash represents contraction with gamma matrices following (1.151).

2.2. Solutions in Type II supergravity

In our original work we explored, among other things, supergravity solutions in Type II. Thus, it is convenient to review the most basic concepts and solutions related to these theories.

First of all, we must talk about branes, which appear as solutions to the Type IIA/B supergravities. As mentioned previously, they are extended objects that arise naturally in non-perturbative string theory, but also play a role here. We already spoke of D-branes when presenting the string boundary conditions in section 1.1 and presented their effective worldvolume action in subsection 1.4.1. Moreover, they are the sources of the RR fluxes, which were introduced in section 1.3. As only certain RR fluxes appeared in each kind of Type II supergravity (either of odd or even rank), something similar happens with the branes as summarised in Table 2.1. In addition, in Type II string theory the so-called NS-branes appear. In the classical theory they are seen as the sources of the NSNS two-form B_2. In more concrete terms, we have the *fundamental strings* F1, which are the sources of the electric components of B_2 and its magnetic counterpart the *NS5-brane*, which are branes with five spacial dimensions and which are the sources of the magnetic part of B_2. Let us now review the Type II supergravity solutions associated to a single

	Type IIA	Type IIB
NS-branes	F1, NS5	
D-branes	D0, D2, D4, D6, D8	D(−1), D1, D3, D5, D7

Table 2.1: Table summarising the branes that appear in Type II string theory.

stationary brane. The metric of these backgrounds can be written either in the string or Einstein frames so we present both. For a set of fundamental strings expanding along the x direction, we have the following fields,

$$\begin{aligned} ds_E^2 =& H_{\mathrm{F1}}^{-\frac{3}{4}}(-dt^2+dx^2)+H_{\mathrm{F1}}^{\frac{1}{4}}ds_{\mathbb{R}^8}^2\,,\\ ds_s^2 =& H_{\mathrm{F1}}^{-1}(-dt^2+dx^2)+ds_{\mathbb{R}^8}^2\,,\\ e^{-2\Phi} =& H_{\mathrm{F1}}\,,\quad B_2=H_{\mathrm{F1}}^{-1}dt\wedge dx\,, \end{aligned} \tag{2.14}$$

where the equations of motion and Bianchi identities boil down to the following,

$$\nabla_{\mathbb{R}^8}^2 H_{\mathrm{F1}}=0\,. \tag{2.15}$$

In other words, the function H_{F1} which parametrises these solutions must be a harmonic function of the coordinates of the $\mathbb{R}^8$. For instance, we can take

$$H_{\mathrm{F1}}=1+\frac{q_{\mathrm{F1}}}{r^6}\,, \tag{2.16}$$

where r is the radial coordinate of $\mathbb{R}^8$ and q_{F1} is an integration constant. This function describes the effect of a single fundamental string localised in the origin of $\mathbb{R}^8$. As we explained before, this solution contains an electric NSNS two-form potential. On the other hand, if we consider parallel NS5 branes extending along directions parametrised by $\vec{x}_5$, the following Type II supergravity solution arises

$$
\begin{aligned}
ds_E^2 =& H_{\mathrm{NS5}}^{-\frac{1}{4}}(-dt^2 + d\vec{x}_5{}^2) + H_{\mathrm{NS5}}^{\frac{3}{4}} ds_{\mathbb{R}^4}^2 \,, \\
ds_s^2 =& - dt^2 + d\vec{x}_5{}^2 + H_{\mathrm{NS5}} ds_{\mathbb{R}^4}^2 \,, \\
e^{-2\Phi} =& H_{\mathrm{NS5}}^{-1} \,, \quad B_6 = H_{\mathrm{NS5}}^{-1} dt \wedge dx^1 \wedge \ldots \wedge dx^5 \,,
\end{aligned}
\tag{2.17}
$$

where, analogously to the previous case, we have the condition

$$
\nabla_{\mathbb{R}^4}^2 H_{\mathrm{NS5}} = 0 \,. \tag{2.18}
$$

The solution corresponding to a single NS5-brane set in the origin of the $\mathbb{R}^4$ is given by

$$
H_{\mathrm{NS5}} = 1 + \frac{q_{\mathrm{NS5}}}{r^2} \,, \tag{2.19}
$$

where r is the radial coordinate of said space. In this case the NSNS three-form flux is purely magnetic and can be obtained as follows,

$$
H_3 = e^{2\Phi} \star dB_6 \,, \tag{2.20}
$$

where $\star$ is the ten-dimensional Hodge dual operator.

As for the Dp-brane solutions, they all display a similar form, only changing slightly according to the concrete value of p,

$$
\begin{aligned}
ds_E^2 =& H_{\mathrm{D}p}^{\frac{p-7}{8}}(-dt^2 + d\vec{x}_p{}^2) + H_{\mathrm{D}p}^{\frac{p+1}{8}} ds_{\mathbb{R}^4}^2 \,, \\
ds_s^2 =& H_{\mathrm{D}p}^{-\frac{1}{2}}(-dt^2 + d\vec{x}_p{}^2) + H_{\mathrm{D}p}^{\frac{1}{2}} ds_{\mathbb{R}^{9-p}}^2 \,, \\
e^{-2\Phi} =& H_{\mathrm{D}p}^{\frac{p-3}{2}} \,, \quad C_{p+1} = H_{\mathrm{D}p}^{-1} dt \wedge dx^1 \wedge \ldots \wedge dx^p \,,
\end{aligned}
\tag{2.21}
$$

where, as always, $\vec{x}_p$ is the vector of spacial coordinates tangent to the Dp-branes and we have that

$$
\nabla_{\mathbb{R}^{9-p}}^2 H_{\mathrm{D}p} = 0 \,. \tag{2.22}
$$

We have that C_p is the RR p-form potential and its relation to the corresponding RR flux for the solutions (2.21) is given below

$$
F_{p+1} = dC_p \,. \tag{2.23}
$$

As before, if r is the radial coordinate of $\mathbb{R}^{9-p}$, the single brane solution localised at its centre is given by one of the following harmonic functions,

$$
H_{\mathrm{D}p} = 1 + \frac{q_{\mathrm{D}p}}{r^{7-p}} \quad \text{for} \quad p \leq 6 \,, \quad H_{\mathrm{D7}} = 1 + q_{\mathrm{D7}} \log|r| \,, \quad H_{\mathrm{D8}} = 1 + q_{\mathrm{D8}} |r| \,. \tag{2.24}
$$

As we saw in section 1.3, when $p \geq 4$ the contribution of the Dp-brane to the RR fluxes is taken to be magnetic in the following manner,

$$\begin{aligned} &F_4 = -\star F_6\,, \quad F_2 = \star F_8\,, \quad F_0 = \star F_{10} \quad \text{in} \quad \text{Type IIA}\,, \\ &F_3 = -\star F_7\,, \quad F_1 = \star F_9 \quad \text{in} \quad \text{Type IIB}\,. \end{aligned} \tag{2.25}$$

One last remark about brane solutions (either Neveu-Schwarz or Ramond) is that the H functions of said objects are defined so that they go to 1 in a certain limit ($r \to \infty$ in our case) and to infinity in the opposite limit ($r \to 0$). This can be physically interpreted as the observer either approaching the brane or distancing from it. In the latter case, the warping associated to the brane disappears, as expected. On the other hand, when $r \to 0$, which is often referred to in the literature as the *near-horizon limit*, one is zooming in the brane, thus observing it as a singularity.

There are a couple of objects appearing in Type II backgrounds, which are interesting in spite of not being branes. They are the *KK-monopoles* and *pp-waves*. As we see below, they only contribute to the metric, but not to the other fields.

A Kaluza-Klein monopole (or KK-monopole) is a six-dimensional object that becomes a monopole upon Kaluza-Klein compactification on a 6d fibre, hence the name. The metric of a spacetime that only contains a KK-monopole localised at $r = 0$ reads

$$ds^2 = ds^2_{\mathbb{R}^{1,5}} + H_{\mathrm{KK}}^{-1}\left(dz + A_m dx^m\right)^2 + H_{\mathrm{KK}} ds^2_{\mathbb{R}^3} \quad \text{with} \quad H_{\mathrm{KK}} = \frac{q_{\mathrm{KK}}}{r}\,, \tag{2.26}$$

where the monopole extends along $\mathbb{R}^{1,5}$ and z. We also have that $\{x^m\}_{m=1}^3$ parametrise the $\mathbb{R}^3$, $A_m = A_m(x^n)$ is a vector field on said space, their indices being uplifted and lowered with the Euclidean metric, and $r = \sqrt{\sum_{m=1}^3 (x^m)^2}$. The $\{z, x^m\}$ coordinates parametrise a so-called *Taub-NUT space*, which displays a singularity at $r = 0$, where the monopole is localised. We highlight that z is an isometry direction of this space. Additionally, we observe that, in the absence of a dilaton, the metric is the same in both the string and Einstein frames.

A pp-wave (or plane-fronted wave with parallel rays) is a kind of gravitational wave characterised by a metric which admits a covariantly constant null Killing vector l_M, meaning that

$$\nabla_M l_N = 0\,, \quad l^2 = l_M l^M = 0\,. \tag{2.27}$$

The metric of a pp-wave propagating along the z direction is given by

$$ds^2 = -H_{\mathrm{W}}^{-1} dt^2 + H_{\mathrm{W}}(dz - H_{\mathrm{W}}^{-1} dt)^2 + ds^2_{\mathbb{R}^8}\,, \tag{2.28}$$

where the equations of motion and Bianchi identities boil down to

$$\nabla^2_{\mathbb{R}^8} H_{\mathrm{W}} = 0\,. \tag{2.29}$$

When we consider the solution associated to several of these branes and other objects, we have to take into account the individual contributions coming from each one. For instance, the resulting metric comes from multiplying the H functions related to each object and taking into account the non-diagonal terms (in the case of waves or KK-monopoles). The dilaton is just the product of the individual ones, the NSNS three-form flux is given by $H_3 = dB_2$ and the RR fluxes are given by [1]

$$\begin{aligned} F_0 =& m\,, \quad F_2 = dC_1 + mB_2\,,\ F_4 = dC_3 - H_3 \wedge C_1 + \frac{m}{2} B_2 \wedge B_2 \quad \text{in Type IIA}\,, \\ F_1 =& dC_0\,,\ F_3 = dC_2 - C_0 H_3\,,\ F_5 = dC_4 - H_3 \wedge C_2 \ \text{ in Type IIB}\,, \end{aligned} \tag{2.30}$$

where the gauge potentials B_2 and C_p are the sum of all the contributions (either electric or magnetic). However the contribution to the RR fluxes coming from each brane may be modified by the backreaction of the others. These ideas will be further exemplified when our original work is presented.

There are a set of quantised charges associated to the D-branes, called Page charges [93]. The value of each kind of Page charge in a region of space measures the number of Dp-branes of a certain kind in said region. In order to compute them, we first need to compute the Page fluxes, which are defined as

$$\hat{F} = F \wedge e^{-B_2}\,, \tag{2.31}$$

where

$$F = \sum_i F_i \quad \text{and} \quad e^{-B_2} = 1 - B_2 + \frac{1}{2!} B_2 \wedge B_2 - \frac{1}{3!} B_2 \wedge B_2 \wedge B_2 + \dots \tag{2.32}$$

From now on we will use the notation

$$B_2^n = B_2 \wedge \overset{n)}{\cdots} \wedge B_2 \tag{2.33}$$

in order to simplify the expressions. Thus, we have the following definitions for the Page fluxes,

$$\begin{aligned} \hat{F}_0 =& F_0\,, \quad \hat{F}_2 = F_2 - F_0 B_2\,, \quad \hat{F}_4 = F_4 - F_2 \wedge B_2 + \frac{F_0}{2!} B_2^2\,, \\ \hat{F}_6 =& F_6 - F_4 \wedge B_2 + \frac{1}{2!} F_2 \wedge B_2^2 - \frac{F_0}{3!} B_2^3\,, \\ \hat{F}_8 =& F_8 - F_6 \wedge B_2 + \frac{1}{2!} F_4 \wedge B_2^2 - \frac{1}{3!} F_2 \wedge B_2^3 + \frac{F_0}{4!} B_2^4\,, \\ \hat{F}_{10} =& F_{10} - F_8 \wedge B_2 + \frac{1}{2!} F_6 \wedge B_2^2 - \frac{1}{3!} F_4 \wedge B_2^3 + \frac{1}{4!} F_2 \wedge B_2^4 - \frac{F_0}{5!} B_2^5 \end{aligned} \tag{2.34}$$

for massive Type IIA and

$$\begin{aligned} \hat{F}_1 =& F_1\,, \quad \hat{F}_3 = F_3 - F_1 \wedge B_2\,, \quad \hat{F}_5 = F_5 - F_3 \wedge B_2 + \frac{1}{2!} F_1 \wedge B_2^2\,, \\ \hat{F}_7 =& F_7 - F_5 \wedge B_2 + \frac{1}{2!} F_3 \wedge B_2^2 - \frac{1}{3!} F_1 \wedge B_2^3\,, \\ \hat{F}_9 =& F_9 - F_7 \wedge B_2 + \frac{1}{2!} F_5 \wedge B_2^2 - \frac{1}{3!} F_3 \wedge B_2^3 + \frac{1}{4!} F_1 \wedge B_2^4 \end{aligned} \tag{2.35}$$

[1]We have adopted the notations of [76].

for Type IIB. In the case that these new fluxes may be confused with the ones in (2.30), the latter ones will be referred as Maxwell fluxes. The Page charge of a Dp-brane is thus given by [93, 94]

$$Q_{\mathrm{D}p} = \frac{1}{(2\pi)^{7-p} g_s \alpha'^{(7-p)/2}} \int_{\Sigma_{8-p}} \hat{F}^m_{8-p} \,, \tag{2.36}$$

where $\hat{F}^m_k$ is the magnetic part of the corresponding Page flux. On the other hand, the integration of H_3 results directly into quantised charges,

$$Q_{\mathrm{F1}} = \frac{1}{(2\pi)^6 g_s^2 \alpha'^3} \int_{\Sigma_7} H^m_7 \,, \quad Q_{\mathrm{NS5}} = \frac{1}{(2\pi)^2 \alpha'} \int_{\Sigma_3} H^m_3 \,, \tag{2.37}$$

where the higher-ranked NSNS flux is given by

$$H_7 = e^{-2\Phi} \star H_3 \,. \tag{2.38}$$

All the previous charges are called *magnetic*, as they have being defined as the integral of a magnetic flux. This mirrors how the electric charge is defined in classical electrodynamics as the integral of the Hodge dual of the electromagnetic tensor. However, sometimes the computation of their electric counterparts is more appropriate. The electric Page charges are defined as follows,

$$Q^e_{\mathrm{D}p} = \frac{1}{(2\pi)^{p+1} g_s \alpha'^{(p+1)/2}} \int_{M_{p+2}} \hat{F}^e_{p+2} \,, \tag{2.39}$$

where M_{p+2} is the worldvolume of the Dp-brane and the superscript e is used to show that the charge and flux are electric this time. As for the electric quantised charges of the NS-branes, they are given by

$$Q_{\mathrm{F1}} = \frac{1}{(2\pi)^2 \alpha'} \int_{M_3} H^e_3 \,, \quad Q_{\mathrm{NS5}} = \frac{1}{(2\pi)^6 g_s^2 \alpha'^3} \int_{M_7} H^e_7 \,. \tag{2.40}$$

2.3. Basic solutions in 11d supergravity

When compared to Type II, supergravity in eleven dimensions seems far simpler and more elegant. For instance, only two kinds of branes appear in this case, as opposed to the many that appear in the ten-dimensional cases. These new branes are called *M-branes* and have either two or five spatial dimensions. Furthermore, we now have a metric and a 4-form flux G_4 as the two only fields of interest. Concretely, the M2-branes couple electrically to the G_4, while the M5-branes couple magnetically to it.

Thus, the solution for M2-branes extending along the directions $\vec{x}_2$ is given by the following metric and 3-form gauge potential,

$$ds^2 = H_{\mathrm{M2}}^{-\frac{2}{3}}(-dt^2 + d\vec{x}_2{}^2) + H_{\mathrm{M2}}^{\frac{1}{3}} ds^2_{\mathbb{R}^8} \,, \quad A_3 = H_{\mathrm{M2}}^{-1}\, dt \wedge dx^1 \wedge dx^2 \,, \tag{2.41}$$

where $G_4 = dA_3$ and the equations of motion and Bianchi identities are equivalent to

$$\nabla^2_{\mathbb{R}^8} H_{\mathrm{M2}} = 0\,. \tag{2.42}$$

As before, we can pick the solution of a single M2-brane disposed in the origin of $\mathbb{R}^8$ by setting

$$H_{\mathrm{M2}} = 1 + \frac{q_{\mathrm{M2}}}{r}\,, \tag{2.43}$$

where r is the radial coordinate of $\mathbb{R}^8$. As for the case of M5-branes, we have something similar,

$$ds^2 = H_{\mathrm{M5}}^{-\frac{1}{3}}(-dt^2 + d\vec{x}_5{}^2) + H_{\mathrm{M5}}^{\frac{2}{3}} ds^2_{\mathbb{R}^5}\,, \quad A_6 = H_{\mathrm{M5}}^{-1}\, dt \wedge dx^1 \wedge \ldots \wedge dx^5\,, \tag{2.44}$$

with $G_4 = \star dA_6$, $\star$ being the eleven-dimensional Hodge star operator. The condition now is

$$\nabla^2_{\mathbb{R}^5} H_{\mathrm{M5}} = 0\,. \tag{2.45}$$

As always, a single M5-brane in the centre of $\mathbb{R}^5$ is given by the choice

$$H_{\mathrm{M5}} = 1 + \frac{q_{\mathrm{M5}}}{r}\,, \tag{2.46}$$

with r the radial coordinate of $\mathbb{R}^5$ in this case. The discussion regarding the limits of the Type II brane solutions is also valid here.

Although there are only this two kinds of branes in 11d supergravity, other objects may appear. One of this is the KK-monopole, which is 7-dimensional in this case and warps the spacetime giving rise to the following metric,

$$ds^2 = ds^2_{\mathbb{R}^{1,6}} + H_{\mathrm{KK}}^{-1}\left(d\psi + 2^{-1} q_{\mathrm{KK}}\omega\right)^2 + H_{\mathrm{KK}}\left(dr^2 + r^2 ds^2_{S^2}\right)\,, \tag{2.47}$$

where, as in the ten-dimensional case, ψ parametrises a circle, $d\omega = \mathrm{vol}_{S^2}$ and

$$H_{\mathrm{KK}} = \frac{q_{\mathrm{KK}}}{2r} \tag{2.48}$$

We also have a wave solution, which is very similar to that of the ten-dimensional case. It presents a propagation direction z and the associated metric can be written as below,

$$ds^2 = -H_{\mathrm{W}}^{-1} dt^2 + H_{\mathrm{W}}(dz - H_{\mathrm{W}}^{-1} dt)^2 + ds^2_{\mathbb{R}^9}\,, \tag{2.49}$$

where the equations of motion and Bianchi identities imply that

$$\nabla^2_{\mathbb{R}^9} H_{\mathrm{W}} = 0\,. \tag{2.50}$$

2.4. G-structure

In this section, we introduce and explore the concept of G-structures and their relation to supersymmetry. Using G-structure as a solution generating technique in supergravity has being quite fruitful in the last years. In more concrete terms, one considers a certain family of solutions by fixing part of the geometry and fluxes and looks for a set of conditions that ensures a certain amount of supersymmetry for said family.

Let us consider a frame bundle FM, i.e. a bundle where the fibre is an ordered basis. A *G-structure* is defined as a spinor or tensor field on a FM whose stabiliser is G everywhere [95]. The main property of a G-structure is that it reduces the structure group of FM[2] from the whole $\mathrm{GL}(d,\mathbb{R})$ to one of its subgroups G. A G-structure can also be thought as defining a subbundle of FM.

A basic example is when the base manifold is Riemannian, as a metric can be used to define a G-structure. This is because one can think of the subbundle $OFM \subset FM$ that has the frames that are orthogonal with respect to the metric as their fibres.

However, for the purposes of this thesis, we are not interested in the general formulation of G-structure. On the contrary, we want to understand how it can be applied to finding supersymmetry conditions for Type IIA/B AdS_3 backgrounds.

For this goal, we use yet another interpretation of G-structure: it can be thought of as a generalisation of G-holonomy [96]. Let (M,g) be a D-dimensional Riemannian manifold, let E be a vector bundle over (M,g) and ∇, a connection over E. We can think of a point $x \in M$ and the parallel transport maps along the fibres $p : E_x \to E_x$ corresponding to closed loops with x at its base. The set of all these maps has a group structure and it is called the *Holonomy group of M under the connection ∇ based at $x \in M$*, which we denote by $\mathrm{Hol}(M,x)$. In our case, we take the spin bundle as our vector bundle and the spin connection as our connection. As the parallel transport preserves the length of the vectors, the holonomy group we are considering must be a subgroup of $\mathrm{Spin}(D)$.

In our case, we are considering a compactification space of dimension seven, which we denote by M_7. Taking a look to the Berger Classification, we observe that the only viable candidate for 7d holonomy is the exceptional group G_2. Such manifolds are called *G_2 holonomy manifolds* or simple *G_2 manifolds*. Being a G_2 manifold is equivalent to having a nowhere vanishing, globally defined 3-form Φ_3, which is both closed and co-closed, i.e.

$$d\Phi_3 = 0\,, \quad d \star_7 \Phi_3 = 0\,, \tag{2.51}$$

where $\star_7$ is the Hodge dual in M_7. G_2 manifolds first appeared in the context of 11d supergravity with a Minkowski space of dimension four as external space. In other words, if we assume that the the only non-trivial bosonic field is a metric of the form

$$ds^2_{11} = ds^2_{\mathbb{R}^{1,3}} + ds^2_{M_7} \tag{2.52}$$

and then we demand 11d $\mathcal{N}=1$ supersymmetry, we have that M_7 must be a G_2 manifold with its associated Φ_3. Curiously, those manifolds whose holonomy group is a subgroup

[2] The structure group is isomorphic to the group of transition functions that relate the open subsets that define the open covering of the bundle.

of G_2 are automatically Ricci-flat, implying that a metric of the form (2.52) satisfies the vacuum Einstein equations if M_7 is a G_2 manifold. Moreover, a G_2 structure generalises this notion for the case where the 3-form Φ_3 is not closed or co-closed. In this case $d\Phi_3$ and $\star_7 d\Phi_3$ can be decomposed into irreducible representations of G_2,

$$d\Phi_3 = \tau_0 \star_7 \Phi_3 + 3\tau_1 \wedge \Phi_3 + \star_7\tau_3\,, \quad d \star_7 \Phi_3 = 4\tau_1 \wedge \star_7\Phi_3 + \star_7\tau_2\,. \tag{2.53}$$

These τ_k are the so-called *torsion classes* [44]. We have that τ_0 transforms in the **1** representation of G_2, τ_1 in the **7**, τ_2 in the **14** and τ_3 in the **27**.

Let us now concentrate in this case and show how supersymmetry and G-structure are equivalent. Following the conventions in [97], a solution of Type II supergravity preserves some amount of supersymmetry if there are Majorana-Weyl spinors $\epsilon_{1,2}$ that solve the following spinorial equations

$$\begin{aligned}
&\left(\nabla^{(10)}_M - \frac{1}{4}(\not{H}_3)_M\right)\epsilon_1 + \frac{e^\Phi}{16}F\Gamma_M\epsilon^2 = 0\,,\\
&\left(\nabla^{(10)}_M + \frac{1}{4}(\not{H}_3)_M\right)\epsilon_2 \pm \frac{e^\Phi}{16}\lambda(F)\Gamma_M\epsilon^1 = 0\,,\\
&\left(\nabla^{(10)} - \frac{1}{4}H_3 - d\Phi\right)\epsilon_1 = 0\,,\\
&\left(\nabla^{(10)} + \frac{1}{4}H_3 - d\Phi\right)\epsilon_2 = 0\,,
\end{aligned} \tag{2.54}$$

where H_3 and Φ are the NSNS 3-form flux and dilaton of Type IIA/B supergravity respectively and the notation in (1.151) has being applied. We also assumed the Clifford map and defined the spin covariant derivatives as

$$\nabla_M = \partial_M + \frac{1}{4}\omega_M{}^{\underline{PQ}}\Gamma_{\underline{PQ}}\,, \quad de^{\underline{M}} + \omega^{\underline{M}}{}_{\underline{N}} \wedge e^{\underline{N}} = 0\,. \tag{2.55}$$

We have denoted by M the curved indices and by $\underline{M}$ the flat ones. Besides, the upper/lower signs are taken in Type IIA/B. We also have the polyform F defined as

$$F = \begin{cases} F_0 + F_2 + F_4 + F_6 + F_8 + F_{10} & \text{IIA}\,,\\ F_1 + F_3 + F_5 + F_7 + F_9 & \text{IIB}\,,\end{cases} \tag{2.56}$$

which satisfies a self-dual constraint

$$\lambda(F) = \star F\,, \tag{2.57}$$

where $\lambda(X_k) = (-1)^{\lfloor \frac{k}{2} \rfloor}X_k$, $\lfloor x \rfloor$ being the floor function. We will consider the following $3+7$ split for the 10d gamma matrices,

$$\begin{aligned}
\Gamma_{\underline{\mu}} =& \gamma_{\underline{\mu}} \otimes \sigma_3 \otimes \mathbb{I}_8 \quad \text{for} \quad \underline{\mu} = 0,1,2\,,\\
\Gamma_a =& \mathbb{I}_2 \otimes \sigma_1 \otimes \gamma_a \quad \text{for} \quad a = 1,\ldots,7\,,\ i\gamma_{1\ldots7} = \mathbb{I}_8\,,
\end{aligned} \tag{2.58}$$

where $\gamma_{\underline{\mu}} = (i\sigma_2, \sigma_1, \sigma_3)_{\underline{\mu}}$ and are thus real.

If the spacetime can be decomposed as M_{10} =AdS$_3 \times M_7$ and taking into account (2.58), we can decompose the 10d spinors as follows,

$$\epsilon_1 = \zeta \otimes \theta_+ \otimes \chi_1 \,, \quad \epsilon_2 = \zeta \otimes \theta_\mp \otimes \chi_2 \,, \tag{2.59}$$

where ζ are real Killing spinors on AdS$_3$ meeting the condition

$$\nabla_\mu^{\mathrm{AdS}_3} \zeta = \frac{m}{2} \gamma_\mu \zeta \,, \tag{2.60}$$

where m is the inverse of the AdS$_3$ radius, the Minkowski case being recovered for $m = 0$. $\chi_{1,2}$ are Majorana spinors on M_7. Moreover, $\theta_\pm$ are auxiliary 2d spinors necessary to obtain the right dimensionality for the 10d spinors when decomposed into the 3d and 7d ones. They are defined as

$$\theta_+ = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -i \end{pmatrix} \,, \quad \theta_- = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ i \end{pmatrix} \tag{2.61}$$

and, therefore, $\epsilon_{1,2}$ are Majorana-Weyl spinors. The $\pm$ signs denotes ten-dimensional chirality and, as before, the upper/lower signs correspond to Type IIA/B. Plugging the ansatz (2.59) in the conditions (2.54) and applying some identities, one ends up with the following equations,

$$\begin{aligned}
&(me^{-A} - id\!\!\!/ A)\chi_1 + \frac{1}{4} e^{\Phi} \beta_\pm f_\pm \chi_2 = 0 \,, \\
&(me^{-A} \pm id\!\!\!/ A)\chi_2 + \frac{1}{4} e^{\Phi} \beta_\pm^* \lambda(f_\pm) \chi_1 = 0 \,, \\
&\left(\nabla_a - \frac{1}{4} (H\!\!\!/_3)_a \right) \chi_1 + \frac{1}{8} e^{\Phi} i \beta_\pm^* f_\pm \gamma_a \chi_2 = 0 \,, \\
&\left(\nabla_a + \frac{1}{4} (H\!\!\!/_3)_a \right) \chi_2 - \frac{1}{8} e^{\Phi} i \beta_\pm^* \lambda(f_\pm) \gamma_a \chi_1 = 0 \,, \\
&\left[\frac{3}{2} m e^{-A} - i \left(\frac{3}{2} d\!\!\!/ A + \nabla\!\!\!\!/ + \frac{i}{4} H\!\!\!/_3 - d\!\!\!/ \Phi \right) \right] \chi_1 = 0 \,, \\
&\left[\frac{3}{2} m e^{-A} \pm i \left(\frac{3}{2} d\!\!\!/ A + \nabla\!\!\!\!/ + \frac{1}{4} H\!\!\!/_3 - d\!\!\!/ \Phi \right) \right] \chi_2 = 0 \,,
\end{aligned} \tag{2.62}$$

where

$$\beta_+ = 1 \,, \beta_- = i \,. \tag{2.63}$$

Let us consider the simplest case, which consists on taking $\chi_1 = \chi_2 \equiv \chi$ and setting $H_3 = f_\pm = 0$ and assume constant dilaton and warp factor e^A. Under these considerations, the conditions (2.62) boil down to

$$\nabla_a \chi = 0 \,, \quad m = 0 \,, \tag{2.64}$$

which, as mentioned before, corresponds to the case where the AdS_3 degenerates into a 4d Minkowski spacetime. Thus, we must have a metric of the form

$$ds_{10}^2 = ds_{\mathbb{R}^{1,3}}^2 + ds_{M_7}^2\,, \tag{2.65}$$

which supports a covariantly constant spinor. It is possible to show that the condition (2.64) is equivalent to (2.51). The main idea behind this equivalence is that a single seven-dimensional Majorana spinor defines the 3-form of the G_2-structure [98],

$$\Phi_3 = -i\frac{1}{3!}\chi^\dagger\gamma_{abc}\chi e^a\wedge e^b\wedge e^c\,. \tag{2.66}$$

Starting with this expression and applying (2.64), one obtains the condition (2.51).

In general, two Majorana spinors $\chi_{1,2}$ define a $G_2\times G_2$-structure in seven dimensions, i.e. a G_2 structure for each spinor. This is usually referred to as their largest common subgroup SU(3). In other words, one can decompose a general G_2-structure in terms of an SU(3) as

$$\Phi_3 = J\wedge U - \mathrm{Im}(\Omega)\,, \tag{2.67}$$

characterised by a real 2-form J and a holomorphic 3-form Ω, both six-dimensional. The two Φ_3 corresponding to $\chi_{1,2}$ share the same (J,Ω) but have different U. The situation where $\chi_1=\chi_2$ is an exception where both Φ_3 coincide and, therefore, the largest common subgroup is the whole G_2. Thus, in that case we have a G_2-structure.

2.4.1. G-structure in $\mathcal{N}=(1,1)$ AdS$_3$ solutions to Type IIA/B supergravity

As an example, we now explore in more detail the case of $\mathcal{N}=(1,1)$ Type IIA/B backgrounds, as presented in [50]. Actually, the solutions that appear in our original work are $\mathcal{N}=(0,1)$ supersymmetric. However, a thorough justification of the equivalence between supersymmetry and G-structure conditions in the $\mathcal{N}=(0,1)$ case is too advanced and exceeds the purposes of this thesis. Thus, we restrict ourselves to the simpler $\mathcal{N}=(1,1)$ scenario for this example. We start by assuming that the fields take the following form,

$$\begin{aligned} ds^2 =& e^{2A}ds^2(\mathrm{AdS}_3)+ds^2(\mathrm{M}_7)\,, \quad H = e^{3A}h_0\mathrm{vol}_{\mathrm{AdS}_3}+H_3\,,\\ F =& f_\pm + e^{3A}\mathrm{vol}(\mathrm{AdS}_3)\wedge\star_7\lambda(f_\pm) \quad \text{with} \quad \lambda(f_n)=(-1)^{\lfloor\frac{n}{2}\rfloor}f_n\,, \end{aligned} \tag{2.68}$$

where we denote by m the inverse of the radius of the AdS_3. We also consider that the warp factor e^{2A}, the dilaton Φ, the magnetic components of the NSNS flux H_3 and the RR polyform $f_\pm$ depend on the coordinates of M_7, but are independent on those of the AdS_3. The sign in $f_\pm$ denotes whether the considered forms are of even (+) or odd (−) rank, i.e. the former corresponds to Type IIA and the latter to Type IIB.

We will realise the $\mathcal{N}=(1,1)$ supersymmetry of our backgrounds through the two ten-dimensional Majorana-Weyl Killing spinors $\epsilon_{1,2}$. In more concrete terms, we assume that they can be decomposed as follows,

$$\epsilon_1=\zeta_+\otimes\theta_+\otimes\chi^1_+ +\zeta_-\otimes\theta_+\otimes\chi^1_-\,,\quad \epsilon_2=\zeta_+\otimes\theta_\mp\otimes\chi^2_+ +\zeta_-\otimes\theta_\mp\otimes\chi^2_-\,, \tag{2.69}$$

where $\zeta_\pm$ are the two independent real Killing spinors on AdS$_3$ realising the $\mathcal{N}=(1,1)$ supersymmetry and which are charged under the group SL$(2)_\pm$ $\subset$SO$(2,2)$. $\chi^{1,2}_\pm$ are four independent Majorana spinors and, without loss of generality, their norms can be taken to satisfy

$$\begin{aligned}|\chi^1_+|^2+|\chi^2_+|^2=&|\chi^1_-|^2+|\chi^2_-|^2=2e^A\,,\\ e^A(|\chi^1_+|^2-|\chi^2_+|^2)=&-e^A(|\chi^1_-|^2-|\chi^2_-|^2)=c\,,\end{aligned} \tag{2.70}$$

where c is an arbitrary real constant. Also the $\theta_\pm$ are two dimensional vectors and codify the ten-dimensional chirality labelled by the subscript (the subscripts in $\zeta_\pm$ and $\chi^{1,2}_\pm$ are just labels). Without entering into details, there are certain geometric necessary and sufficient conditions for the existence of an $\mathcal{N}=(1,1)$ solution. These refer to the following bilinears,

$$\Psi^{st}_+ + i\Psi^{st}_- =\chi^1_s\otimes\chi^{2\dagger}_t\,,\quad s,t=\pm\,, \tag{2.71}$$

where

$$\chi^1_s\otimes\chi^{2\dagger}_t=\frac{1}{8}\sum_{n=1}^{7}\frac{1}{n!}\chi^{2\dagger}_t(\gamma_{a_1\ldots a_n})\chi^1_s e^{a_1}\wedge\ldots\wedge e^{a_n}\,, \tag{2.72}$$

where γ_a are the flat space seven-dimensional gamma matrices, the e^a is a vielbein of said dimension and the subscript in $\Psi^{st}_\pm$ refers to form degree once again. The aforementioned conditions are displayed below,

$$\begin{aligned}&e^{3A}h_0=-mc\,,\quad d_{H_3}(e^{A-\Phi}\Psi^{++}_\mp)\pm\frac{c}{16}f_\pm=0\,,\quad d_{H_3}(e^{A-\Phi}\Psi^{--}_\mp)\mp\frac{c}{16}f_\pm=0,\\ &(\Psi^{++}_\mp,f_\pm)_7=\mp\frac{1}{2}e^{-\Phi}\left(m+\frac{1}{4}e^{-A}ch_0\right)\text{vol}(\text{M}_7)\,,\\ &(\Psi^{--}_\mp,f_\pm)_7=\pm\frac{1}{2}e^{-\Phi}\left(m+\frac{1}{4}e^{-A}ch_0\right)\text{vol}(\text{M}_7)\,,\\ &d_{H_3}(e^{2A-\Phi}\Psi^{++}_\pm)\mp 2me^{A-\Phi}\Psi^{++}_\mp=\frac{1}{8}e^{3A}\star_7\lambda(f_\pm)\,,\\ &d_{H_3}(e^{2A-\Phi}\Psi^{--}_\pm)\pm 2me^{A-\Phi}\Psi^{--}_\mp=\frac{1}{8}e^{3A}\star_7\lambda(f_\pm)\,,\end{aligned} \tag{2.73}$$

where $(X,Y)_7$ is the seven-dimensional Mukai pairing defined as

$$(X,Y)_7=X\wedge\lambda(Y)\Big|_7 \tag{2.74}$$

and $d_{H_3} = d - H_3\wedge$. It can be shown that an $\mathcal{N} = (1,1)$ AdS_3 gets enhanced into AdS_4 at all regular points in internal space unless the following conditions are met,

$$\chi_+^{1\dagger}\chi_-^1 + \chi_+^{2\dagger}\chi_-^2 = 0\,, \quad \chi_+^{1\dagger}\gamma^a\chi_-^1 \mp \chi_+^{2\dagger}\gamma^a\chi_-^2 = 0\,. \tag{2.75}$$

We assume these restrictions in order to obtain bona fide AdS_3 solutions. However, imposing (2.75) gives rise to an additional set of conditions,

$$\begin{aligned}
&d(e^A g) + m\tilde{\xi} = 0\,, \quad d_{H_3}(e^{2A-\Phi}(\Psi_\pm^{+-} + \Psi_\pm^{-+})) = 0\,,\\
&d_{H_3}(e^{-\Phi}(\Psi_\pm^{+-} - \Psi_\pm^{-+})) = \frac{1}{8}\tilde{\xi}\wedge f_\pm,\\
&d_{H_3}(e^{A-\Phi}(\Psi_\mp^{+-} + \Psi_\mp^{-+})) \pm m e^{-\Phi}(\Psi_\pm^{+-} - \Psi_\pm^{-+}) = \mp\frac{1}{8} g f_\pm\,,\\
&d_{H_3}(e^{3A-\Phi}(\Psi_\mp^{+-} - \Psi_\mp^{-+})) \pm e^{3A-\Phi}h_0(\Psi_\pm^{+-} - \Psi_\pm^{-+}) \pm 3me^{2A-\Phi}(\Psi_\pm^{+-} + \Psi_\pm^{-+}) =\\
&= \pm\frac{1}{8}e^{3A}\tilde{\xi}\wedge\star_7\lambda(f_\pm)\,,
\end{aligned} \tag{2.76}$$

where

$$g = \chi_+^{1\dagger}\chi_-^1 - \chi_+^{2\dagger}\chi_-^2\,, \quad \tilde{\xi} = -i(\chi_+^{1\dagger}\gamma^a\chi_-^1 \pm \chi_+^{2\dagger}\gamma^a\chi_-^2)e^a \tag{2.77}$$

cannot vanish globally when $\mathcal{N} = (1,1)$. Some of the equations (2.73) and (2.76) can be shown to be redundant in the light of the relations for 7d spinors. Furthermore, if we consider internal spinors that satisfy (2.70) and (2.75), the $\mathcal{N} = (1,1)$ AdS_3 solutions boils down to the ones below,

$$\begin{aligned}
&e^{3A}h_0 = -mc\,, \quad d(e^A g) + m\tilde{\xi} = 0\,, \quad d_{H_3}(e^{A-\Phi}(\Psi_\mp^{++} + \Psi_\mp^{--})) = 0,\\
&d_{H_3}(e^{2A-\Phi}(\Psi_\pm^{++} - \Psi_\pm^{--})) \mp 2me^{A-\Phi}(\Psi_\mp^{++} - \Psi_\mp^{--}) = 0\,,\\
&d_{H_3}(e^{A-\Phi}(\Psi_\mp^{++} - \Psi_\mp^{--})) \pm \frac{c}{8} f_\pm = 0\,,\\
&d_{H_3}(e^{2A-\Phi}(\Psi_\pm^{++} + \Psi_\pm^{--})) \mp 2me^{A-\Phi}(\Psi_\mp^{++} + \Psi_\mp^{--}) = \frac{1}{4}e^{3A}\star_7\lambda(f_\pm)\,,\\
&d_{H_3}(e^{A-\Phi}(\Psi_\mp^{+-} + \Psi_\mp^{-+})) \pm me^{-\Phi}(\Psi_\pm^{+-} - \Psi_\pm^{-+}) = \mp\frac{1}{8}g f_\pm\,,\\
&(\Psi_\mp^{++} - \Psi_\mp^{--}, f_\pm)_7 = \pm e^{-\Phi}\left(m + \frac{1}{4}e^{-A}ch_0\right)\mathrm{vol}(\mathrm{M}_7)\,.
\end{aligned} \tag{2.78}$$

From now on, we fix $c = 0$ and so, according to (2.70), both internal bispinors have the same norm. This means that $h_0 = 0$ now solves the first equation in (2.78). Without loss of generality, we can parametrise the four internal spinors by two real unit norm vectors $(V, \hat{V})$, a set of three real functions (a, b, α) and a unit norm spinor χ,

$$\begin{aligned}
\chi_+^1 &= e^{A/2}\chi\,, \quad \chi_+^2 = e^{A/2}(a + ibV)\chi\,,\\
\chi_-^1 &= e^{A/2}(\cos\alpha + i\sin\alpha V)\chi\,, \quad \chi_-^2 = -e^{A/2}(\cos\alpha \mp i\sin\alpha V)(a + ib\hat{V})\chi\,,
\end{aligned} \tag{2.79}$$

where $a^2 + b^2 = 1$. Substituting these spinors in (2.77), we obtain the following expressions,

$$g = 2e^A\cos\alpha\,, \quad \tilde{\xi} = 2e^A\sin\alpha V\,, \tag{2.80}$$

where neither $\cos\alpha$ nor $\sin\alpha$ can be globally zero. A straightforward consequence of this is that the second equation in (2.78) takes the following form,

$$d(2e^{2A}\cos\alpha) + me^{A}\sin\alpha V = 0\,, \tag{2.81}$$

meaning that the internal manifold M_7 may be decomposed via a warped product of an interval spanned by V and a six-dimensional manifold M_6. In general the spinor χ satisfies the relations below,

$$\chi\otimes\chi^\dagger = \Psi_+^{(G_2)} + i\Psi_-^{(G_2)} = \frac{1}{8}(1 - i\Phi_3 - \star_7\Phi_3 + i\text{vol}_7)\,, \quad \Phi_3\wedge\star_7\Phi_3 = 7\text{vol}_7\,, \tag{2.82}$$

where Φ_3 is the 3-form associated to the G_2-structure that a single seven-dimensional Majorana spinor supports. The G-structure of an $\mathcal{N} = (1,1)$ solution will depend on the relative orientation of V and $\hat{V}$. If they are parallel to each other, then the solution presents an SU(3)-structure on the M_6. If they are not parallel and $b \neq 0$, then an SU(2)-structure is supported in the M_6. This reduction can be thought of as each $\chi_\pm$ imposing a restriction on the G-structure. Thus, the maximal group (SU(3) in this case) is obtained only when V and $\hat{V}$ are parallel.

Let us capitulate what we saw in this chapter before we conclude. We provided an interpretation of supergravity as gauged supersymmetry in section 2.1. Then we explored some basic solutions of Type IIA/B and 11d supergravities in sections 2.2 and 2.3, respectively. We closed this chapter by introducing the notion of G-structure and exploring a relevant example in section 2.4.

Chapter 3

Holography

This chapter revolves around a key concept for this thesis, but also for the understanding of the current research in supergravity and CFT. This is no other than the *AdS/CFT correspondence*, which conjectures that a Type IIA/B string theory which lives in an Anti-de Sitter space is related to a CFT whose fields are defined over the conformal boundary of said space. It is a remarkable application of the *holographic principle*, which tells us that the information contained in a volume V_{d+1} is actually encoded in its boundary A_d. The motivation behind this principle is the so-called Bekenstein bound for the maximum amount of entropy enclosed within a certain volume: $S_{max} = A_d/(4G_N)$, where A_d is the area of the boundary (measured in units of the Planck area l_p^d) and G_N is the Newton's constant.

Although this may be surprising the first time one hears about it, there is a historical background suggesting that the conjecture is quite likely to be true. In particular, the holographic principle first appeared in the context of the *black hole information paradox*. In 1975 Hawking published a paper suggesting that black holes are not completely black, but radiate due to quantum effects in their horizons [99]. This result raised some controversy as it seemed to violate the unitarity of time evolution, a fundamental postulate of quantum mechanics that estates that quantum systems evolve according to unitary operators and, therefore, information cannot be destroyed when a system changes from one state to another. For this reason, this contradiction was called the black hole information paradox. Almost two decades later, in 1993, Stephens, t' Hooft and Whiting proposed a solution to this paradox: the entropy of a black hole must be proportional to the surface of its horizon because the degrees of freedom of a black hole are duplicated in the Hawking radiation [100]. This statement is the seed of the holographic principle, which was later formulated in general by 't Hooft [101]. Maldacena's 1998 paper [17] then introduced the AdS/CFT correspondence with the support of the particular example of D3-branes in Type IIB string theory, constituting a very exciting example of the holographic principle. Moreover, in the past decades, other such examples regarding different brane configurations in Type IIA/B have been studied. Another hint that suggests that the AdS/CFT duality should be correct is related to the isometry group of AdS_{d+1}, which is the Lorentz group $\mathrm{SO}(d, 2)$. This group is isomorphic to the conformal group $Conf(d-1, 1)$, which is the symmetry group of a CFT living in $\mathbb{R}^{1,d-1}$.

Furthermore, the AdS/CFT correspondence is a strong-weak duality. This means that it relates a strongly coupled field theory with a classical and weakly curved gravity theory. This provides a path for the study of strongly coupled field theories, as some observables can be computed on the gravity side. This also applies the other way around, relating weakly coupled field theories to strongly coupled and heavily curved string theory. Roughly speaking, at the heart of the correspondence lies the dual nature of D-branes. On the one hand, they are objects on which the open strings must end. However, they are also extremal p-branes, which are supergravity solutions.

The main sources consulted for this chapter were [102, 103].

3.1. The original $\mathrm{AdS}_5/\mathrm{CFT}_4$ correspondence

We consider the original case of D3-branes, which was devised by Maldacena in 1997 [17]. In the D3-brane case, the correspondence relates 4d $\mathcal{N}=4$ Super Yang-Mills (SYM) and Type IIB string theory on $\mathrm{AdS}_5\times S^5$. In its strongest formulation, the $\mathrm{AdS}_5/\mathrm{CFT}_4$ conjecture states that both theories are dynamically equivalent and matches their free parameters in the following way,

$$g_{\mathrm{YM}}^2 = 2\pi g_s \quad \text{and} \quad 2g_{\mathrm{YM}}^2 N = L^4/\alpha'^2\,. \tag{3.1}$$

Here g_{YM} is the coupling constant of the gauge group $\mathrm{SU}(N)$ that describes the interactions in the SYM theory and $\lambda = g_{\mathrm{YM}}^2 N$ is referred to as the 't Hooft coupling. We also have that g_s is the string coupling constant over $\mathrm{AdS}_5\times S^5$, the radius of curvature of both the AdS_5 and the S^5 is L and there are N units of F_5 flux on the S^5. Thus, we observe that g_{YM} and N of the SYM theory are mapped to g_s and L^2/α' on the string theory side. The two theories being dynamically equivalent implies that they describe the same physics from two different perspectives. But this implies that string theory, a theory of quantum gravity, is mapped to a quantum field theory where gravity does not appear at all. In other words, the gravitational degrees of freedom of the former theory have to transform into something else in the latter one. Furthermore, as we presented in the introduction to this section, the information of the 5d theory obtained upon Kaluza-Klein reducing the Type IIB solution on the 5-sphere is actually encoded within the conformal boundary of the AdS_5.

It is quite difficult to perform general computations applying the AdS_5/CFT_4 correspondence. The idea then consists on taking certain limits on both sides of the correspondence. This provides a more applicable version of the conjecture, albeit a weaker one. In particular, as string theory is not fully understood at the non-perturbative level, it is reasonable to take the weak coupling regime ($g_s \ll 1$ and constant $L/\sqrt{\alpha'}$) on that side as our start point. By considering only the leading order in g_s, we obtain classical string theory, meaning that only the tree level diagrams are regarded instead of the whole genus expansion. We now turn our attention to CFT side of the correspondence. If the mapping in (3.1) holds, we must have $g_{\mathrm{YM}} \ll 1$ and $g_{\mathrm{YM}}^2 N$ must stay finite. In other words, we are in the large N limit ($N \to \infty$) for a fix value of the 't Hooft coupling λ. This corresponds to the so-called *planar limit* of the gauge theory, meaning that the scattering amplitudes are dominated by Feynman diagrams that can be described by planar graphs. This implies that the AdS/CFT conjecture is an example of a conjecture proposed by 't Hooft, which states that the planar limit of a gauge theory is a string theory [104]. We conclude that a $1/N$ expansion of the CFT can be mapped into a genus expansion of the string theory, as $1/N \propto g_s$ for constant λ, according to (3.1).

We must focus on the strongly coupled limit on the CFT side, which corresponds to sending $\alpha'/L^2 \to 0$, as it is the one we can work with. In this regime, we are considering that the string length is very small when compared to the AdS curvature radius. Thus, we are in the point particle limit. This fact jointly with the tree level truncation we already considered implies that we are dealing with classical supergravity. On the field theory side, we are in the strongly coupled, $\lambda \to \infty$, regime. This is the *weak form* of the AdS/CFT correspondence, with relates Type IIB supergravity on a weakly curved $\text{AdS}_5 \times S^5$ to strongly coupled $\mathcal{N} = 4$ Super Yang-Mills in four dimensions. For our particular example, we will consider this form of the conjecture.

As we anticipated in the introduction to this chapter, the AdS/CFT correspondence arises in the context of string theory as a consequence of the dual nature of D-branes. We will address these two points of view as the *open string* and *closed string* perspectives. Which one of these two cases is applicable relies completely on the value of g_s, which determines the strength of the interaction between strings and, therefore, the regime we are in.

The open string perspective corresponds to the viewpoint where we consider D-branes as the region of space where open strings may end. For this interpretation to make sense, we must be in the perturbative string regime, i.e. $g_s \ll 1$. In particular, we are interested in the low-energy case ($E \ll \alpha'^{-1/2}$), when the dynamics of the open strings can be described via a supersymmetric gauge theory defined within the worldvolume of the D-branes. The gauge potential in play, A_μ, accounts for the open string excitations along the worldvolume directions of the D-branes, while scalar fields ϕ^i describe those excitations transverse to the D-branes. The case where N D-branes coincide gives rise to a $\text{U}(N)$ gauge group. Therefore, we conclude that the effective coupling must be $g_s N$ in this case. Furthermore, the condition $g_s N \ll 1$ must be met for the open string viewpoint to be reasonable considered.

As for the closed string point of view, it is the completely opposite one. In this case, D-branes are regarded as solutions to supergravity, i.e. as p-branes, as we presented in section 2.2. In this case, we must consider that D-branes curve the surrounding spacetime, which we completely ignored in the open string perspective. The radius of curvature L is taken to be large so that we are in the low curvature scenario and we can apply the supergravity approximation. When N D-branes coincide, we observe that $L^4/\alpha'^2 \propto g_s N$ according to (3.1). Thus, this perspective is only trustworthy when $g_s N \gg 1$.

These two perspectives motivate the AdS_5/CFT_4 duality when applied to a stack of N D3-branes in Type IIB string theory embedded in flat spacetime. As we explained earlier, this will provide a connection between strongly coupled $\mathcal{N} = 4$ Super Yang-Mills

Theory in 4 dimensions and a Type IIB supergravity solution displaying an $\mathrm{AdS}_5 \times S^5$ geometry. The first step is to specify how the D3-branes are embedded in ten-dimensional spacetime. We will consider that they extend along x^0, x^1, x^2 and x^3 and, without loss of generality, we also consider that they satisfy $x^4 = \ldots = x^9 = 0$. These specifications are summarised in Table 3.1.

branes	x^0	x^1	x^2	x^3	x^4	x^5	x^6	x^7	x^8	x^9
D3	$\times$	$\times$	$\times$	$\times$	$-$	$-$	$-$	$-$	$-$	$-$

Table 3.1: Description of the worldvolume and transversal directions of the D3-branes. The former ones are represented by $\times$ and correspond to Neumann boundary conditions for open strings, while the latter ones are depicted by $-$ and coincide with the Dirichlet ones.

3.1.1. The open string interpretation

We start with the open string perspective, thus considering the perturbative case $g_s N \ll 1$. In principle, for the considered kind of backgrounds in perturbative string theory, we can have open strings with both ends attached to the stack of D3-branes and closed strings. The former ones can be regarded as perturbations of the D3-branes, while the latter ones are viewed as excitations of the 10-dimensional flat spacetime. We will only consider massless excitations and ignore the higher energy ones by assuming $E \ll \alpha'^{-1/2}$. As the brane configuration preserves half of the supersymmetries, we have sixteen supercharges. Because of this, the massless closed string states are grouped into a 10d $\mathcal{N} = 1$ supergravity multiplet. As for the massless open string modes, they combine into a four-dimensional $\mathcal{N} = 4$ supermultiplet consisting of a gauge field A_μ, six real scalar fields ϕ^i and their corresponding fermionic superpartners. As a consequence of the D3 coinciding in a single stack, the strings ending on them are massless and all the open string modes must be in the adjoint representation of $\mathrm{U}(N)$.

After the previous discussion, we can write down the general form of the action describing all massless excitations,

$$S = S_{\mathrm{cl}} + S_{\mathrm{op}} + S_{\mathrm{int}} \, , \tag{3.2}$$

where S_{cl} contains the closed strings alone, S_{op} is the analogous for the open ones and S_{int} describes the interactions between both kinds of modes. We remark that S_{cl} is the supergravity action with some extra higher derivative corrections,

$$S_{\mathrm{cl}} = \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{-g} e^{-2\Phi} [R + 4\partial_M \Phi \partial^M \Phi] + \cdots \sim -\frac{1}{2} \int d^{10}x \partial_M h \partial^M h + \mathcal{O}(\kappa) \, , \tag{3.3}$$

where h is the lowest order fluctuation of the metric, i.e. $g = \eta + \kappa h$. The actions S_{op} and S_{int} can be derived from the DBI and Wess-Zumino actions. The former one can be written as follows for the case of a single D3-brane,

$$S_{\text{DBI}} = -\frac{1}{(2\pi)^3\alpha'^2 g_s}\int d^4x e^{-\Phi}\sqrt{-\det(\mathcal{P}[g] + 2\pi\alpha' F)}\,, \tag{3.4}$$

where B_2 has been set to zero and F is the field strength of the gauge potentials A_μ living on the worldvolume of the D3-branes. We now consider the static gauge. This means that we will denote by x^μ with $\mu = 0, 1, 2, 3$ the worldvolume coordinates and identify the remaining ones, x^i, with six real scalar fields ϕ^i,

$$x^{i+3} = 2\pi\alpha'\phi^i \quad \text{for} \quad i = 1, \ldots, 6\,. \tag{3.5}$$

We also have that $\mathcal{P}$ represents the pullback to the worldvolume and, according to what we have just seen, the pullback of the metric takes the following form,

$$\mathcal{P}[g]_{\mu\nu} = g_{\mu\nu} + (2\pi\alpha')(g_{i+3\ \nu}\partial_\mu\phi^i + g_{\mu\ j+3}\partial_\nu\phi^j) + (2\pi\alpha')^2 g_{i+3\ j+3}\partial_\mu\phi^i\partial_\nu\phi^j\,. \tag{3.6}$$

If we expand the metric and dilaton in (3.4) at leading order in α', we get the following effective actions,

$$\begin{aligned} S_{\text{op}} &= -\frac{1}{2\pi g_s}\int d^4x\left(\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}\eta^{\mu\nu}\partial_\mu\phi^i\partial_\nu\phi^i + \mathcal{O}(\alpha')\right)\,, \\ S_{\text{int}} &= -\frac{1}{8\pi g_s}\int d^4x\Phi\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \cdots\,. \end{aligned} \tag{3.7}$$

These actions are for a single D3-brane. If we want to generalise to the case of N coinciding D3-branes, we have to take into account that the scalars and gauge potentials are U(N)-valued in that case. In other words, in general we have

$$\phi^i = \phi^{ia}T_a\,, \quad A_\mu = A^a_\mu T_a\,, \tag{3.8}$$

where T_a are the Lie algebra generators. We also have to take the trace whenever the T_a appear in order to have gauge invariance. For instance, $F_{\mu\nu}F^{\mu\nu}$ becomes $F^a_{\mu\nu}F^{a\mu\nu}$. One last consideration, we need to replace the partial derivatives with covariant ones and add an scalar potential

$$V = \frac{1}{2\pi g_s}\sum_{i,j}\text{Tr}\left[\phi^i, \phi^j\right]^2 \tag{3.9}$$

to S_{op} to lowest order in α'.

Finally, if we take $\alpha' \to 0$, we observe that S_{op} becomes the bosonic part of the $\mathcal{N} = 4$ Super Yang-Mills action with

$$2\pi g_s = g^2_{\text{YM}}\,. \tag{3.10}$$

For S_{cl}, we observe that $\kappa_{10} \propto \alpha'^2 \to 0$ in this limit so we recover the action of free 10d supergravity. As for the interaction term, we have that Φ must be rescaled by κ_{10} for renormalisation reasons. We conclude that S_{int} vanishes in the $\alpha' \to 0$ limit and both open and closed strings decouple.

3.1.2. The closed string interpretation

Now we explore the opposite limit by considering a stack of N D3-branes in the strongly coupled regime ($g_s N \to \infty$). Thus, we must switch to the closed string perspective and regard the D3-branes as massive sources of the Type IIB supergravity fields. The supergravity solution preserves once again half of the supercharges (sixteen) and, as we saw in section 2.2, it takes the following form,

$$\begin{aligned} ds^2 =& H(x^i)^{-1/2}\eta_{\mu\nu}dx^\mu dx^\nu + H(x^i)^{1/2}\delta_{ij}dx^i dx^j \,, \\ C_{(4)} =& H(x^i)^{-1}\, dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3 \,, \end{aligned} \tag{3.11}$$

where $H(x^i)$ is a harmonic function in the x^i coordinates and the dilaton is constant so we can take it to vanish. In our case, we are interested in the most symmetric solution, which consists in taking

$$H = 1 + \frac{L^4}{r^4} \,, \tag{3.12}$$

where r is the radial coordinate of the $\mathbb{R}^6$ parametrised by the x^i. As we already explained in section 2.2, this particular choice of H interpolates between two different behaviours in two separate regions of spacetime. First we have that $H(r) \sim 1$ when $r \gg L$; the observer is far away from the sources so the spacetime is asymptotically flat. On the other hand, $H \sim \frac{L^4}{r^4}$ when $r \ll L$, i.e. when the near-horizon limit is taken; this region is shaped like a throat. In this latter regime, the metric reduces to

$$\begin{aligned} ds^2 =& \frac{r^2}{L^2}\eta_{\mu\nu}dx^\mu dx^\nu + \frac{L^2}{r^2}\delta_{ij}dx^i dx^j = \\ =& \frac{L^2}{z^2}\left(\eta_{\mu\nu}dx^\mu dx^\nu + dz^2\right) + L^2 ds^2_{S^5} = \\ =& ds^2_{\mathrm{AdS}_5} + L^2 ds^2_{S^5} \,, \end{aligned} \tag{3.13}$$

where we have gone to spherical coordinates in the second line and defined $z \equiv L^2/r$. We highlight the fact that we have arrived at the conclusion that the geometry of our solution is $\mathrm{AdS}_5 \times S^5$ in the near-horizon limit and flat in the outermost region. Thus, we have to differentiate between closed strings propagating in each of these two regions. Furthermore, in the low-energy limit, both kinds of closed strings decouple. This can be reasoned by considering a string excitation. An observer positioned at a fixed r will measure an energy E_r for the excitation, while another one at infinity will measure E_∞. These two measures of the energy are related as below,

$$E_\infty = H^{-1/4} E_r \,. \tag{3.14}$$

As $H^{-1/4} \sim r/L$ when $r \to 0$, we conclude that E_∞ goes to zero in the throat region independently on the value of E_r. Therefore, these string excitations are in the low-energy regime for an observer at infinity. They would conclude that there are two decoupled low-energy modes: the supergravity modes propagating in flat ten-dimensional spacetime and the string excitations in the throat corresponding to the $\mathrm{AdS}_5 \times S^5$ geometry.

3.1.3. The comparison of both interpretations

If we compare the two perspectives, we observe that both present two decoupled effective theories in their low-energy regimes. Both perspectives are equivalent in the sense that the underlying physics are the same in spite of being described in two very different ways. As expected, they share the presence of Type IIB supergravity on $\mathbb{R}^{1,9}$. However, while in the open string perspective we also observe $\mathcal{N} = 4$ SYM theory in $\mathbb{R}^{1,3}$, the closed string perspective contains Type IIB supergravity on $\text{AdS}_5 \times S^5$. Thus, these two theories have to be related in some way. This is what motivated Maldacena to propose the AdS/CFT conjecture [17]. If we ignore the low-energy condition, the conjecture proposes the equivalence between $\mathcal{N} = 4$ SYM in flat 4d spacetime and Type IIB string theory on $\text{AdS}_5 \times S^5$.

Without entering into details, this correspondence also proposes a map between the operators of the 4d $\mathcal{N} = 4$ Super Yang-Mills theory and the spectrum of states of string theory. The underlying reason for this map to exist is that the symmetry group of both theories is the same, namely $\text{PSU}(2, 2|4)$, allowing certain representations of said group that correspond to operators in the gauge theory to be mapped into string states on the AdS in the same representation. In the weak form of the conjecture, the map relates the operators of the CFT to supergravity fields. This field-operator map enables a formulation of the AdS/CFT correspondence as as a relation between generating functionals in both theories.

All these results can be extended to other dimensions by considering different branes or brane configurations. In more concrete terms, whenever an AdS_{p+1} solution arises in the near-horizon limit of some brane set-up, one can study if it is holographically dual to some CFT_p which lives in said brane set-up. To illustrate this, we present an example of the $p = 2$ case in the next section.

3.2. An example of $\text{AdS}_3/\text{CFT}_2$ correspondence

In this section, we will explore a D1-D5 bound state. Its holographic properties were already studied by Maldacena in [17] and it gives rise to the simplest example of $\text{AdS}_3/\text{CFT}_2$ duality. This case of the correspondence ($p = 2$) is particularly relevant for our purposes, as we explored it in our work. We start by considering the brane set-up depicted in Table 3.2. In this brane configuration, the D1- and D5-branes are taken to

branes	t	x^1	x^2	x^3	x^4	x^5	x^6	x^7	x^8	x^9
D1	×	–	–	–	–	×	–	–	–	–
D5	×	–	–	–	–	×	×	×	×	×

Table 3.2: D1-D5 brane configuration. The (x^6, x^7, x^8, x^9) coordinates parametrise a four-fold M_4, which is taken to be either a $\mathbb{T}^4$ or a $K3$.

share a worldvolume direction, x^5. Besides, we have a four-dimensional internal compact manifold $M_4 = \mathbb{T}^4$ or $K3$, and the D5-branes are wrapped around it.

On the field theory side, we have Type IIB string theory on $\mathbb{R}^{1,4} \times S^1 \times M_4$ if we take x^5 to parametrise a circle. This theory preserves 16 out of the 32 supercharges of Type IIB string theory. When the size of M_4 is small compared with the length of S^1, the low-energy dynamics are given by a field theory living in the $(1+1)$-dimensional brane intersection. Standard weak coupling open string quantisation determines that this is a $\mathrm{U}(N_1)\times\mathrm{U}(N_5)$ supersymmetric gauge theory that flows in the IR to a non-trivial CFT. Here N_1 and N_5 are the number of D1- and D5-branes in the considered intersection, respectively. The theory presents $\mathcal{N} = (4,4)$ supersymmetry in $(1+1)$ dimensions, meaning that it displays 4 right-handed and 4 left-handed supercharges. The central charge of the associated superconformal algebra can be proven to be

$$c = 6N_1N_2\,. \tag{3.15}$$

On the supergravity side, the D1-D5 brane system depicted in Table 3.2 gives rise to the following solution to Type IIB supergravity,

$$\begin{aligned} ds^2 =& H_{\mathrm{D1}}^{-1/2} H_{\mathrm{D5}}^{-1/2}(-dt^2 + dx_5^2) + H_{\mathrm{D1}}^{1/2} H_{\mathrm{D5}}^{1/2} dx^l dx^l + H_{\mathrm{D1}}^{1/2} H_{\mathrm{D5}}^{-1/2} ds^2_{M_4}\,, \\ F_3 =& \partial_r H_{\mathrm{D1}}^{-1}\, dt \wedge dx^5 \wedge dr - \partial_r H_{\mathrm{D5}}\, r^3 \mathrm{vol}_{S^3}\,, \quad e^{\Phi} = H_{\mathrm{D1}}^{1/2} H_{\mathrm{D5}}^{-1/2}\,. \end{aligned} \tag{3.16}$$

We have parametrised the transversal space to the D1-D5 system by x^l with $l = 1,2,3,4$. We have also used spherical coordinates for this space, denoting the radial direction by $r = \sqrt{\sum_{l=1}^4 (x^l)^2}$ and by vol_{S^3} the volume form of the associated 3-sphere. Besides, we assumed $H_{\mathrm{D1}} = H_{\mathrm{D1}}(r)$ and $H_{\mathrm{D5}} = H_{\mathrm{D5}}(r)$, which must be harmonic functions in $\mathbb{R}^4_r$ in order to satisfy the Bianchi identities and equations of motion. Let us consider the particular solution

$$\begin{aligned} H_{\mathrm{D1}}(r) =& 1 + \frac{q_{\mathrm{D1}}}{r^2} \quad \text{with} \quad q_{\mathrm{D1}} = \frac{(2\pi)^4 g_s N_1 \alpha'^3}{\mathrm{Vol}_{M_4}}\,, \\ H_{\mathrm{D5}}(r) =& 1 + \frac{q_{\mathrm{D5}}}{r^2} \quad \text{with} \quad q_{\mathrm{D5}} = g_s N_5 \alpha'\,,\,, \end{aligned} \tag{3.17}$$

where Vol_{M_4} is the volume of M_4. Besides, q_{D1} and q_{D5} are two integration constants which provide the length scale $L = (q_{\mathrm{D1}} q_{\mathrm{D5}})^{1/4}$. We can now take the near-horizon limit $r \to 0$, which yields the following fields,

$$\begin{aligned} ds^2 =& ds^2_{\mathrm{AdS}_3} + L^2 ds^2_{S^3} + q_{\mathrm{D1}}^{1/2} q_{\mathrm{D5}}^{-1/2} ds^2_{M_4}\,, \\ F_3 =& 2q_{\mathrm{D5}}\,(\mathrm{vol}_{\mathrm{AdS}_3} + \mathrm{vol}_{S^3})\,, \quad e^{\Phi} = q_{\mathrm{D1}}^{1/2} q_{\mathrm{D5}}^{-1/2}\,, \end{aligned} \tag{3.18}$$

where

$$ds^2_{\mathrm{AdS}_3} = \frac{r^2}{L^2}(-dt^2 + dx_5^2) + \frac{L^2}{r^2} dr^2\,, \quad \mathrm{vol}_{\mathrm{AdS}_3} = \frac{r}{L} dt \wedge dx^5 \wedge dr \tag{3.19}$$

are the metric and element of volume of an AdS_3 with radius L. We can compute the central charge from this supergravity solution using holographic techniques. The holographic central charge is thus given by the formula below [105, 106],

$$c_{\text{hol}} = 3\frac{(p-1)^{p-1}}{G_N}\frac{b(r)^{(p-1)/2}(\hat{H})^{\frac{2p-1}{2}}}{(\hat{H}')^{p-1}}, \tag{3.20}$$

where $G_N = 8\pi^6\alpha'^4 g_s^2$ is the ten-dimensional Newton's constant and we are considering a generic metric and dilaton of the form

$$ds^2 = a(r, y^k)(ds^2_{\mathbb{R}^{1,p-1}} + b(r)dr^2) + g_{ij}(r, y^k)dy^i dy^j, \quad \Phi = \Phi(r, y^k) \tag{3.21}$$

and

$$\hat{H} = \left(\int dy^{9-p}\sqrt{e^{-4\Phi}\det(g_{ij})a(r, y^k)^{p-1}}\right)^2 . \tag{3.22}$$

Here $\{y^i\}$ parametrise the space transversal to the AdS_{p+1}. Applying (3.20)-(3.22) to our background (3.18), we obtain the central charge below,

$$c_{\text{hol}} = \frac{3L^4 \text{Vol}_{S^3}\text{Vol}_{M_4}}{2G_N} . \tag{3.23}$$

This can be further simplified taking into account the values of q_{D1} and q_{D5} that appear in (3.17). The final result is

$$c_{\text{hol}} = 6N_1N_5 , \tag{3.24}$$

which coincides exactly with the central charge obtained via the field theory computation. This further supports the AdS/CFT conjecture.

We point out that this comparison between the central charge computed from the supergravity solution and the dual field theory has being used in our work. In particular, the agreement of both quantities in the IR has being used to support the hypothesis that we will introduce in the next chapter, which states that the quiver field theories we built flowed in said regime into the CFTs dual to the AdS supergravity solutions.

In this chapter we have introduced the notion of holographic duality. We concentrated in particular in the weak form of the correspondence, which relates a strongly coupled field theory with supergravity. We then illustrated the correspondence with the $\text{AdS}_5/\text{CFT}_4$ case that was originally devised by Maldacena [17]. The idea was to consider D3-branes from two different perspectives. From the so-called open string perspective, D-branes are a region of space where open strings may end, which is only valid in the perturbative limit of string theory ($g_s \ll 1$). On the other hand, D-branes can be regarded as objects that warp the spacetime they live in. From the so-called closed string perspective, a brane set-up is described by a supergravity solution. The comparison of both perspectives provides a link between the CFT that arises from the open string viewpoint and the supergravity solution that appears in the closed string one. Furthermore, these ideas can be used to build a map that relates the fields of the supergravity solution to the operators in the CFT. This conjecture can be generalised to other dimensions by taking into consideration different brane set-ups, as we saw for the $\text{AdS}_3/\text{CFT}_2$ through a D1-D5 bound state. Furthermore, one can compute the central charge of the CFT and then compare it to the one obtained from the supergravity solution in order to obtain evidence for the Maldacena conjecture. As we explain in the next chapter, the brane solutions to Type II supergravity are dual to quiver field theories. In some regime these field theories are expected to flow into CFTs which are dual to the AdS solutions that appear in the near-horizon limit of the brane ones.

Chapter 4

Quiver field theories on brane systems

In the previous chapter, we introduced the idea of holography as a duality that links Type II string theory in an AdS space with CFTs living in the boundary of said space. However, AdS solutions usually appear only as near-horizon limits of more general ones. Thus, the next logical step is to wonder whether there is a way of relating supergravity solutions associated to certain brane set-ups with quantum field theories living in their worldvolume. This idea has being explored in the literature (in our work in particular) by building *quiver field theories* living in brane systems, which are dual to the supergravity solutions in this sense. These are theories which can be described in a graphic manner through quivers where the nodes and edges represent the different multiplets that encode the field theory. In more concrete terms, the best understood case are Dp-NS5-D$(p+2)$ brane intersections for some non-negative integer p. They have being found to realise p-dimensional field theories with 8 supercharges that flow to CFTs in the IR (for $p<4$), in the UV (for $p>4$), or are conformal per se (for $p=4$) [107]. These CFTs are dual to AdS$_{p+1}$ solutions in Type IIB supergravity with 16 supercharges that arises as near-horizons of the aforementioned brane intersection. Thus, these constructions have being useful in the study of the AdS/CFT correspondence.

For more complicated brane set-ups, the dual CFT may be difficult to describe, sometimes even lacking a Lagrangian. Following what was explained above, the solution consists on building a quiver field theory, which does admit a Lagrangian and provides an easy description of the fermions. In our papers we considered the AdS$_{p+1}$/CFT$_p$ correspondence with $p=1,2$ and, therefore, it is hypothesised that the quiver field theory flows in the IR into the CFT dual to the AdS supergravity solution, as happened in the case of Dp-NS5-D$(p+2)$ intersections. In order to support this assertion one computes the central charge both from the field theory and the supergravity solution in said limit and compares both results.

The basics for the building of quiver quantum theories in brane set-ups were first explored by Hanany and Witten in the case of a D3-D5-NS5 intersection in Type IIB string theory [108]. We revisit said paper in section 4.1 in order to provide the background to understand the construction of quivers in our original work. The references [91, 107, 109, 110] were also consulted.

There are other Hanany-Witten brane set-ups that are relevant for this thesis, as they appear as mother theories in our works. In particular, the D6-NS5-D8 brane intersection, which is reviewed in subsection 6.2.1, and the D4-NS5-D6 brane set-up, which we revisit in subsection 7.5.1. The former one gives rise to a $\mathcal{N} = (1, 0)$ 6d quiver field theory, while the latter one corresponds to an $\mathcal{N} = 2$ 4d quiver field theory.

4.1. Hanany-Witten brane set-ups

We start by reviewing the original Hanany-Witten brane setup. The ten-dimensional coordinates are denoted to be $x^0, x^1, \dots x^9$. All considered branes extend along the $\mathbb{R}^{1,2}$ associated to x^0, x^1 and x^2. The D3-branes also span the x^6 direction, which is transversal to both D5- and NS5-branes. The remaining worldvolume directions for the NS5-branes are x^3, x^4 and x^5, while for the D5-branes they are x^7, x^8 and x^9. Table 4.1 should serve as a summary for this brane setup. This configuration displays an interesting property: it remains invariant upon performing S-duality and a rotation interchanging (x^3, x^4, x^5) with (x^7, x^8, x^9). This combined transformation is often referred to as RS transformation.

If we take a look to the supersymmetry, of the solution at hand, it is a known fact that Type II supergravity theories can present a maximum of 32 supercharges. It is also known that a D-brane breaks half of the supersymmetries, as it imposes the following condition on the left- and right-handed components of a Killing spinor,

$$\epsilon_L = \Gamma^0 \Gamma^{a_1} \dots \Gamma^{a_p} \epsilon_R \,, \tag{4.1}$$

where $x^{a_1}, \dots, x^{a_p}$ are the worldvolume directions of the considered Dp-brane. Thus, the presence of D5-branes in our brane system halves the number or supersymmetries according to

$$\epsilon_L = \Gamma^0 \Gamma^1 \Gamma^2 \Gamma^7 \Gamma^8 \Gamma^9 \epsilon_R \,. \tag{4.2}$$

Analogously, the presence of NS5-branes halves again the number of supersymmetries, as they are only invariant under those satisfying

$$\epsilon_L = \Gamma^0 \Gamma^1 \Gamma^2 \Gamma^3 \Gamma^4 \Gamma^5 \epsilon_L \,, \quad \epsilon_R = -\Gamma^0 \Gamma^1 \Gamma^2 \Gamma^3 \Gamma^4 \Gamma^5 \epsilon_R \,. \tag{4.3}$$

We also know that Type IIB is a chiral theory and therefore

$$\Gamma_{11} \epsilon_R = \epsilon_R \,, \quad \Gamma_{11} \epsilon_L = \epsilon_L \,. \tag{4.4}$$

Combining (4.2), (4.3) and (4.4), the following condition can be derived,

$$\epsilon_L = \Gamma^0 \Gamma^1 \Gamma^2 \Gamma^6 \epsilon_R \,, \tag{4.5}$$

which corresponds to D3-branes stretching along x^0, x^1, x^2 and x^6. It is quite straightforward to conclude that the addition of D3-branes in said orientation breaks no extra

branes	x^0	x^1	x^2	x^3	x^4	x^5	x^6	x^7	x^8	x^9
D3	×	×	×	−	−	−	×	−	−	−
D5	×	×	×	−	−	−	−	×	×	×
NS5	×	×	×	×	×	×	−	−	−	−

Table 4.1: The $\frac{1}{4}$-BPS intersection of D3-, D5- and NS5-branes originally studied by Hanany and Witten in [108].

supersymmetries. Thus, one ends up naturally with the brane configuration depicted in Table 4.1 preserving 8 supercharges.

The presence of branes also breaks the isometry group from SO(9, 1), the Lorentz group corresponding to empty Minkowski space, to SO(2, 1)×SO(3)×SO(3). The first SO(3) acts on (x^3, x^4, x^5) and is labelled as $\mathrm{SO}(3)_V$, while the second one acts on (x^7, x^8, x^9) and it is labelled as $\mathrm{SO}(3)_H$. Their double covers $\mathrm{SU}(2)_V$ and $\mathrm{SU}(2)_H$ are symmetries of the Coulomb and Higgs branches of the field theory. Thus, the R-symmetry group turns out to be $\mathrm{SO}(4)\simeq\mathrm{SU}(2)_V\times\mathrm{SU}(2)_H$.

The field content of a theory living in a brane configuration is given by the quantisation of open strings ending on the different branes. We consider only D3-branes that are finite in the x^6, ending on D5- or NS5-branes. Thus, the field theories living on these D3-branes is macroscopically $2+1$-dimensional and the fields are independent on x^6. In order to see which are these theories, we start with a single infinite D3-brane. The theory living in said branes is 4d $\mathcal{N}=4$ Super Yang-Mills, as we saw in chapter 3. It presents a U(1) gauge group with an irreducible supermultiplet under the $\mathcal{N}=4$ supersymmetry transformations. Now, when the four-dimensional $\mathcal{N}=2$ (or three-dimensional $\mathcal{N}=4$) subalgebra given by equations (4.2), (4.3) and (4.5), the supermultiplet decomposes into a vector multiplet and a hypermultiplet. When the D3-brane ends on one of the other branes, the resulting boundary conditions set to zero half of the massless fields that live in the worldvolume of the D3-brane. The possible boundary conditions are described below:

1. A $3+1$-dimensional scalar field can obey either Dirichlet or Neumann boundary conditions. The former ones require that the field vanishes at the boundary, while the latter ones set its normal derivative to zero.

2. For $3+1$-dimensional vector fields, we can also have either Dirichlet or Neumann boundary conditions, but they imply something different in this case. If we impose Dirichlet boundary conditions on vector field A with field strength $F=dA$, then we must set $F_{\mu\nu}$ to zero at the boundary, where $\mu,\nu=0,1,2$ represent the directions tangent to the boundary. On the other hand, Neumann boundary conditions imply that the components $F_{\mu 6}$ and $F_{6\mu}$ vanish at the boundary; the direction x^6 acts as the normal direction of the boundary. A $3+1$-dimensional vector field A decomposes into a $2+1$-dimensional scalar field b and a $2+1$-dimensional vector field a. The new fields are related to the old one by $\partial_\mu b = F_{\mu 6}$ and $a_\mu = A_\mu$. Thus, Dirichlet boundary conditions set a to zero in the boundary, while Neumann boundary condition do the

analogous for b.

We thus observe that the possible massless modes of the $2+1$-dimensional field theory are a, b and the fluctuations of the D3-branes along (x^3, x^4, x^5) and (x^7, x^8, x^9). In particular, the fluctuations along (x^3, x^4, x^5) are set to zero at D5-branes and also a. On the other hand, both fluctuations along (x^7, x^8, x^9) and b are set to zero at NS5-branes. Under our algebra, (x^3, x^4, x^5) and a_μ form the bosonic part of the vector multiplet, while (x^7, x^8, x^9) and b form the bosonic part of the hypermultiplet. From now on, we refer to the three-dimensional perspective when we speak about supersymmetry. These facts can be used to identify the effective $2+1$-dimensional field theory depending on the two branes on which the D3 brane ends:

1. If the D3-brane terminates on two NS5-branes, then the theory is that of an $\mathcal{N}=4$ U(1) vector multiplet. In general, for a configuration with n_v parallel D3-branes suspended between two NS5-branes, the field content comes from considering strings stretching between the different branes. The resulting field theory is that of n_v $\mathcal{N}=4$ vector multiplets and the gauge group is $\mathrm{U}(1)^{n_v}$, but gets enhanced at the classical level into $\mathrm{U}(n_v)$ when the D3-branes coincide.

2. When a single D3-brane is delimited by two D5-branes, then an $\mathcal{N}=4$ hypermultiplet arises. If we now consider n_h parallel D3-branes stretched between the NS5-branes, the resulting field theory consists of n_h hypermultiplets in the fundamental representation of $\mathrm{U}(1)^{n_h}$ if there is enough distance between the branes. When the D3-branes coincide, the hypermultiplets are in the fundamental representation of $\mathrm{U}(n_h)$. This can be seen as a consequence of mirror symmetry interchanging the Coulomb and Higgs branches. This is because a combination of S-duality and a rotation transforms the NS5-branes into D5-branes and, therefore, the vector multiplets that appeared in the previous case (where the D3-branes were stretched between two NS5-branes) are now hypermultiplets.

3. Let us now consider a D3-brane delimited by a D5-brane on one side and by an NS5-brane on the other. In this case, the (x^3, x^4, x^5) coordinates of the D3-brane are fixed to match those of the D5-brane. Similarly, the (x^7, x^8, x^9) coordinates of the D3-brane are fixed to match those of the NS5-brane. Also a_μ and b are set to zero. Thus, there are no massless modes in the worldvolume theory.

We now explore the particular brane picture in Figure 4.1 as an example. In this case we have a chain of NS5-branes, a set of K_j parallel D3-branes linking the j-th and $(j+1)$-th NS5-branes. We assume that the D3-branes of each interval are close enough to one another to consider them coincident. Additionally, F_j D5-branes are added in the space between the j-th and $(j+1)$-th NS5-branes. In this scenario, the open strings that have both ends on the same stack of D3-branes contributes, as in the case of a single stack, with an $\mathcal{N}=4$ vector multiplet with $\mathrm{U}(K_j)$ as gauge group. Thus, the gauge group of the whole theory is $\prod_{j=1}^{n-1}\mathrm{U}(K_j)$. Besides, open strings can also end on adjacent stacks of D3-branes, going through an NS5-brane. The result is an $\mathcal{N}=4$ hypermultiplet, which becomes

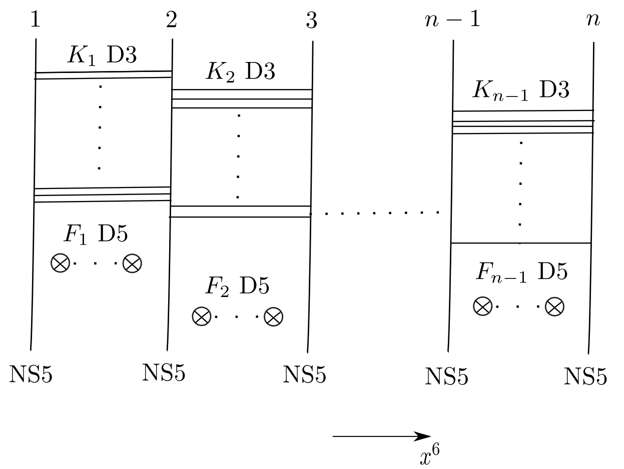


Figure 4.1: A brane picture for the D3-D5-NS5 brane setup.

massless when the two stacks meet at the NS5-brane. The open strings with one end on a D5-brane and the other in the stack of D3-branes contribute in each interval with F_j $\mathcal{N}=4$ hypermultiplets. Finally, open strings with both ends on D5-branes of the same interval give rise to massive modes, which decouple in the low-energy limit. Thus, the gauge group corresponding to the D5-branes of each interval becomes an $\mathrm{U}(F_j)$ global (or flavour) group. This information was summarised in a graphic manner in Figure 4.2. Circles represent $\mathcal{N}=4$ vector multiplets in the adjoint representation of the corresponding $\mathrm{U}(K_j)$ colour group. Boxes, on the other hand, represent F_j hypermultiplets in the fundamental representation of $\mathrm{U}(K_j)$. Thus, each colour group has a flavour one attached to it. Besides, lines are $\mathcal{N}=4$ hypermultiplets in the bifundamental representation of the groups at their ends. As for the coupling constants g_j of the gauge groups, let x_j^6 be the x^6 coordinate of

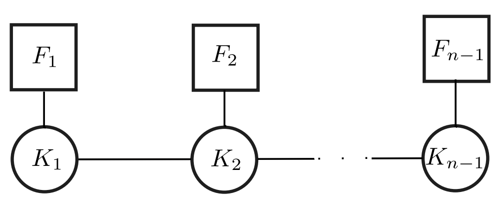


Figure 4.2: Quiver representation of the quantum field theory living in the D3-D5-NS5 brane setup depicted in figure 4.1.

the j-th NS5-brane, we have the following,

$$\frac{1}{g_j^2} = \frac{x_{j+1}^6 - x_j^6}{2\pi g_s} \,. \tag{4.6}$$

This effective coupling can be obtained by integrating along the x^6 direction in the first action in (3.7). As we have assumed that all the fields are independent on that direction, the result is just $x_{j+1}^6 - x_j^6$. Following the same reasoning as in section (3.1) one then finds that the coupling is (4.6).

In [111], Gaiotto and Witten explored the fact that a choice of two partitions ρ and $\hat{\rho}$ of an arbitrary positive integer N completely determine the data of the field theory, namely $\{K_j, F_j\}$. In particular, they conjectured that the 3d field theory associated to the quiver in Figure 4.1 flows to a non-trivial CFT in the IR if the following inequalities hold,

$$\hat{\rho}^T > \rho \iff \rho^T > \hat{\rho}\,. \tag{4.7}$$

Let us start by writing two useful parametrisations of ρ,

$$\begin{aligned} \rho: \quad N =& l_1 + l_2 + \ldots + l_k = \\ =& \underbrace{1 + \ldots + 1}_{F_1} + \underbrace{2 + \ldots + 2}_{F_2} + \ldots\,, \end{aligned} \tag{4.8}$$

where the l_i are a set of integers such that $l_1 \geq l_2 \geq \ldots \geq l_k > 0$ and F_j is the number of times the integer j appears in the decomposition. The F_j are then non-negative integers satisfying $\sum jF_j = N$. One can associate a Young tableaux to ρ with rows of lengths $l_1\,, l_2\,, \ldots\,, l_k$. Something similar can be done to the partition $\hat{\rho}$,

$$\begin{aligned} \hat{\rho}: \quad N =& \hat{l}_1 + \hat{l}_2 + \ldots + \hat{l}_k = \\ =& \underbrace{1 + \ldots + 1}_{\hat{F}_1} + \underbrace{2 + \ldots + 2}_{\hat{F}_2} + \ldots\,, \end{aligned} \tag{4.9}$$

With these parametrisations, we observe that the F_j are precisely the number of hypermultiplets in the fundamental representation of the jth gauge group, while the rank of each of these groups reads

$$K_1 = k - \hat{l}_1 \quad \text{and} \quad K_j = K_{j-1} + m_j - \hat{l}_j \quad j = 2\,, \ldots\,, \hat{k} - 1\,. \tag{4.10}$$

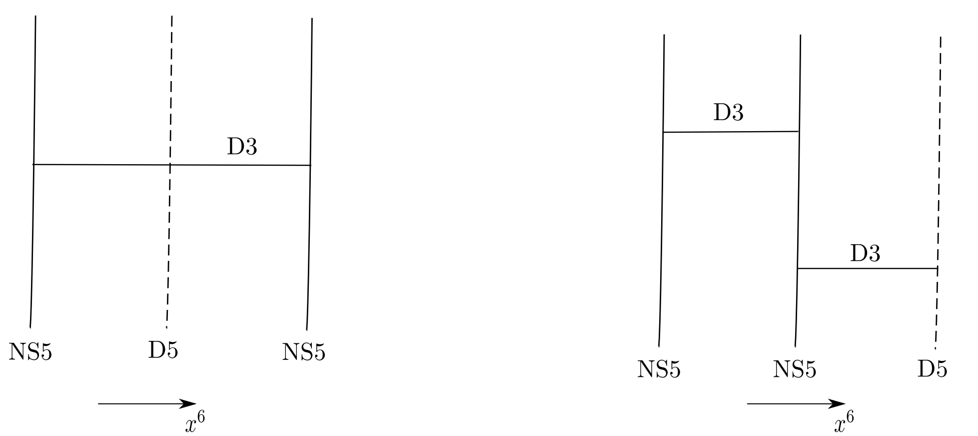
Here m_j is the number of l_i greater than or equal to j in the first line of (4.8). Consequently, $m_1 = k$ and $m_{l+1} = m_l - K_l$. It can be seen that the m_l are a non-increasing sequence of positive integers that define a partition ρ^T, which associated Young tableau is the transpose of that of the partition ρ. The condition (4.7) is a shorthand way of writing the following strict inequalities,

$$\sum_{s=1}^{i} m_s > \sum_{s=1}^{i} \hat{l}_s \quad \forall\; i = 1\,, \ldots\,, l_1\,. \tag{4.11}$$

We observe that, due to (4.10), these inequalities are equivalent to imposing that the ranks K_j of the gauge groups are positive integers. This condition also implies that $F_j = 0$ when $j \geq \hat{k}$ so that there are not hypermultiplets associated to empty gauge group factors. We conclude that (4.11) is necessary for $(\rho, \hat{\rho}, N)$ to properly define a quiver field theory. We observe that if one of the inequalities in (4.10) is replaced by an equality, then the quiver splits into two disconnected components. We conclude that what Gaiotto and Witten proposed is that all such quiver field theories flow in the IR into non-trivial CFTs.

4.1.1. Hanany-Witten brane creation effect

Before this chapter concludes, we must review a very important phenomenon concerning the study of Hanany-Witten-like brane set-ups and their associated quivers. This is no other than the so-called Hanany-Witten brane creation effect depicted in Figure 4.3. Let us consider a brane configuration where a D3-brane connects two NS5-branes with a D5-brane between them as in Figure 4.3a. The branes are oriented as in Table 4.1. Open strings connecting the D5-brane with the right portion of the D3-brane give rise to hypermultiplets with mass proportional to $|\vec{m}-\vec{x}|$, where $\vec{x}$ and $\vec{m}$ are the (x^3, x^4, x^5) coordinates of the transverse position of the D5- and D3-branes respectively. We want to interchange the D5-brane with the NS5-brane on the right. In order to do this, we move the D5-brane to the right and the NS5-brane to the left so they intersect at some point in this procedure. In that position, the hypermultiplet becomes massless. Finally, the two branes "go through" each other creating a second D3-brane linking the interchanged branes. Thus, we end up with Figure 4.3b. A way to give a logical explanation to why the described intersection should generate a new brane is that only in this way we have a massless multiplet when we approach the NS5- and D5-branes in Figure 4.3b. Without the new D3-brane, there would be no reason to have any kind of singularity upon intersecting these two branes.



(a) Original set-up **(b)** Final set-up

Figure 4.3: Example of a Hanany-Witten move.

This phenomenon has been observed in more general situations with other branes and it has proven useful when it comes to finding the quiver for the quantum field theory living in a brane set-up. Furthermore, if we consider a brane system in Type II including an NS5-brane and a Dp-brane under analogous circumstances as before, then a chain of T-dualities shows that a D$(p-2)$-brane linking them is generated if they cross each other[1]. S-duality also suggests that, in the same conditions, the crossing of a D3- an a D5-brane

[1]Regarding the circumstances, no D$(p-2)$-brane can end on both branes before the crossing. Also the orientations of the NS5- and Dp-branes cannot be arbitrary, but they must be related via the chain of T-dualities to the NS5- and D5-branes depicted in Table 4.1, respectively

produces a fundamental string. In general, these crossings of branes are called *Hanany-Witten moves* and can be used to relate a certain brane set-up to another one where the branes are located at the positions we need.

Part II

Original Work

Chapter 5

Outline of the Original Results

This chapter may serve as an introduction to the original work developed for this thesis and explored in further detail in chapters 6 and 7. The papers that collect this work are the ones below:

[65] Y. Lozano, N. Petri and C. Risco, "New AdS_2 supergravity duals of 4d SCFTs with defects," JHEP **10** (2021), 217,
doi:10.1007/JHEP10(2021)217 [arXiv:2107.12277 [hep-th]].

[66] Y. Lozano, N. T. Macpherson, N. Petri and C. Risco, "New $\mathrm{AdS}_3/\mathrm{CFT}_2$ pairs in massive IIA with (0, 4) and (4, 4) supersymmetries," JHEP **09** (2022), 130,
doi:10.1007/JHEP09(2022)130 [arXiv:2206.13541 [hep-th]].

[67] Y. Lozano, N. Petri and C. Risco, "Line defects as brane boxes in Gaiotto-Maldacena geometries," JHEP **02** (2023), 193,
doi:10.1007/JHEP02(2023)193 [arXiv:2212.10398 [hep-th]].

[68] Y. Lozano, N. Petri and C. Risco, "AdS2 near-horizons, defects, and string dualities," Phys. Rev. D **107** (2023) no.10, 106012,
doi:10.1103/PhysRevD.107.106012 [arXiv:2212.11095 [hep-th]].

They share common goals: building new AdS_2 and AdS_3 solutions of Type IIA/B supergravity and, if possible, their holographically dual CFTs. Roughly speaking, they were achieved by following some general steps:

1. Obtention of a new AdS_2 or AdS_3 Type IIA/B background.
2. Computation of spatial distribution of the quantised charges and derivation of the underlying brane set-up.
3. Construction of the dual field theory living in the brane set-up.
4. Computation of the central charge and test of the duality.

We highlight that not all the steps are always attained and, even when they are, the path taken and the concrete tools employed depend greatly on the case. For instance, we were not always able to give a brane interpretation to the solutions nor in all cases we were able to derive the quiver field theory. In the next sections, we will explore these steps and briefly comment on the particular results we obtained.

5.1. Construction of new low-dimensional AdS solutions

The new AdS solutions have being obtained in several ways:

- Starting with a known solution describing a brane intersection, one can sometimes add extra branes in order to obtain a new solution. The AdS solution then arises in the so-called *near-horizon limit*.
- If a Type IIA/B or 11d background is known, one can derive new ones by performing T- or S-duality or applying the truncation/uplifted formulae that relate eleven-dimensional and Type IIA supergravities, as explained in section 1.4.
- From a general class of AdS solutions with a certain amount of supersymmetry (usually computed using G-structure technology), one can either study a certain subclass of it or generalise the geometry.

In the first case, the new branes were taken to be fully localised within the worldvolume of the original system of *background branes*, but smeared along the other directions and they are called *defect branes*. As shown in [46, 58], this requirement is crucial (at least for AdS_2 and AdS_3 vacua) in order to decouple the field equations of the defect branes from those of the background ones.

In Figure 5.1 we present a summary of the AdS_2 solutions of Type IIA/B computed in this thesis. As for the new AdS_3 classes of solutions we obtained a single one of massive Type IIA, characterised by an $\mathrm{AdS}_3 \times S^3 \times M_4$ geometry. In particular, we constructed two subclasses: the first one corresponds to $M_4 = S^2 \times \Sigma_2$, with Σ_2 a Riemann surface, and appears in the near-horizon limit of D2-D4 branes ending on a D6-NS5-D8 bound state; while the second one satisfies $M_4 = \mathbb{T}^3 \times I$ and arises in the massless limit when one takes the near-horizon limit of D2-D4-NS5.

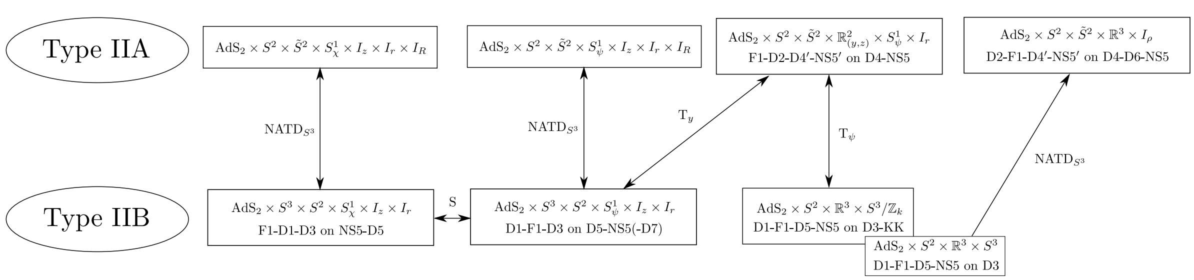


Figure 5.1: Summary of the AdS_2 solutions built in the thesis.

5.2. Computation of quantised charges and derivation of the dual field theory

As explained in section 2.2, once one has a supergravity solution, the fluxes can be used to compute quantised charges that reveal the presence and distribution of the branes and other objects that underlie the background. In particular, we are interested in Hanany-Witten brane set-ups, as these have been mapped to quiver field theories. The key feature of these considered theories is that they can be seen as defects embedded in higher-dimensional field theories dual to the background brane set-up. The multiplets of these defect theories can be obtained by considering the massless modes of open strings ending on the branes of the set-up[1]. The main idea is to take into account which multiplets are compatible with the dimension and supersymmetry of the considered theory and then impose the boundary conditions associated to the branes on which the strings end. This determines the multiplet associated to each set of boundary conditions, as we saw in chapter 4. We remark that the branes themselves can end on other branes, which limits and breaks down the multiplets living in them. In particular, we explore the $\mathcal{N}=(0,4)$ two-dimensional case. We also study the $\mathcal{N}=4$ one-dimensional case, but we consider the notation and knowledge of the $\mathcal{N}=(0,4)$ two-dimensional one.

On the other hand, sometimes, the massive open F1-strings present in the brane set-up admit a so-called *baryon vertex* interpretation. For this to be possible, we need to have a stack of extra D-branes at a distance L from the stack of colour D-branes with both kinds of branes oriented in such a way that we can stretch F1-strings between them. A set of $(l_1, l_2, \ldots, l_M)$ F1-strings stretched between the stacks have as their lowest energy excitation a fermionic field. It was shown in [113, 114] that this produces a half-BPS Wilson loop in an antisymmetric representation labelled by a Young tableau (depicted in Figure 5.2).

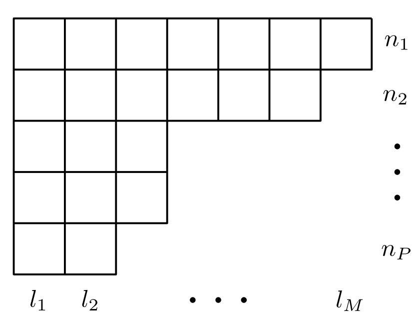


Figure 5.2: Young tableau labelling the irreducible representations of $\mathrm{U}(N)$.

The coupling describing a baryon vertex is an effective version of the Wess-Zumino action, which reads [115]

$$S_{\mathrm{D}p} = T_p \int_{M_{p+1}} \hat{F}_p \wedge A_t \,, \tag{5.1}$$

[1]See, for instance, [49, 108, 112].

where $\hat{F}_p$ is the appropriate Page flux, A_t is the time component of the gauge potential living in the string and M_{p+1} is the worldvolume of the Dp-brane. We now assume that M_{p+1} can be locally written as $M_p \times \mathbb{R}_t$, where $\mathbb{R}_t$ parametrises time. The action (5.1) tells us that $\hat{F}_p$ contributes with N units of A_t-charge, where

$$\int_{M_p} \hat{F}_p = N \tag{5.2}$$

is the amount of Page charge produced by the considered Dp-brane. As the total amount of A_t-charge must vanish in a closed universe, there must be another source which produces a charge of $-N$. As we said above, the source turns out to be N fundamental strings ending on the Dp-brane. As they are charged under A_t with either 1 or -1 depending on the orientation, we must have N fundamental strings with the same orientation in order to cancel the contribution of $\hat{F}_p$. The fundamental strings are then perceived as a gauge-invariant combination of N quarks within the worldvolume of the Dp-brane. This explains why we say that the Dp-brane is called a baryon or anti-baryon vertex, depending on the orientation.

5.3. Computation of the central charge and test of the duality

We are interested in AdS$_2$ and AdS$_3$ backgrounds, which are holographically dual to SCQMs and CFT$_2$s, respectively. In order to study these dualities, we constructed quiver field theories living in the underlying brane set-ups. As explained in chapter 4, one such theory is expected to flow in the IR into the CFT dual to the considered AdS solution. In order to support this hypothesis, we can compute the central charge of the field theory in two ways and compare them in the IR. The first way consists on computing the holographic central charge from the AdS supergravity solution via the formula given by (3.20). On the other hand, one can compute the central charge directly from the field theory, as we explain below. In the IR, both results should coincide with the central charge of the CFT.

Let us start by considering AdS$_3$ backgrounds. In particular, in our papers we computed the central charge of 2d $\mathcal{N} = (0,4)$ CFTs from the R-symmetry anomaly (see for instance [116]). This is a so-called *t' Hooft anomaly*, meaning that it is scale-independent. Therefore, it makes sense for us to compute the central charge of the quiver field theory, which is a UV deformation of the CFT, and then study its IR limit. This is because its expression must remain valid at all scales/energies. The formula we considered is give by

$$c_R = 3\ \mathrm{Tr}\big(\gamma^3 Q_R^2\big)\,, \tag{5.3}$$

where the trace is over the Weyl fermions of the theory, γ^3 is the 2d chirality matrix and Q_R is the R-charge under $\mathrm{U}(1)_R$. We recalls the following facts:

- $\mathcal{N} = (0,4)$ vector multiplets contain two left-moving fermions with R-charge 1,

- $\mathcal{N}=(0,4)$ twisted hypermultiplets contain two right-moving fermions with R-charge 0,
- $\mathcal{N}=(0,4)$ hypermultiplets contain two right-moving fermions with R-charge -1,
- $\mathcal{N}=(0,2)$ Fermi multiplets contain one left-moving fermion with R-charge 0,
- $\mathcal{N}=(4,\,4)$ hypermultiplets contain an $\mathcal{N}=(0,4)$ hypermultiplet plus an $\mathcal{N}=(0,4)$ Fermi multiplet and also provide 2 units of charge to the R-symmetry anomaly.

Substituting these R-charges in (5.3), one gets the well-known expression below [116],

$$c_R = 6\,(n_{hyp} - n_{vec}), \tag{5.4}$$

where n_{hyp} stands for the number of $\mathcal{N}=(0,4)$ untwisted hypermultiplets and n_{vec} for the number of $\mathcal{N}=(0,4)$ vector multiplets.

In the AdS_2 case this computation involves some caveats, as defining a central charge for a one-dimensional CFT is known to be a subtle issue [2], which we did not had in the two-dimensional case. The main difficulty lies in its interpretation, as the central charge of a CFT is its free energy. However, as the trace of the stress-energy tensor of a conformal field theory must vanish, the free energy of a one-dimensional CFT (its central charge) must be zero. It is, in principle, not clear if we can compute a non-vanishing central charge or what its interpretation should be. The central charge in this case has being argued to be counting the degeneracy of ground states of the system. Some proposals exist in the literature for computing this degeneracy. In [119–121] the number of ground states of quiver quantum mechanics with gauge group $\prod_v \mathrm{U}(N_v)$ with the $\mathrm{U}(N_v)$ subgroups connected by bifundamentals (so-called Kronecker quivers) was computed by quantising the classical moduli space in the Higgs branch. The result is

$$\mathcal{M} = \sum_{v,w} N_v N_w - \sum_v N_v^2 + 1\,, \tag{5.5}$$

where N_w stands for the rank of the gauge groups adjacent to a given colour group of rank N_v.

Alternatively, it was shown in [62, 64] that when the AdS_2 solution dual to an $\mathcal{N}=4$ SCQM can be obtained from an AdS_3 space through a null compactification (the so-called null orbifold construction) the dual SCQM corresponds to the chiral half of the 2d CFT dual to the AdS_3 solution. The AdS_3 spaces considered in [62, 64] preserve $\mathcal{N}=(4,4)$ supersymmetries, but the result can be extrapolated to the case in which they preserve $\mathcal{N}=(0,4)$, where the SCQM simply arises upon compactification of the 2d dual $\mathcal{N}=(0,4)$ CFT. In these situations the obvious interpretation of the central charge of the SCQM is as counting the excitations of the 2d CFT, and the caveats mentioned above do not apply. Moreover, one can use the expression (5.4) to obtain the central charge of the SCQM. This was done explicitly for the class of AdS_2 solutions constructed in [42], obtained by

[2] See for instance [63, 117, 118]

T-dualising a sub-class of the AdS_3 solutions to Type IIA supergravity with $\mathcal{N} = (0,4)$ supersymmetries constructed in [38][3]. The central charge computed this way was shown to agree with the holographic calculation in the holographic limit.

Remarkably, in [43,45,65] other classes of AdS_2 solutions with $\mathcal{N} = 4$ supersymmetry were constructed that do not bear any relation with AdS_3. Still, the expression that gives the central charge of a 2d $\mathcal{N} = (0,4)$ CFT was used to compute the central charge and it was shown to agree with the holographic result. This agreement is a remarkable result and deserves further research. It could be related to the fact that the 2d expression can be shown to agree to leading order with the 1d expression given by (5.5), when applied to the same type of quivers.

There are several arguments supporting the fact that the central charge of a SCQM can always be computed by applying (5.3) or (5.4), even when it cannot be obtained via compactifying a two-dimensional CFT [63, 117]. For instance, we have that the R-symmetry anomaly comes from the superconformal symmetry. In this respect, we have that the superconformal algebra of an AdS_3 geometry consists of two copies of the Virasoro algebra, while that of an AdS_2 vacuum is one copy of the Virasoro algebra. These seems to suggest that the tools used in the AdS_3 case should still be valid when we consider an AdS_2 geometry.

Besides, in [117] two actions we considered: both of them admitted an AdS_2 vacuum, but while the first one could not be lifted up to an AdS_3, the second one could. For both of them it was observed that a twisted energy-momentum tensor was required in order to have consistent boundary condition. The presence of said tensor then gave rise to a non-vanishing central charge in both models. This seems to suggest that the 2d expression for the central charge may be always valid for the SCQMs dual to consistent quantum gravities in AdS_2.

[3] It is straightforward to see that the Abelian T-duality transformation performed in [42] is equivalent to the null orbifold construction in [62, 64].

Chapter 6

AdS$_3$/CFT_2 in Type II

This chapter presents the new results published in [66]. In said paper, the AdS$_3$/CFT_2 case of the holographic conjecture was explored. We start by presenting a new class of $\mathcal{N} = (0,4)$ AdS$_3$ solutions of massive IIA supergravity in section 6.1, as well as its interpretation as brane intersection. We then move on to building the quiver field theory living on said brane set-up in section 6.2. Section 6.3 is then devoted to the study of the supergravity solution and its dual quiver in the particular case in which the Romans mass vanishes and part of the spacetime is given by a $\mathbb{T}^3$. Finally, in section 6.4 we go back to the massive case, embed the supergravity solution in Type I' and explore its dual quiver field theory.

6.1. A new class of $\mathcal{N} = (0,4)$ AdS$_3$ solutions in massive IIA

We started by generalising a class of $\frac{1}{4}$-BPS solutions describing the D8-D6-NS5 intersection developed in [122,123]. The original class presents a flat six-dimensional Minkowski space as external space and no F_4. The generalisation was done by adding an RR four-form flux that further breaks the isometries of the external space as follows,

$$\mathbb{R}^{1,5} \to \mathrm{AdS}_3 \times S^3 \,. \tag{6.1}$$

The general form of the fields of solutions in the new class are given below,

$$\begin{aligned}
ds^2 =& \frac{q}{\sqrt{h}}\left[ds^2(\mathrm{AdS}_3) + ds^2(\mathrm{S}^3)\right] + g\left[\frac{1}{\sqrt{h}}d\rho^2 + \sqrt{h}\Big(dz_1^2 + dz_2^2 + dz_3^2\Big)\right], \\
e^{-\Phi} =& \frac{h^{\frac{3}{4}}}{\sqrt{g}}, \quad F_0 = \frac{\partial_\rho h}{g}, \quad F_4 = 2\,q\Big(\mathrm{vol}(\mathrm{AdS}_3) + \mathrm{vol}(\mathrm{S}^3)\Big) \wedge d\rho\,, \\
F_2 =& -\left(\partial_{z_1} h dz_2 \wedge dz_3 + \partial_{z_2} h dz_3 \wedge dz_1 + \partial_{z_3} h dz_1 \wedge dz_2\right), \\
H_3 =& -\left(\partial_{z_1} g dz_2 \wedge dz_3 + \partial_{z_2} g dz_3 \wedge dz_1 + \partial_{z_3} g dz_1 \wedge dz_2\right) \wedge d\rho + \\
& + \partial_\rho(hg) dz_1 \wedge dz_2 \wedge dz_3\,,
\end{aligned} \tag{6.2}$$

where the AdS$_3$ and S^3 are of unit radius and q is a redundant constant that will be useful later. Away from sources, the Bianchi identities for the fluxes demand that the F_0 is constant and

$$(\partial_{z_1}^2+\partial_{z_2}^2+\partial_{z_3}^2)g+\partial_\rho^2(gh)=0\,,\quad (\partial_{z_1}^2+\partial_{z_2}^2+\partial_{z_3}^2)h+F_0\partial_\rho(gh)=0\,, \tag{6.3}$$

which are the same PDEs that appeared in [123]. These constraints give rise locally to two classes of solutions depending on whether F_0 vanishes or not. If $F_0=0$, then $\partial_\rho h=0$ and the PDEs are those of a flat D6-NS5 intersection. On the other hand, in the $F_0\neq 0$ scenario, we can clear g from the definition of F_0,

$$g=\frac{\partial_\rho h}{F_0}\,. \tag{6.4}$$

Taking this into account, the equations in (6.3) boil down to a single one,

$$(\partial_{z_1}^2+\partial_{z_2}^2+\partial_{z_3}^2)h+\frac{1}{2}\partial_\rho^2(h^2)=0\,. \tag{6.5}$$

6.1.1. Supersymmetry

Next we studied the amount of supersymmetry preserved by this new class of solutions. Our solutions (6.2) are within the class (2.68), whose amount of supersymmetry was studied in section 2.4 through the tools of G-structure for the case of $\mathcal{N}=(1,1)$. We recall that we considered $\mathcal{N}=(1,1)$ as an example in section 2.4 instead of the more general $\mathcal{N}=(0,1)$ because the latter case needed tools and concepts that lie beyond the purposes of this thesis. In any case, for our solutions (6.2), we applied the bi-spinor relations for $\mathcal{N}=(0,1)$ first introduced in [33] and generalised in [124]. The 10d Majorana-Weyl spinor for $\mathcal{N}=(0,1)$ supersymmetric AdS$_3$ can be decomposed as follows,

$$\epsilon_1=\zeta_+\otimes\theta_+\otimes\chi^1\,,\quad \epsilon_2=\zeta_+\otimes\theta_-\otimes\chi^2\,, \tag{6.6}$$

where $\theta_\pm$ are two-dimensional vectors and their subscript denotes the 10d chirality, ζ_+ is an AdS$_3$ Killing spinor realising the $\mathcal{N}=(0,1)$ supersymmetry and its label denotes the SL(2)$_+$ subgroup of SO(2,2) = SL(2)$_+\times$SL(2)$_-$ and $\chi^{1,2}$ are two real Killing spinors with support on M_7. The aforementioned bi-spinor relations then take the following form,

$$\begin{aligned}&d_{H_3}(e^{A-\Phi}\Psi_-)=0\,,\quad d_{H_3}(e^{2A-\Phi}\Psi_+)-2e^{A-\Phi}\Psi_-=\frac{1}{8}e^{3A}\star_7\lambda(f_+)\,,\\ &(\Psi_-\wedge\lambda f_+)\Big|_7=-\frac{1}{2}e^{-\Phi}\mathrm{vol}(\mathrm{M}_7)\,,\end{aligned} \tag{6.7}$$

where

$$\Psi_++i\Psi_-\equiv\chi_1\otimes\chi_2^\dagger \tag{6.8}$$

and the tensor product is defined as in (2.72). However, the explicit expression of $\Psi_\pm$ does not matter, as we only need them to realise a $G_2\times G_2$ structure. For this paper, we assumed that the intersection of the two G_2 was SU(3), which lets us parametrise the $\Psi_\pm$ as

$$\Psi_+ = -\text{Im}\left(e^{-iJ}\right) + V \wedge \text{Re}\Omega\,, \quad \Psi_- = -\text{Im}\Omega - V \wedge \text{Re}\left(e^{-iJ}\right)\,, \tag{6.9}$$

where V is a real one-form whose dual vector is taken to define a direction in M_7 and (J, Ω) are defined in terms of another 3 complex vielbein directions E_1, E_2, E_3,

$$J = E_1 \wedge \overline{E}_1 + E_2 \wedge \overline{E}_2 + E_3 \wedge \overline{E}_3\,, \quad \Omega = E_1 \wedge E_2 \wedge E_3\,. \tag{6.10}$$

We remark that the class of solutions (6.2) preserves $\mathcal{N} = (0,4)$ supersymmetry if it preserves 4 independent SU(3)-structures satisfying (6.7). Exploiting the fact that the class presents an S^3 factor, we defined 1-forms (L_a, R_a) (with $a = 1,2,3$) such that

$$dL_a = \frac{1}{2}\epsilon_{abc}L_b \wedge L_c\,, \quad dR_a = -\frac{1}{2}\epsilon_{abc}R_b \wedge R_c\,, \quad ds^2(\text{S}^3) = \frac{1}{4}(L_a)^2 = \frac{1}{4}(R_a)^2\,. \tag{6.11}$$

L_a behaves as a singlet/triplet under the $\text{SO}(3)_{L/R}$ subgroup of $\text{SO}(4)=\text{SO}(3)_L\times\text{SO}(3)_R$, while R_a transforms the other way round. We then considered the SU(3)-structure defined through the vielbein

$$E_a = -\sqrt{g}h^{\frac{1}{4}}dx_a + i\frac{1}{2\mu h^{\frac{1}{4}}}L_a\,, \quad V = \frac{\sqrt{g}}{h^{\frac{1}{4}}}d\rho\,, \tag{6.12}$$

which can be shown to solve (6.7), realising $\mathcal{N} = (0,1)$ explicitly. We highlight that $\Psi_\pm$ depends on the 3-sphere through L_a, dL_a (which are $\text{SO}(3)_R$ triplets) and $\text{vol}(\text{S}^3)$, which is SO(4)-invariant, with only the latter one affecting the physical fields. This means that, if (6.12) solves (6.7), then so does the SU(3)-structure obtained via performing a generic constant $\text{SO}(3)_R$ rotation in (6.12), which only transforms the L_a. One can apply this to three independent $\text{SO}(3)_R$ rotations in order to generate three further independent SU(3)-structures which solve (6.7) for the same physical fields. We then concluded that $\mathcal{N} = (0,1)$ supersymmetry is always enhanced to small $\mathcal{N} = (0,4)$ in our class (6.2). The fact that this one and no other superconformal group is the one realised for (6.2) comes from the realisation that any other such group would require the presence of other isometries that do not appear in the class in general. Another way to see this would be to realise that the class of solutions can be related through Abelian T-duality to that of section 3.3 of [51], for the particular case where the coordinate x parametrises an isometry directions.

The presence of the round 3-sphere in (6.2) raises the question of whether or not an enhancement to $\mathcal{N} = (4,4)$ is possible and under which restrictions. We noticed that said enhancement, if possible, would require four additional $\mathcal{N} = (1,0)$ SU(3)-structures to be supported by the background. They should satisfy the equation which results after performing the substitution $\Psi_- \to -\Psi_-$ in (6.7) [1]. Analogously to the $\mathcal{N} = (0,1)$ case, the new SU(3)-structures must span the S^3 in terms of R_a. This is because each $\mathcal{N} = 4$

[1]It must be noticed that this map does not hold in general, but only for the restricted case at hand. See [50] for full details.

sub-sector must be a singlet with respect to the R-symmetry group of the other one. Let us consider the following vielbein,

$$E_a = \sqrt{g}h^{\frac{1}{4}}dx_a + i\frac{1}{2\mu h^{\frac{1}{4}}}R_a\,, \quad V = -\frac{\sqrt{g}}{h^{\frac{1}{4}}}d\rho\,. \tag{6.13}$$

It can be shown to give rise to an SU(3)-structure that satisfies the $\mathcal{N} = (1, 0)$ conditions. The same train of thought used before implies that another three SU(3)-structures can be built by using SO(3)$_L$ rotations. However the F_2 now changes sign with respect to the one in (6.2). This implies that the physical fields of our class of solutions are only compatible with both sub-sectors if they satisfy $dh = 0$, rendering F_2, F_0 trivial. However, in the general case only $\mathcal{N} = (0, 4)$ supersymmetry is preserved. A last and interesting remark is that, in the case where $h \neq$ constant S^3 can be replaced by the lens space S$^3/\mathbb{Z}_k$ without breaking any further supersymmetry. Nevertheless, when $h =$ constant the Lens space does break $\mathcal{N} = (4,\ 4)$ into $\mathcal{N} = (0, 4)$.

6.1.2. The brane picture

The next matter that we addressed is that of the interpretation of (6.2) as the near-horizon limit of a brane intersection defined by D2-D4 branes ending on D6-NS5-D8 bound states, as depicted in Table 6.1.

Branes	x^0	x^1	r	φ^1	φ^2	ρ	ζ	θ^1	θ^2	θ^3
D2	×	×	–	–	–	×	–	–	–	–
D4	×	×	×	×	×	–	–	–	–	–
NS5	×	×	–	–	–	–	×	×	×	×
D6	×	×	–	–	–	×	×	×	×	×
D8	×	×	×	×	×	–	×	×	×	×

Table 6.1: $\frac{1}{8}$-BPS brane intersection giving rise in the near-horizon limit to the $\mathcal{N} = (0, 4)$ AdS$_3$ solutions described by (6.2). (x^0, x^1) parametrise an $\mathbb{R}^{1,1}$, where the 2d dual CFT lives, (r, φ^i) are spherical coordinates that describe the same space that the Cartesian (z_1, z_2, z_3), ζ is the radial coordinate of AdS$_3$ and the θ^i parametrise the S^3.

The supergravity solution for D6-NS5-D8 had already being explored by Imamura in [122]. By adding the D2- an D4-branes to said solution, we arrived at the following fields,

$$
\begin{aligned}
ds^2 =& h^{-1/2}\left[H_{\mathrm{D4}}^{-1/2}\,H_{\mathrm{D2}}^{-1/2}\,ds^2(\mathbb{R}^{1,1})+H_{\mathrm{D4}}^{1/2}\,H_{\mathrm{D2}}^{1/2}\,(d\zeta^2+\zeta^2 ds^2(\mathrm{S}^3))\right]\\
&+h^{-1/2}\,g\,H_{\mathrm{D4}}^{1/2}\,H_{\mathrm{D2}}^{-1/2}\,d\rho^2+h^{1/2}\,g\,H_{\mathrm{D4}}^{-1/2}\,H_{\mathrm{D2}}^{1/2}(dr^2+r^2ds^2(\mathrm{S}^2))\,,\\
e^{\Phi} =& h^{-3/4}\,g^{1/2}\,H_{\mathrm{D2}}^{1/4}\,H_{\mathrm{D4}}^{-1/4}\,,\\
H_3 =& -\partial_r g\,r^2\,d\rho\wedge\mathrm{vol}(\mathrm{S}^2)+H_{\mathrm{D2}}\,H_{\mathrm{D4}}^{-1}\,\partial_\rho\,(h\,g)\;r^2\,dr\wedge\mathrm{vol}(\mathrm{S}^2)\,,\\
F_2 =& -\partial_r h\,r^2\,\mathrm{vol}(\mathrm{S}^2)\,,\\
F_4 =& \partial_\zeta H_{\mathrm{D2}}^{-1}\,\mathrm{vol}(\mathbb{R}^{1,1})\wedge d\zeta\wedge d\rho-\partial_\zeta H_{\mathrm{D4}}\,\zeta^3\,\mathrm{vol}(\mathrm{S}^3)\wedge d\rho
\end{aligned}
\tag{6.14}
$$

with the addition of a Romans mass F_0. The two-dimensional Minkowski space is described by coordinates (x^0, x^1), the space transverse to D2-D4 by (ζ, θ^i) and (r, φ^i) are spherical coordinates parametrising the 3d space previously codified by (z_1, z_2, z_3). We assume that the charges associated to D2-D4 are completely localised within the worldvolume of the D6-NS5-D8 bound system. This is implemented by imposing that the corresponding warp factors satisfy $H_{\mathrm{D2}} = H_{\mathrm{D2}}(\zeta)$ and $H_{\mathrm{D4}} = H_{\mathrm{D4}}(\zeta)$. Besides, the D6-NS5-D8 intersection is codified by $h = h(\rho, r)$ and $g = g(\rho, r)$. The Bianchi identities for the D2-D4 system decouple from those of the D6-NS5-D8 bound state giving rise to

$$
H_{\mathrm{D2}} = H_{\mathrm{D4}} \quad \text{and} \quad \nabla^2_\zeta\, H_{\mathrm{D4}} = 0\,,
\tag{6.15}
$$

are those for the former and

$$
\partial_\rho h = F_0\, g \quad \text{and} \quad \nabla^2_r\, h + \frac{1}{2}\partial^2_\rho\, h^2 = 0\,,
\tag{6.16}
$$

for the latter. In the previous PDEs, we have denoted by ∇^2_r and ∇^2_ζ the Laplacian in spherical coordinates on the spaces transverse to the D6-NS5-D8 and D2-D4 systems, respectively. We notice that (6.16) is nothing else that (6.3) in the massive case, i.e. after imposing (6.4) and (6.5). On the other hand, equation (6.15) can be easily solved by the simplest 4d harmonic function,

$$
H_{\mathrm{D4}}(\zeta) = H_{\mathrm{D2}}(\zeta) = 1 + \frac{q}{\zeta^2}
\tag{6.17}
$$

with q an integration constant.

If we take the limit $\zeta \to 0$, then ζ becomes the radial coordinate of an AdS_3, and the fields of the brane solution (6.14) take the form below,[2]

$$
\begin{aligned}
ds^2_{10} =& q\,h^{-1/2}\left[ds^2(\mathrm{AdS}_3)+ds^2(S^3)\right]+h^{-1/2}g\,d\rho^2+h^{1/2}g\left(dr^2+r^2ds^2(S^2)\right)\,,\\
e^{\Phi} =& h^{-3/4}g^{1/2}\,,\quad H_3=-\partial_r g\,r^2\,d\rho\wedge\mathrm{vol}(S^2)+\partial_\rho\,(h\,g)\;r^2\,dr\wedge\mathrm{vol}(s^2)\,,\\
F_2 =& -\partial_r h\,r^2\,\mathrm{vol}(S^2)\,,\quad F_4 = 2q\,\mathrm{vol}(\mathrm{AdS}_3)\wedge d\rho+2q\,\mathrm{vol}(S^3)\wedge d\rho
\end{aligned}
\tag{6.18}
$$

[2]The Minkowski coordinates have been rescaled as $(t, x^1) \to q\,(t, x^1)$ in order to obtain an AdS_3 of unitary radius.

with the presence of the F_0 and (h, g) solving (6.16). We observe that this is precisely (6.2) with the three-dimensional space transverse to the D6-NS5-D8 intersection parametrised in spherical coordinates (r, φ^i) instead of the Cartesian (z^1, z^2, z^3). This allows for an interpretation of our new solutions (6.2) as the low-energy regime of the D6-NS5-D8 bound states wrapping an AdS$_3$ $\times$ S^3 geometry, completely determined by the choice of h and g, and the D2-D4 intersection is fully resolved into said geometry. Thus, the addition of the D2-D4 branes break the isometries of the six-dimensional worldvolume common to the D6-NS5-D8 intersection, as in (6.1).

6.1.3. An uplift of 6d minimal $\mathcal{N} = 2$ ungauged supergravity

One last detail that we found interesting is the fact that governing PDEs (6.3) support solutions with either a warped Mink$_6$ or an AdS$_3 \times S^3$ factor. This led us to the idea that it should actually work for any solution to 6d $\mathcal{N} = 2$ ungauged supergravity with SU(2) R-symmetry. The pseudo-action of said theory is displayed below,

$$S_6 = \int d^6x\sqrt{-g_6}\Big(R - \frac{1}{3}H^{(6)}_{abc}H^{(6)abc}\Big), \tag{6.19}$$

where $H^{(6)}$ is a closed self-dual 3-form, the latter constraint needing to be imposed after varying the action. This theory can be embedded into massive IIA supergravity through the following expressions for the 10d fields,

$$\begin{aligned}
ds^2 =& \frac{1}{\sqrt{h}}\Big[c^{-2}ds_6^2 + g d\rho^2\Big] + g\sqrt{h}\Big(dz_1^2 + dz_2^2 + dz_3^2\Big)\,, \quad e^{-\Phi} = \frac{h^{\frac{3}{4}}}{\sqrt{g}}\,,\\
H_3 =& -(\partial_{z_1} g dz_2\wedge dz_3 + \partial_{z_2} g dz_3\wedge dz_1 + \partial_{z_3} g dz_1\wedge dz_2)\wedge d\rho\\
&+ \partial_\rho(hg) dz_1\wedge dz_2\wedge dz_3\,,\\
F_0 =& \frac{\partial_\rho h}{g}\,, \quad F_2 = -(\partial_{z_1} h dz_2\wedge dz_3 + \partial_{z_2} h dz_3\wedge dz_1 + \partial_{z_3} h dz_1\wedge dz_2)\,,\\
F_4 =& 2c^2 H^{(6)}\wedge d\rho\,,
\end{aligned} \tag{6.20}$$

where c is an arbitrary constant. We verified that the solutions of the form (6.20) satisfy the 10d equations of motion provided that they meet (6.3) and ds_6^2, $H^{(6)}$ solve the 6d equations of motion derived from (6.19). A complete classification for such supersymmetric solutions was presented in [125].

6.2. Defects within $\mathcal{N} = (1, 0)$ 6d CFTs

In this section, we expound a family within the class of solutions (6.2) characterised by an asymptotically locally AdS$_7$ geometry and their dual interpretation as surface defects within the 6d $\mathcal{N} = (1, 0)$ CFTs dual to the AdS$_7$ solutions of massive Type IIA supergravity constructed in [126].

Our first aim was to derive the particular set of coordinates for which the AdS_7 asymptotics is manifest. This can be done by direct calculation in ten dimensions or by making use of the consistent truncation of massive IIA supergravity to minimal 7d $\mathcal{N}=1$ gauged supergravity [127]. From the latter viewpoint, these supergravity solutions take the form of a domain wall with $\text{AdS}_3 \times S^3$ worldvolume with a locally AdS_7 vacuum at infinity, which arises upon consistent truncation from the $\text{AdS}_7 \times S^2 \times I$ solutions of [126]. In ten dimensions one can see from the brane picture studied in subsection 6.1.2 that D2-D4 branes break the isometries of the $\mathbb{R}^{1,5}$ worldvolume common to the D6-NS5-D8 intersection, as

$$\mathbb{R}^{1,5} \quad \longrightarrow \quad \text{AdS}_3 \times S^3 \,, \tag{6.21}$$

leaving intact the conformal symmetries of AdS_3. In the UV the AdS_7 vacuum emerges as a foliation of the $\text{AdS}_3 \times S^3$ subspace over an interval.

With the insight coming from the supergravity analysis, we constructed 2d $\mathcal{N}=(0,4)$ quiver gauge theories that flow in the IR to the CFTs dual to the AdS_3 solutions and showed that they can be embedded within the 6d quivers constructed in [128, 129], dual to the AdS_7 solutions in [126].

6.2.1. The AdS_7 vacua of massive IIA and their dual 6d CFTs

Let us briefly review the main properties of the AdS_7 solutions of massive IIA supergravity and of their 6d dual CFTs.

The solutions in [126] are described by $\text{AdS}_7 \times S^2$ foliations over an interval preserving 16 supercharges. They arise in the near horizon limit of a D6-NS5-D8 intersection, constructed in [130]. In the parametrisation of [129] they take the form

$$\begin{aligned}
ds^2_{10} =& \pi\sqrt{2}\Bigg[8\Big(-\frac{\alpha}{\alpha''}\Big)^{1/2} ds^2_{\text{AdS}_7} + \Big(-\frac{\alpha''}{\alpha}\Big)^{1/2} dy^2 + \\
& + \Big(-\frac{\alpha}{\alpha''}\Big)^{1/2} \frac{(-\alpha\alpha'')}{\alpha'^2 - 2\alpha\alpha''} ds^2_{S^2}\Bigg] \,, \\
e^{2\Phi} =& 3^8 2^{5/2} \pi^5 \frac{(-\alpha/\alpha'')^{3/2}}{\alpha'^2 - 2\alpha\alpha''} \,, \\
B_2 =& \pi\Big(-y + \frac{\alpha\alpha'}{\alpha'^2 - 2\alpha\alpha''}\Big) \text{vol}_{S^2} \,, \\
F_2 =& \Big(\frac{\alpha''}{162\pi^2} + \frac{\pi F_0 \alpha\alpha'}{\alpha'^2 - 2\alpha\alpha''}\Big) \text{vol}_{S^2} \,.
\end{aligned} \tag{6.22}$$

The solutions are specified by the function $\alpha(y)$, which satisfies the differential equation

$$\alpha''' = -162\pi^3 F_0 \,. \tag{6.23}$$

Let us now recall the main ingredients of the 6d quivers dual to these solutions. We will follow [129] and [131]. The B_2 in (6.22) (see below) implies that there are (colour) NS5-branes located at given positions in the y-direction, which can be labelled by an integer number k. Piecewise $\alpha(y)$ functions defined in intervals $[k, k+1]$ between NS5-branes can then be constructed, with continuous first and second derivatives, and third derivative satisfying

$$\alpha_k''' = -81\pi^2\beta_k\,. \tag{6.24}$$

We thus have on a given $[k, k+1]$ interval

$$Q_{NS5}^{(k)} = \frac{1}{4\pi^2}\int H_3 = \frac{1}{4\pi^2}\int_{\mathrm{S}^2}\Big(B_2(y=k+1) - B_2(y=k)\Big) = 1\,, \tag{6.25}$$

where we have chosen units where $\alpha' = g_s = 1$. Moreover, given that $Q_{D8} = 2\pi F_0$, equation (6.24) implies that

$$Q_{D8}^{(k)} = \beta_k \tag{6.26}$$

on each $[k, k+1]$ interval. β_k are therefore integer numbers, and $(\beta_{k-1} - \beta_k)$ is the number of D8-branes introduced at each $y = k$ position. Integrating (6.24) one finds

$$\alpha_k(y) = -\frac{27}{2}\pi^2\beta_k(y-k)^3 + \frac{1}{2}\gamma_k(y-k)^2 + \delta_k(y-k) + \mu_k \quad \text{for} \quad y \in [k, k+1]\,, \tag{6.27}$$

where $(\gamma_k, \delta_k, \mu_k)$ are constants that are determined by imposing continuity of $\alpha, \alpha', \alpha''$. The condition that $\alpha_k'' = \alpha_{k-1}''$ at $y = k$ implies that

$$\gamma_k = -81\pi^2\beta_{k-1} + \gamma_{k-1} = -81\pi^2(\beta_0 + \beta_1 + \cdots + \beta_{k-1})\,. \tag{6.28}$$

This means that the D6-brane charge at each interval is given by

$$Q_{D6}^{(k)} = \frac{1}{2\pi}\int_{\mathrm{S}^2}\hat{F}_2, = -\frac{\gamma_k}{81\pi^2}\,, \tag{6.29}$$

where $\hat{F}_2 = F_2 - F_0 \wedge B_2$ is the Page flux, defining a charge that should be integer. In turn, the conditions $\alpha_k' = \alpha_{k-1}'$ and $\alpha_k = \alpha_{k-1}$ at $y = k$ impose, respectively,

$$\delta_k = -\frac{81}{2}\pi^2\beta_{k-1} + \gamma_{k-1} + \delta_{k-1}\,, \quad \mu_k = -\frac{27}{2}\pi^2\beta_{k-1} + \frac{1}{2}\gamma_{k-1} + \delta_{k-1} + \mu_{k-1}\,. \tag{6.30}$$

The continuity conditions need to be supplemented by conditions at the boundaries of the y-interval. For this to be geometrically well-defined, the asymptotic form of the metric needs to approach one of 4 physical behaviours compatible with the metric factors, namely a regular zero or singular D6, O6 or D8/O8 behaviour. Two of these arise generically: one can choose the integration constants so that $\alpha = 0$ at a boundary of the space, in which case the behaviour corresponds to fully localised D6-branes, or one can impose that $\alpha'' = 0$, in which case one finds fully localised O6-planes. The other behaviours are possible with specific tunings of α when $F_0 \neq 0$. One can tune α so that in the boundary interval $\alpha = -q_2(y)\alpha''$, for $q_n = q_n(y)$ an order n polynomial; then, as long as q_2 has non-degenerate zeros, the zero of α'' is regular. Likewise one can simultaneously impose $\alpha'' = 0$ and $(\alpha')^2 - 2\alpha\alpha'' = q_3\alpha''$, then the behaviour at the zero of $\alpha'' = 0$ is that of a localised O8, which may be coincident to additional D8s.

	x^0	x^1	x^2	x^3	x^4	x^5	x^6	x^7	x^8	x^9
D6	×	×	×	–	–	–	×	×	×	×
D8	×	×	–	×	×	×	×	×	×	×
NS5	×	×	–	–	–	–	×	×	×	×

Table 6.2: $\frac{1}{4}$-BPS brane intersection underlying the 6d $(1,0)$ CFTs living in D6-NS5-D8 brane intersections. The directions $(x^0, x^1, x^6, x^7, x^8, x^9)$ are the directions where the 6d CFT lives. x^2 is the field theory direction, along which the D6-branes are stretched. (x^3, x^4, x^5) are the directions realising the SO(3) R-symmetry.

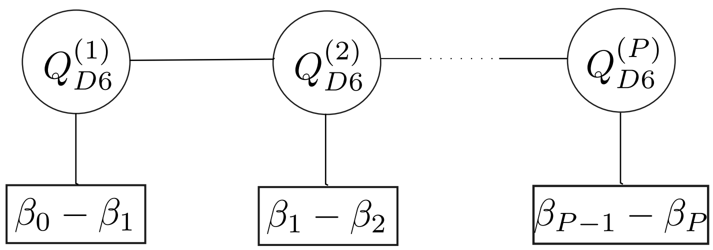


Figure 6.1: Quiver describing the field theory living in D6-NS5-D8 intersections. The circles denote $\mathcal{N} = (1,0)$ vector multiplets and the lines $\mathcal{N} = (1,0)$ bifundamental matter fields. The quiver has been terminated with $(\beta_{P-1} - \beta_P)$ D8-branes at the end of the space, with $\beta_P = \frac{\gamma_P}{81\pi^2}$ and $\gamma_P = -81\pi^2 \sum_{l=1}^{P-1} \beta_l$.

The D6-NS5-D8 brane set-up associated to the solutions is the one depicted in Table 6.2. Here the D6-branes play the role of colour branes while the D8-branes play the role of flavour branes [132, 133]. In 6d language the quantised charges give rise to the quiver depicted in Figure 6.1, as discussed in [129, 131]. One can check that 6d anomaly cancellation is fulfilled given that at each gauge node of the quiver

$$2N_k = 2Q_{D6}^{(k)} = N_f^k = Q_{D6}^{(k-1)} + Q_{D6}^{(k+1)} + \Delta Q_{D8}^{(k)}, \tag{6.31}$$

with $\Delta Q_{D8}^{(k)} = \beta_{k-1} - \beta_k$.

6.2.2. The surface defect ansatz

In this subsection we search for a solution within the class constructed in section 6.1 that is asymptotically AdS_7. The first step is to decide on the form of the external 7d and internal 3d spaces. For this purpose, we assumed the following form for the ten-dimensional metric,

$$\begin{aligned} \frac{1}{\sqrt{2}\pi} ds^2 =& L^2 \sqrt{-\frac{\alpha}{\alpha''}} ds^2(\text{M}_{1,6}) + \Delta_1 dy^2 + \Delta_2 ds^2(\text{S}^2)\,, \\ ds^2(\text{M}_{1,6}) =& P^2 \left[ds^2(\text{AdS}_3) + \frac{1}{m^2} ds^2(\text{S}^3) \right] + Q^2 dx^2, \end{aligned} \tag{6.32}$$

where P, Q depend on x alone, while $\Delta_{1,2}$ are functions of both x and y. In this section, we fix without loss of generality the constant q that appears in (6.18) to 1. Imposing SO(3) symmetry in (6.2), we were able to substitute $(z_1, z_2, z_3) \to (r, \mathrm{S}^2)$. The next step consists on performing a change of coordinates $(r, \rho) \to (x, y)$ such that (6.32) emerges. It turned out to be the one below,

$$r = q_1(x)\alpha\,, \quad \rho = -q_2(x)\alpha'\,. \tag{6.33}$$

This variable change was performed in (6.2) and the result was compared to (6.32), leading us to certain constrains. In order for them to be met, we had to fix

$$h = \frac{1}{2P^4L^4\pi^2}\left(-\frac{\alpha''}{\alpha}\right)\,, \quad g = \frac{4L^8\pi^4P^6q_2^2Q^2}{(\dot{q}_1)^2(q_1^2(\alpha')^2 - 2L^4\pi^2P^4q_2^2\alpha\alpha'')} \tag{6.34}$$

and take into account the condition

$$q_1\dot{q}_1 = 2L^4\pi^2P^4q_2\dot{q}_2\,. \tag{6.35}$$

With respect to the Bianchi identities, the condition $F_0 =$ constant, jointly with equations (6.23) and (6.35), gives rise to the following conditions

$$4q_1\dot{P} = P\dot{q}_1\,, \quad (\dot{q}_1)^2 = \frac{2\pi L^8}{3^4}P^6Q^2q_2\,, \tag{6.36}$$

and implies the remaining Bianchi identities. Up to diffeomorphisms, one can solve (6.35) and (6.36) without loss of generality with

$$P = 2^{3/2}x\,, \quad Q = -\frac{2^{3/2}}{(c+x^4)^{\frac{1}{4}}}\,, \quad q_1 = \frac{64L^6}{3^4}x^4\,, \quad q_2 = \frac{8L^4}{3^4\pi}\sqrt{c+x^4}\,, \quad dc = 0\,. \tag{6.37}$$

With all the previous considerations in mind, we were able to write our class of solutions (6.2) as displayed below,

$$\begin{aligned}
\frac{ds^2}{8\sqrt{2}\pi L^2} =& \Bigg[\sqrt{-\frac{\alpha}{\alpha''}}\left(x^2\big(ds^2(\mathrm{AdS}_3) + ds^2(\mathrm{S}^3)\big) + \frac{dx^2}{\sqrt{c+x^4}}\right) + \\
&+ \frac{\sqrt{c+x^4}}{x^2}\sqrt{\frac{-\alpha''}{\alpha}}\left(dy^2 + \frac{\alpha^2x^4}{\Delta}ds^2(\mathrm{S}^2)\right)\Bigg]\,,\\
e^{-\Phi} =& \frac{L\sqrt{\Delta}}{3^4 2^{\frac{5}{4}}\pi^{\frac{5}{2}}x(c+x^4)^{\frac{1}{4}}}\left(-\frac{\alpha''}{\alpha}\right)^{\frac{3}{4}}\,, \quad B_2 = -L^2\pi\left(-y + \frac{x^4\alpha\alpha'}{\Delta}\right)\mathrm{vol}(\mathrm{S}^2)\,,\\
F_0 =& -\frac{1}{162\pi^3}\alpha'''\,, \quad F_2 = F_0B_2 - \frac{L^2}{162\pi^2}(162F_0\pi^3y + \alpha'')\mathrm{vol}(\mathrm{S}^2),\\
F_4 =& -\frac{2^4L^4}{3^4\pi}d(\sqrt{c+x^4}\alpha') \wedge \Big(\mathrm{vol}(\mathrm{AdS}_3) + \mathrm{vol}(\mathrm{S}^3)\Big),\\
F_6 =& -\frac{2^4L^6}{3^4}d(\sqrt{c+x^4}(\alpha - y\alpha')) \wedge \Big(\mathrm{vol}(\mathrm{AdS}_3) + \mathrm{vol}(\mathrm{S}^3)\Big) \wedge \mathrm{vol}(\mathrm{S}^2)\\
&+ F_4 \wedge B_2\,,
\end{aligned} \tag{6.38}$$

where we have defined

$$\Delta = x^4\left((\alpha')^2 - 2\alpha\alpha''\right) - 2c\alpha\alpha''\,. \tag{6.39}$$

In the $x \to \infty$ limit, we have that $x^{-4}\Delta \to 1$ and the NSNS sector of the purely AdS_7 solutions described in (6.22) is recovered, where the AdS_7 radius is set to 1. As for the RR sector, F_0 and F_2 also behave as in the purely AdS_7 solutions, but the 4-form flux does not vanish, indicating the presence of a D2-D4 defect. It can be checked that the directions $(\mathrm{AdS}_3, \mathrm{S}^3, x)$ tend to AdS_7 by computing the Riemann curvature tensor.

The solution is bounded from below, but its behaviour at the lower bound depends on c. In the $c \geq 0$ case, x is bounded to the interval $[0, \infty)$. For $c = 0$ there is a curvature singularity at the lower bound $x = 0$ that we do not recognise as physical. On the other hand, when $c > 0$ the metric around the bound behaves as

$$\begin{aligned}\frac{ds^2}{8\sqrt{2}\pi L^2} =& \sqrt{-\frac{\alpha}{\alpha''}}\left[\sqrt{z}\Big(ds^2(\mathrm{AdS}_3) + ds^2(\mathrm{S}^3)\Big) + \frac{1}{16\sqrt{c}z^{\frac{3}{2}}}(dz^2 + z^2 ds^2(\mathrm{S}^2))\right] + \\ &+ \frac{\sqrt{c}}{8\sqrt{z}}\sqrt{-\frac{\alpha''}{\alpha}}dy^2\,,\end{aligned} \tag{6.40}$$

where $x = z^{\frac{1}{4}}$. If we had $-\alpha/\alpha'' = 1$, this would be the behaviour of a stack of D6 branes localised within $(\mathrm{AdS}_3, \mathrm{S}^3, y)$, with NS5-branes wrapped along (AdS_3, S^3) and smeared in the y direction. However, $-\alpha/\alpha'' \neq 1$ in general and, therefore, we have a generalisation of this instead. In our case, the NS5-branes are not smeared along y, making said direction an isometry, but they form a y-dependent distribution. Finally, for $c < 0$ we can fix $c = -b^4$ and the metric is bounded from below at $x = b$, where one sees the behaviour of ONS5 fixed planes[3] that are smeared along y. The most interesting behaviour is that of $c > 0$ and, therefore, we assumed it for the rest of the paper[4].

Before constructing the 2d quivers dual to the solutions defined by (6.38), we present the value of the holographic central charge computed using (3.20) for later comparison with the field theory result,

$$c_{\mathrm{hol}} = \frac{2^6}{3^7\pi^4}\int dx dy\, x^3\,(-\alpha\alpha''). \tag{6.41}$$

6.2.3. Surface defect CFTs

In this subsection the 2d quivers that flow in the IR to the CFTs dual to the solutions defined by (6.38) are presented. It is also shown how in a certain limit these quivers can be embedded in the 6d quivers living on the D6-NS5-D8 intersection.

We start by analysing the brane charges associated to the D2-D4-D6-NS5-D8 brane set-up underlying the solutions. One can see from the expressions for F_0 and F_2 in (6.38) that the D8 and D6 quantised charges of the AdS_3 solutions coincide with those of the AdS_7 backgrounds, given by equations (6.26) and (6.29). In turn, for finite x there are

[3] The S-dual of O5-planes.

[4] See our discussion on smeared ONS5s below around equation (6.64).

NS5-branes located at fixed values in y and also in x. Since we are interested in embedding the 2d CFT in the 6d CFT associated to the D6-NS5-D8 subsystem, we will take x large enough such that we can neglect the $(H_3)_{x\mathrm{S}^2}$ component of the NSNS 3-form flux and take the NS5-branes located at fixed positions in y, as in the D6-NS5-D8 subsystem. The fluxes associated to the AdS$_3$ solutions are then compatible with the brane intersection depicted in Table 6.1, that we repeat in Table 6.3 below in a generic system of coordinates for a better reading. Note that the R-symmetry of the 2d field theory living in the brane set-up

	x^0	x^1	x^2	x^3	x^4	x^5	x^6	x^7	x^8	x^9
D2	×	×	×	–	–	–	–	–	–	–
D4	×	×	–	×	×	×	–	–	–	–
D6	×	×	×	–	–	–	×	×	×	×
D8	×	×	–	×	×	×	×	×	×	×
NS5	×	×	–	–	–	–	×	×	×	×

Table 6.3: $\frac{1}{8}$-BPS brane intersection underlying the AdS$_3$ solutions (6.38). (x^0, x^1) are the directions where the 2d dual CFT lives. x^2 is the field theory direction, which we identify with y, where the NS5-branes are located (for x sufficiently large). The D2- and D6-branes are stretched in this direction. (x^3, x^4, x^5) are the directions associated to the isometries of the S^2 while (x^6, x^7, x^8, x^9) are those associated to the S^3.

is the SO(3)$_R$ ⊂SO(4) symmetry group of the S^3, while for the 6d field theory living in the D6-D8-NS5 brane intersection it is identified with the SO(3) symmetry group of the S^2. This is exactly what happens for 2d $\mathcal{N} = (4, 4)$ field theories arising upon compactification from 6d $(1, 0)$ CFTs, where the SO(3) R-symmetry of the 6d theory becomes the R-symmetry of the Coulomb branch of the 2d theory, and the SO(3)$_L$×SO(3)$_R$ R-symmetry of the Higgs branch of the 2d theory arises in the dimensional reduction [134–136]. In our $\mathcal{N} = (0, 4)$ theories there is just a Higgs branch, since the Coulomb branch contains no scalars, and the R-symmetry is just the SO(3)$_R$ arising in the dimensional reduction.

Let us now consider the Hanany-Witten brane set-up depicted in Figure 6.2. The D2-branes in it play the role of colour branes. They are extended along the y-direction, which is split into intervals of length 1 in our units, where the NS5-branes are located. The D6-branes are stretched along the x and y directions, the former one coordinate being non-compact and rendering these branes flavour ones. The D4- and D8-branes lie as well along the x direction and, therefore, they also behave as be flavour branes. The quiver that lives in this set-up is determined by the quantisation of the open strings stretched between the different branes[5]. In our case and due to the presence of non-compact dimensions, there are only four types of massless modes:

- D2-D2 strings: First we have to specify whether the two end-points of the string lie on the same stack of D2-branes or on two different stacks, separated by an NS5-brane. In the former case, D2-branes stretched between NS5-branes give rise to

[5] This quiver gauge theory has been studied before in detail. The reader may consult [49], for instance.

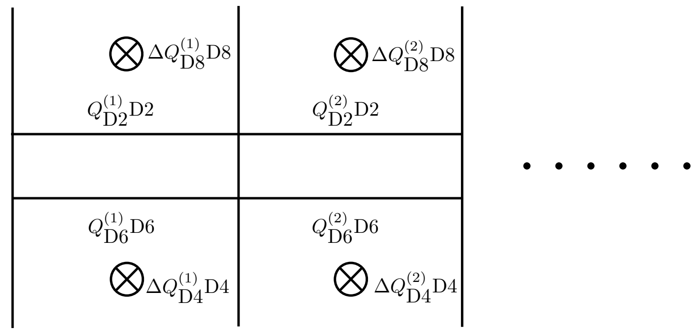


Figure 6.2: Hanany-Witten brane set-up associated to the AdS_3 solutions (6.38).

an $\mathcal{N} = (0,4)$ vector multiplet and an $\mathcal{N} = (0,4)$ adjoint twisted hypermultiplet, coming from the motion of the D2-branes along the (x^6, x^7, x^8, x^9) directions. The scalars within the hypermultiplet are charged under the R-symmetry and, therefore, they combine into a twisted one. The $\mathcal{N} = (0,4)$ vector multiplet and the $\mathcal{N} = (0,4)$ adjoint twisted hypermultiplet then combine into an $\mathcal{N} = (4, 4)$ vector multiplet.

Now we discuss the case in which the end-points of the string lie on adjacent stacks of D2-branes, separated by an NS5-brane. The massless modes arise from the intersection of the two stacks of D2-branes and the NS5-brane. This fixes the degrees of freedom moving along the (x^6, x^7, x^8, x^9) directions, leaving only the perturbations in the (x^3, x^4, x^5) directions and the A_2 component of the gauge field. These fields combine into an $\mathcal{N} = (4, 4)$ untwisted hypermultiplet in the bifundamental representation, since the scalars are not charged under the R-symmetry of the solution.

- D2-D4 strings: Strings with one end on D2-branes and the other end on D4-branes in the same interval contribute with fundamental $\mathcal{N} = (4, 4)$ hypermultiplets. They come from the degrees of freedom associated to the motion of the strings along the (x^3, x^4, x^5) directions and the A_2 component of the gauge field.

- D2-D6 strings: Strings with one end on D2-branes and the other end on D6-branes in the same interval between NS5-branes contribute with fundamental $\mathcal{N} = (0,4)$ twisted hypermultiplets, codifying the motion of the string along the (x^6, x^7, x^8, x^9) directions. These coordinates are charged under the R-symmetry of the solution, thus explaining that the hypermultiplets are twisted. Strings with one end on D2-branes and the other end on D6-branes in adjacent intervals contribute with $\mathcal{N} = (0,2)$ Fermi multiplets in the fundamental representation.

- D2-D8 strings: Strings with one end on D2-branes and the other end on orthogonal D8-branes in the same interval contribute with fundamental $\mathcal{N} = (0,2)$ Fermi multiplets.

These multiplets are gathered in Table 6.4 for easier consult.

String	Interval	Multiplet	Representation
D2-D2	Same	$\mathcal{N}=(4,4)$ vector	Adjoint
D2-D2	Adjacent	$\mathcal{N}=(4,4)$ hyper	Bifundamental
D2-D4	Same	$\mathcal{N}=(4,4)$ hyper	Bifundamental
D2-D6	Same	$\mathcal{N}=(0,4)$ twisted hyper	Bifundamental
D2-D6	Adjacent	$\mathcal{N}=(0,2)$ Fermi[6]	Bifundamental
D2-D8	Same	$\mathcal{N}=(0,2)$ Fermi	Bifundamental

Table 6.4: Summary of the multiplets associated to the brane intersection in Table 6.3. The column interval indicates whether the branes on which the string ends are on the same or adjacent intervals $[k,k+1]$ interval in $y\equiv x^2$.

The previous fields give rise to a field theory that can be described via the quivers represented in Figure 6.3. In these quivers the D6 and D8 brane charges are the ones

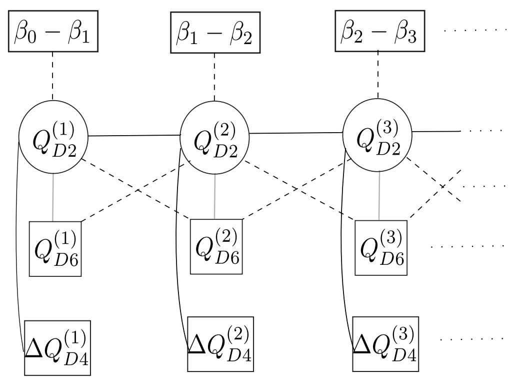


Figure 6.3: 2d quivers associated to the AdS$_3$ solutions (6.38). Circles denote $\mathcal{N}=(4,4)$ vector multiplets, black lines $\mathcal{N}=(4,4)$ bifundamental hypermultiplets, grey lines $\mathcal{N}=(0,4)$ bifundamental twisted hypermultiplets and dashed lines $\mathcal{N}=(0,2)$ bifundamental Fermi multiplets.

given by equations (6.29) and (6.26), while the D2 and D4 brane charges at each interval are given by

$$Q^{(k)}_{D2}=\frac{1}{(2\pi)^5}\int_{I_x,\mathrm{S}^2,\mathrm{S}^3}\hat{F}_6=\frac{4}{3^4\pi^2}\int_{I_x}dx\,\frac{2x^3}{\sqrt{c+x^4}}\alpha_k \tag{6.42}$$

and

$$\Delta Q^{(k)}_{D4}=\frac{1}{(2\pi)^3}\int_{I_y,\mathrm{S}^3}\hat{F}_4=\frac{4}{3^4\pi^2}\sqrt{c+x^4}\int_k^{k+1}dy\,\alpha_k''\,. \tag{6.43}$$

As the x-direction is semi-infinite the D2-brane charges diverge, as one would have foreseen due to their interpretation as defect branes. As for the anomaly cancellation for the gauge groups associated to them, it is given by the following condition at each of these nodes,

$$2Q^{(k)}_{D6}=Q^{(k-1)}_{D6}+Q^{(k+1)}_{D6}+\Delta Q^{(k)}_{D8}\,. \tag{6.44}$$

[6]Only when vol($\mathbb{T}^3$) is of stringy size.

In order to obtain this condition, we took into account that $\mathcal{N} = (0,4)$ fundamental multiplets contribute 1 to the gauge anomaly, $\mathcal{N} = (0,2)$ fundamental Fermi multiplets contribute $-1/2$ and the remaining vector and matter fields do not contribute since they are $\mathcal{N} = (4,4)$[7].

Let us now explore the computation of the central charge. For a 2d $\mathcal{N} = (0,4)$ CFT it can be computed away from criticality, since it equals the anomaly in the two-point function of the R-symmetry current, as given by (5.4), which is a t' Hooft anomaly. In order to compute n_{hyp} and n_{vec}, we first need to choose the precise way in which we would like to close the y interval. The conditions we picked consist on setting $\alpha = \alpha' = \alpha''$ to zero at both ends of the interval, and to glue the quiver to a symmetric version of itself at a given value $y = P+1$, in a continuous way. The resulting quivers are the ones depicted in Figure 6.4, where the notation is that of Figure 6.3. We remark that this is just one of

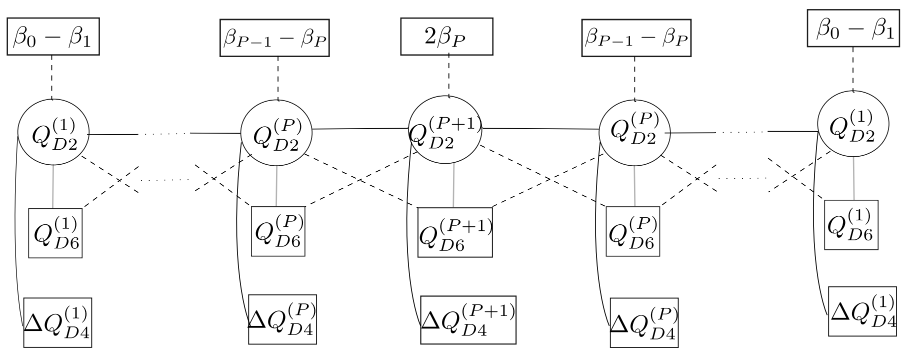


Figure 6.4: 2d quivers completed in a symmetric way.

many possible ways in which the y-direction can be globally defined. For this quiver, we have the numbers below,

$$n_{hyp} = 2\sum_{k=1}^{P} Q_{D2}^{(k)} Q_{D4}^{(k)} + Q_{D2}^{(P+1)} Q_{D4}^{(P+1)} + 2\sum_{k=1}^{P} Q_{D2}^{(k)} Q_{D2}^{(k+1)} \tag{6.45}$$

and

$$n_{vec} = 2\sum_{k=1}^{P} (Q_{D2}^{(k)})^2 + (Q_{D2}^{(P+1)})^2 \,, \tag{6.46}$$

which lead to

$$c_R = 6\Big[\Big(2\sum_{k=1}^{P} Q_{D2}^{(k)} Q_{D4}^{(k)} + Q_{D2}^{(P+1)} Q_{D4}^{(P+1)}\Big) + \Big(2\sum_{k=1}^{P} Q_{D2}^{(k)} (Q_{D2}^{(k+1)} - Q_{D2}^{(k)}) - (Q_{D2}^{(P+1)})^2\Big)\Big] \,. \tag{6.47}$$

A prescription is needed in order to regularise the infinite D2-brane charge and render it finite. The one we chose consists on evaluating all charges at a given value of x and summing over all of them. Doing this, one can see that the contribution of the second

[7]The reader is referred to [49, 94] for more details.

big bracket in (6.47) to c_R is subleading in x compared to that of the first big bracket. The expression we arrived at diverges with the same power of x as the holographic central charge computed in (6.41), and agrees with it to leading order in P, i.e. for long quivers. More concretely, the leading order in P of (6.47) turns out to be

$$c_R = \frac{2^7}{3^7\pi^4}\int_{I_x} dx\, x^3 \sum_{k=1}^{P} \mu_k\gamma_k\,. \tag{6.48}$$

Before showing the matching with the holographic central charge, we recall that it is given by [137]

$$c_{\rm hol} = \frac{c_L + c_R}{2}\,. \tag{6.49}$$

Thus, one needs to compute c_L before comparing the central charges. The following formula is useful for this purpose,

$$c_L - c_R = {\rm Tr}\gamma^3\,, \tag{6.50}$$

which leads to [49]

$$c_L - c_R = 2n_H^{(0,4)} - n_F^{(0,2)}\,, \tag{6.51}$$

where $n_H^{(0,4)}$ refers to the number of isolated[8] $\mathcal{N} = (0,4)$ hypermultiplets and $n_F^{(0,2)}$ to the number of isolated $\mathcal{N} = (0,2)$ Fermi multiplets. It can be proven that for our quivers the expression (6.51) identically vanishes because of the anomaly cancellation condition (6.44) and, therefore, $c_{\rm hol} = c_R$. Consequently, the leading order in P is given by the following formula,

$$c_{\rm hol} = \frac{2^6}{3^7\pi^4}\int dx\, x^3\Big[2\sum_{k=0}^{P}\int_k^{k+1} dy(-\alpha\alpha'')\Big] = \frac{2^7}{3^7\pi^4}\int_{I_x} dx\, x^3 \sum_{k=1}^{P}\mu_k\gamma_k + \dots\,, \tag{6.52}$$

which exactly matches (6.48) to leading order.

As expected, these quantities diverge in x due to its non-compact character. This shows that the 2d quiver QFTs associated to the (6.38) solutions are not well-defined by themselves. Only in the UV, they have a physical meaning when the extra dimensions where the 6d CFTs live emerge. Nevertheless, the previous analysis shows that, for x large enough, the D2 and D4 defect branes can be non-anomalously embedded within the 6d quiver theories living in the D6-NS5-D8 *mother* brane intersection, giving rise to 2d quiver theories.

6.3. $\mathcal{N} = (4, 4)$ AdS$_3$ from D2-D4-NS5 branes

In this section we explore the subclass of the solutions (6.2) where the coordinates (z_1, z_2, z_3) describe a 3-torus $\mathbb{T}^3$ that the warp factors are independent of. We will see

[8] By isolated, we mean that they do not combine into a larger multiplet.

that the brane intersection in Table 6.3 boils down to the $\mathcal{N} = (4,\, 4)$ D2-D4-NS5 Hanany-Witten brane set-ups studied in [138, 139]. As we explained at the beginning of chapter 4, the Dp-NS5-D$(p+2)$ brane intersections describe Hanany-Witten brane set-ups which realise p-dimensional field theories with 8 supercharges that flow to CFTs in certain limits. Type II solutions displaying AdS$_{p+1}$ geometries with 16 supercharges dual to these CFTs have been constructed in the literature for $p = 6, 5, 4, 3$ (see [110, 126, 128, 129, 140–147]) and partially for $p = 1$ (see [40]). As for the $p = 2$ case, it was not fully understood before[9]. This section is thus dedicated to solving this problem by presenting explicit AdS$_3 \times S^3$ duals to the quivers living in the $\mathcal{N} = (4,\, 4)$ D2-D4-NS5 brane set-ups, albeit with additional O-planes.

In subsection 6.3.1 we study the supergravity solution in (6.2) for the particular case where (z_1, z_2, z_3) parametrise a $\mathbb{T}^3$ and particularise for the massless case. In turn, the 2d quivers living in the brane set-up are built in subsection 6.3.2. The duality of these quivers and the supergravity solutions is supported by the agreement between the central charges computed from both sides of the duality. Subsection 6.3.3 is dedicated to the discussion of the the M-theory realisation of these solutions, which allows us to relate them to the AdS$_3 \times S^2 \times \mathbb{T}^4 \times I$ solutions of massless Type IIA supergravity constructed in [38]. We show that they share a common M-theory origin, implying that they flow to the same 2d dual CFT in the IR. This can be interpreted as a manifestation of mirror symmetry, as discussed in [138, 139]. Finally, in subsection 6.3.4 we apply T-duality in order to derive new $\mathcal{N} = (0, 4)$ solutions of Type IIB supergravity from the previous ones. One such class is holographically dual to D3-brane boxes constructions [149] with small $\mathcal{N} = (0, 4)$ supersymmetry.

6.3.1. AdS$_3 \times S^3 \times \mathbb{T}^3$ solutions with $\mathcal{N} = (4,\, 4)$ supersymmetries

Let us consider the condition that the coordinates (z_1, z_2, z_3) of the solutions given by (6.2) span a 3-torus $\mathbb{T}^3$ of which the warp factors are independent. This leads one to the following subclass of solutions,

$$\begin{aligned}
ds^2 =& \frac{q}{\sqrt{h}}\Big[ds^2(\mathrm{AdS}_3) + ds^2(\mathrm{S}^3)\Big] + \frac{g}{\sqrt{h}}\, d\rho^2 + g\sqrt{h}\, ds^2(\mathbb{T}^3)\,,\\
e^{-\Phi} =& \frac{h^{\frac{3}{4}}}{\sqrt{g}}\,, \quad H_3 = \partial_\rho(hg)\,\mathrm{vol}(\mathbb{T}^3)\,,\\
F_0 =& \frac{\partial_\rho h}{g}\,, \quad F_4 = 2q\,\mathrm{vol}(\mathrm{AdS}_3)\wedge d\rho + 2q\,\mathrm{vol}(\mathrm{S}^3)\wedge d\rho\,,\\
F_6 =& 2q\, gh\,\mathrm{vol}(\mathbb{T}^3)\wedge(\mathrm{vol}(\mathrm{S}^3) + \mathrm{vol}(\mathrm{AdS}_3))\,,
\end{aligned} \tag{6.53}$$

where g, h are functions of ρ and the Bianchi identities boil down to

$$\partial_\rho\left(\frac{\partial_\rho h}{g}\right) = 0\,, \quad \partial_\rho^2(gh) = 0\,, \quad F_0\partial_\rho(gh) = 0 \tag{6.54}$$

[9] In [148] a probe brane analysis revealed an AdS$_3\times$S^3 geometry as gravity dual of an $\mathcal{N} = (4,\, 4)$ CFT.

and imply the equations of motion. We remark that both g and h have being delocalised along the $\mathbb{T}^3$, thus simplifying the underlying brane intersection. As for the solution (6.53), we no longer have an RR 2-form flux and the NSNS 3-form flux is simpler than in (6.2). For the remainder of this section, we consider the massless limit $F_0 = 0$, while the non-vanishing Romans' mass case is expounded in section 6.4. Once more, we point out that $F_0 = 0$ implies

$$h = h_0 = \text{constant}, \tag{6.55}$$

and the second PDE in (6.54) yields the constraint

$$g'' = 0\,. \tag{6.56}$$

As in the considered subclass of solutions both F_0 and F_2 vanish, we are excluding the D8- and D6-branes from the set-up of Table 6.1, thus obtaining the D2-NS5-D4 brane intersection we are interested in. We recall that (6.55) implies that supersymmetry is enhanced to $\mathcal{N} = (4,\,4)$, as discussed at the end of subsection 6.1.1. We thus obtain a class of $\mathcal{N} = (4,\,4)$ AdS$_3 \times S^3 \times \mathbb{T}^3$ backgrounds fibred over an interval whose underlying brane intersection is the one depicted in Table 6.5. The quantised charges of the D2-

branes	x^0	x^1	z_1	z_2	z_3	ρ	ζ	θ^1	θ^2	θ^3
D2	×	×	–	–	–	×	–	–	–	–
D4	×	×	×	×	×	–	–	–	–	–
NS5	×	×	–	–	–	–	×	×	×	×

Table 6.5: $\frac{1}{4}$-BPS brane intersection underlying the solution (6.53) with $F_0 = 0$. (x^0, x^1) are the directions where the 2d dual CFT lives, (z_1, z_2, z_3) describe the $\mathbb{T}^3$ along which the D4-branes are wrapped, ζ is the radial coordinate of the AdS$_3$, the θ^i parametrise the S^3 and ρ is the field theory direction.

D4-NS5 branes are computed from the F_4, H_3 and F_6 magnetic fluxes. However, one encounters a problem with the Page fluxes: a B_2 cannot be globally defined, as it should be invariant under the isometries of the torus, i.e. it should be proportional to vol$(\mathbb{T}^3)$, which is impossible. We then realised that the flux we had to use in order to compute the quantised D2-brane charges is given by

$$\hat{f}_6 = f_6 - C_3 \wedge H_3 = 2\,q\,h_0(g - \rho\,g')\,\text{vol}(\mathbb{T}^3) \wedge \text{vol}(S^3)\,, \tag{6.57}$$

where $\hat{f}_p$ stands for the magnetic component of F_p. We assume that $h_0 = 1$ without loss of generality, as it can be absorbed by rescaling ρ and the Anti-de Sitter radius. We observe that the $\hat{f}_6$, and consequently the Page charge associated to the D2-branes, is gauge dependent because it is sensitive to the choice of C_3. This is accounted for by picking as representative of C_3 the one satisfying the following condition in units with $\alpha' = g_s = 1$,

$$\frac{1}{(2\pi)^3}\int_{\text{S}^3} C_3 \in [0, 1]\,. \tag{6.58}$$

We observe that (6.58) is analogous to the more familiar requirement of the NSNS 2-form potential lying in the fundamental region. It can be implemented by the choice below,

$$C_3 = -2\,q\left(\rho - \frac{2\pi}{q}k\right)\,\mathrm{vol}(\mathrm{S}^3) \tag{6.59}$$

for $\rho \in [\frac{2\pi}{q}k, \frac{2\pi}{q}(k+1)]$. With these considerations the D4-brane charge can be computed,

$$Q_{D4}^{(k)} = \frac{1}{(2\pi)^3}\int_{I_\rho,\mathrm{S}^3} \hat{F}_4\,, \tag{6.60}$$

which gives $Q_{D4}^{(k)} = 1$ for $I = [\frac{2\pi}{q}\,k,\, \frac{2\pi}{q}\,(k+1)]$. This means that there is a single D4-brane within each interval. If we take the whole interval spanned by ρ to be $[0, \frac{2\pi}{q}(P+1)]$ for some integer P, then the brane set-up contains a total of $(P+1)$ D4-branes.

The next step is to solve (6.56). In order to achieve this, we need to take into consideration that g must be continuous, but g' may present discontinuities at the locations of the D4-branes, where $\rho = \frac{2\pi}{q}k$. The most general solution is obtained by solving the equation (6.56) at each interval $\rho \in [\frac{2\pi}{q}k, \frac{2\pi}{q}(k+1)]$,

$$g_k = \alpha_k + \frac{\beta_k}{2\pi}\Big(\rho - \frac{2\pi}{q}k\Big) \quad \text{for} \quad \rho \in \left[\frac{2\pi}{q}\,k,\, \frac{2\pi}{q}\,(k+1)\right]. \tag{6.61}$$

Imposing that the space begins and ends at $\rho = 0, \frac{2\pi}{q}(P+1)$, where g vanishes, we find

$$g(\rho) = \begin{cases} \frac{\beta_0}{2\pi}\rho\,, & 0 \le \rho \le \frac{2\pi}{q}\,, \\ \alpha_k + \frac{\beta_k}{2\pi}(\rho - \frac{2\pi}{q}k)\,, & \frac{2\pi}{q}k \le \rho \le \frac{2\pi}{q}(k+1)\,, \quad k = 1, ..., P-1 \\ \alpha_P + \frac{\beta_P}{2\pi}(\rho - \frac{2\pi}{q}P)\,, & \frac{2\pi}{q}P \le \rho \le \frac{2\pi}{q}(P+1)\,. \end{cases} \tag{6.62}$$

The condition $g\left(\frac{2\pi}{q}(P+1)\right) = 0$ imposes that

$$\beta_P = -q\,\alpha_P\,, \tag{6.63}$$

while continuity across the different intervals gives rise to the following set of constraints,

$$\alpha_k = \frac{1}{q}\sum_{j=0}^{k-1}\beta_j\,, \quad k = 1, \dots, P\,. \tag{6.64}$$

At the boundaries of the interval $\rho = 0, \frac{2\pi}{q}(P+1)$, the solution behaves as an ONS5 plane (the S-dual of an O5 plane) smeared over the $\mathbb{T}^3$. Said smearing is not really physically allowed in string theory, as the plane should lie at the fixed point of the orientifold involution. This is relevant because, although we are working in the supergravity regime, when we approach the ONS5 the curvature becomes large and that description must be supplemented with α' corrections. There is still the hope that these higher order effects conspire, producing a localised ONS5 in string theory [10]. If one insists in having fully localised O-planes in the supergravity solution, there is still hope. Indeed, the compatibility of a class of solutions with smeared O-planes often suggests that it is also compatible with localised ones. Such solutions are obviously harder to derive, but the one at hand (6.53) provides a good foundation for this purpose. We expect such generalisations to exhibit qualitatively similar physical behaviour, although the subtle effects caused by the boundaries of the space should be taken into account.

The quantised charges in each interval $\rho \in \left[\frac{2\pi}{q}k, \frac{2\pi}{q}(k+1)\right]$ are thus given by

$$\begin{aligned} Q_{D2}^{(k)} =& \frac{1}{(2\pi)^5}\int_{\mathbb{T}^3,\mathrm{S}^3} \hat{F}_6 = q\Big(g - g'(\rho - \frac{2\pi}{q}k)\Big) = q\,\alpha_k = \sum_{j=0}^{k-1}\beta_j\,,\\ Q_{NS5}^{(k)} =& \frac{1}{(2\pi)^2}\int_{\mathbb{T}^3} H_3 = \beta_k\,,\quad Q_{D4}^{(k)} = \frac{1}{(2\pi)^3}\int_{I_\rho,\mathrm{S}^3}\hat{F}_4 = 1\,. \end{aligned} \tag{6.65}$$

Finally, the central charge computed with the formula (3.20) gives, for this class of solutions, the below expression[11]

$$c_{\mathrm{hol}} = \frac{3}{\pi}\,q^2\int d\rho\, h\, g. \tag{6.66}$$

This result will be compared to the field theory one in sections 6.3.2 and 6.4.

6.3.2. 2d dual CFTs

In order to compute the quiver QFTs living in the D2-NS5-D4 intersection underlying the class of solutions (6.53), several considerations must be noticed. We need to take into account the ordering of the NS5-branes along the ρ direction, the total number of D2-branes ending on each of them and the D4-branes orthogonal to both types of branes. As always, the massless modes, the ones that appear in the quiver QFT, are associated to open strings stretching between the D-branes either in the same or adjacent intervals between NS5-branes. We arrived at the three massless modes below:

- D2-D2 strings: This kind of strings behave as the analogous ones in section 6.2. As before, there are two possible cases. Open strings with both end points lying on the same stack of D2-branes give rise to $\mathcal{N} = (4,\,4)$ vector multiplets, while those with end points on stacks in adjacent intervals give rise to $\mathcal{N} = (4,\,4)$ hypermultiplets in the bifundamental representation.

- D4-D4 strings: Whether these strings give rise to massless modes or not depends on the size of the $\mathbb{T}^3$, as the D4-branes are wrapped around it. We remark that there is a single D4-brane at each $\rho = \frac{2\pi}{q}k$ and that D4-D4 strings are T-dual to

[10] Indeed, this argument was pursued in [150], where it is suggested that smeared O-planes are a good approximation to localised ones under certain circumstances.

[11] Note that this result is also valid when $F_0 \neq 0$.

D2-D2 strings. Thus, the considered strings would contribute an $\mathcal{N} = (4,\, 4)$ vector multiplet in the scenario where the size of the $\mathbb{T}^3$ is of the order of magnitude of the string length.

- D2-D4 strings: Strings with an end on D2-branes and another on orthogonal D4-branes in the same interval yield fundamental $\mathcal{N} = (4,\, 4)$ hypermultiplets, associated to the oscillations of the strings along the (z_1, z_2, z_3) directions plus the A_5 component of the gauge field.

In order to build the quivers associated to these massless modes we computed the linking numbers of the D4-branes and the NS5-branes. Said numbers were defined thanks to the fact that our brane intersection, described in Table 6.5, is T-dual to the Type IIB one explored in [108]. The definitions we arrived at are the following,

$$\begin{aligned} l_i &= n_i + L_i^{NS5}, \quad \text{for the D4-branes,} \\ \hat{l}_j &= -\hat{n}_j + R_j^{D4}, \quad \text{for the NS5-branes,} \end{aligned} \tag{6.67}$$

where n_i denotes the number of D2-branes ending on the ith D4-brane from the right minus the number of D2-branes ending on it from the left, $\hat{n}_j$ is the analogous for the jth NS5-brane, L_i^{NS5} is the number of NS5-branes lying on the left of the ith D4-brane, and R_j^{D4} is the number of D4-branes lying on the right of the jth NS5-brane[12]. An important property of these linking numbers is that they are invariant under the interchange via crossing of adjacent D4- and NS5-branes (Hanany-Witten moves) because of the Hanany-Witten brane creation effect[13].

Similarly to what we did in chapter 4, we followed [111] in order to infer the QFT living in the brane set-up from the linking numbers, namely, the gauge group $G =\mathrm{U}(N_1)\times\cdots\times\mathrm{U}(N_k)$; the bifundamental fields transform in the $(N_i, \bar{N}_{i+1})$ representations, and the fundamental matter, under $\mathrm{U}(M_i)$ for each group. The way to proceed is as follows. The linking numbers define an integer N,

$$N = \sum_{i=1}^{p} l_i = \sum_{j=1}^{\hat{p}} \hat{l}_j\,, \tag{6.68}$$

where p and $\hat{p}$ are the numbers of D4-branes and NS5-branes, respectively. In order to understand the meaning of N, we first notice that any brane set-up of the kind we are studying can be related via suitable Hanany-Witten moves to another one where the D4-branes are located on the "left" and the NS5-branes, on the "right". In this new brane configuration, N represents the number of D2-branes that end on the left on the collection of D4-branes and on the right on that of NS5-branes. In order to derive the quiver, we consider the partition $N = \sum_{j=1}^{\hat{p}} \hat{l}_j$, where the order of the NS5-branes have to satisfy $\hat{l}_1 \geq \hat{l}_2 \geq \cdots \geq \hat{l}_{\hat{p}}$, and a second partition defined from a list of positive integer numbers

[12]Our conventions are related through T and S dualities to those assumed in [111].

[13]We observe that the D2-D4-NS5 intersection presented in Table 6.5 is related to a D3-D5-NS5 brane set-up through a T-duality along one of the θ^i coordinates. The original brane set-up explored by Hanany and Witten (see Table 4.1) is the flat space version of this D3-D5-NS5 brane set-up.

satisfying $q_1 \geq q_2 \geq \cdots \geq q_r$ with $N = \sum_{s=1}^{r} M_s q_s$, where M_s are integers counting how many times each q_s appears in the partition. Let m_j be the number of terms in the decomposition equal or bigger than a given integer j, i.e.

$$m_j = \sum_{s=1}^{k} M_s \quad \text{for} \quad q_k \leq j < q_{k+1} \,. \tag{6.69}$$

Then the q_s are chosen to meet

$$\sum_{j=1}^{i} m_j \geq \sum_{j=1}^{i} \hat{l}_j \quad \forall\, i = 1, \ldots, \hat{p} \,. \tag{6.70}$$

The ranks of the different U(N_i) gauge groups of the quiver are given by

$$N_i = \sum_{j=1}^{i} (m_j - \hat{l}_j) \,, \tag{6.71}$$

which are non-negative integers because of condition(6.70). On the other hand, the numbers M_s convey the ranks of the fundamental matter groups that couple to each gauge group [14].

Let us particularise this analysis to our solutions, parametrised by the function $g(\rho)$ in (6.62). The brane set-up we are considering can be built by considering the number of branes at each $\rho \in [\frac{2\pi}{q}k, \frac{2\pi}{q}(k+1)]$ interval, given by equations (6.65). Furthermore, as discussed below equation (6.62), β_P anti-NS5-branes[15] must end the space at $\rho = \frac{2\pi}{q}(P{+}1)$. The resulting brane set-up is then the one depicted in Figure 6.5. From it the linking

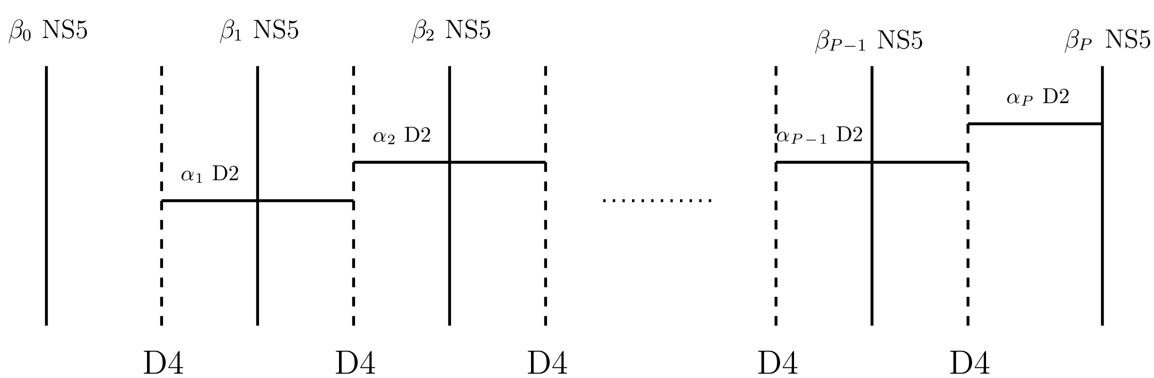


Figure 6.5: Brane set-up associated to the quantised charges (6.65), in units of q.

numbers for the D4-branes turn out to be the following ones,

$$l_i = \sum_{r=0}^{i-2} \beta_r + 2\beta_{i-1} \,, \quad i = 1, \ldots, P \tag{6.72}$$

[14]For a detailed derivation of this result, reference [110] can be consulted.
[15]The fact that we have anti-NS5-branes and not regular ones comes from the negative sign in (6.63).

and for the NS5-branes we have the ones below,

$$
\begin{aligned}
&\hat{l}_1 = \hat{l}_2 = \cdots = \hat{l}_{\beta_0} = P\,, \\
&\hat{l}_{\beta_0+1} = \hat{l}_{\beta_0+2} = \cdots = \hat{l}_{\beta_0+\beta_1} = P-1\,, \\
&\qquad\qquad\qquad \vdots \\
&\hat{l}_{\beta_0+\beta_1+\cdots+\beta_{P-3}+1} = \hat{l}_{\beta_0+\beta_1+\cdots+\beta_{P-3}+2} = \cdots = \hat{l}_{\beta_0+\beta_1+\cdots+\beta_{P-2}} = 2\,, \\
&\hat{l}_{\beta_0+\beta_1+\cdots+\beta_{P-2}+1} = \cdots = \hat{l}_{\beta_0+\beta_1+\cdots+\beta_{P-1}} = 1\,, \\
&\hat{l}_{\beta_0+\beta_1+\cdots+\beta_{P-1}+1} = \cdots = \hat{l}_{\beta_0+\beta_1+\cdots+\beta_{P-1}+\beta_P} = 1\,.
\end{aligned}
\tag{6.73}
$$

From these linking numbers we can compute

$$
N = \sum_{i=1}^{P} l_i = \sum_{j=1}^{2\beta_P} \hat{l}_j = \beta_P\,. \tag{6.74}
$$

This value of N can be used to obtain the quiver field theory. The linking numbers of the NS5-branes in our brane set-up are ordered as $\hat{l}_1 \geq \hat{l}_2 \geq \cdots \geq \hat{l}_{\beta_0+\ldots+\beta_P}$, thus defining a partition $N = \sum_{j=1}^{2\beta_P} \hat{l}_j$. Besides, for the D4-branes we take

$$
\begin{aligned}
N = &\underbrace{\beta_0} + \underbrace{\beta_0+\beta_1} + \underbrace{\beta_0+\beta_1+\beta_2} + \cdots + \underbrace{\beta_0+\beta_1+\cdots+\beta_{P-2}} + \\
&+ 2\,\underbrace{(\beta_0+\beta_1+\cdots+\beta_{P-1})}
\end{aligned}
\tag{6.75}
$$

from where we compute the m_j numbers as in (6.69),

$$
\begin{aligned}
&m_1 = m_2 = \cdots = m_{\beta_0} = P+1\,, \\
&m_{\beta_0+1} = \cdots = m_{\beta_0+\beta_1} = P\,, \\
&\qquad\qquad\qquad \vdots \\
&m_{\beta_0+\beta_1+\cdots+\beta_{P-3}+1} = \cdots = m_{\beta_0+\beta_1+\cdots+\beta_{P-2}} = 3\,, \\
&m_{\beta_0+\beta_1+\cdots+\beta_{P-2}+1} = \cdots = m_{\beta_0+\beta_1+\cdots+\beta_{P-1}} = 2\,.
\end{aligned}
\tag{6.76}
$$

It is clear that these numbers satisfy the condition (6.70) $\forall\ i = 1,\ \ldots,\ (\beta_0 + \cdots + \beta_P)$. The ranks of the gauge groups turn out to be the following ones,

$$
\begin{aligned}
&N_1 = m_1 - \hat{l}_1 = P+1-P = 1\,, \quad N_2 = N_1 + m_2 - \hat{l}_2 = 2\,, \quad \ldots \quad N_{\beta_0} = \beta_0\,, \\
&N_{\beta_0+1} = \beta_0 + 1\,, \quad \ldots \quad N_{\beta_0+\beta_1+\cdots+\beta_{P-1}} = \beta_0 + \beta_1 + \cdots + \beta_{P-1}\,, \\
&N_{\beta_0+\beta_1+\ldots\beta_{P-1}+1} = \beta_0 + \beta_1 + \cdots + \beta_{P-1} - 1\,, \quad \ldots \quad N_{\beta_0+\beta_1+\ldots\beta_{P-1}+\beta_P-1} = 1\,.
\end{aligned}
\tag{6.77}
$$

We observe that these ranks increase in one unit at a time, parallelly to the subscript, till the value $\beta_0+\beta_1+\cdots+\beta_{P-1}$ is reached, to then start decreasing, again in units of one, till the gauge group of rank 1 is reached, corresponding to the D2-branes stretched between the last pair of NS5-branes. From the partition (6.75) we read

$$
M_{\beta_0} = M_{\beta_0+\beta_1} = \cdots = M_{\beta_0+\beta_1+\cdots+\beta_{P-2}} = 1\,, \quad M_{\beta_0+\beta_1+\cdots+\beta_{P-1}} = 2\,, \tag{6.78}
$$

which, as explained above, correspond to the number of times each l_j appears in the partition. We conclude from this that the gauge groups with ranks $\beta_0 = q\,\alpha_1$, $\beta_0 + \beta_1 = q\,\alpha_2\,, \dots\ \beta_0 + \cdots + \beta_{P-2} = q\,\alpha_{P-1}$ have U(1) flavour groups attached to them, while the gauge group with rank $\beta_0 + \beta_1 \cdots + \beta_{P-1} = q\,\alpha_P$ has flavour group U(2). The rest of gauge groups have no flavour symmetries. In Figure 6.6 the quiver is displayed. It can be

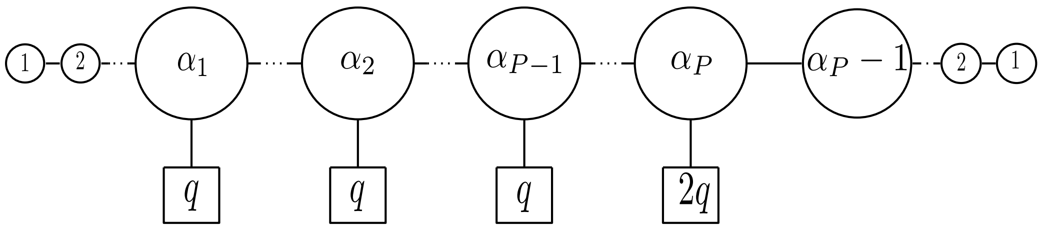


Figure 6.6: 2d quiver associated to the brane set-up in Figure 6.5. Circles denote $\mathcal{N} = (4,\,4)$ vector multiplets and black lines $\mathcal{N} = (4,\,4)$ bifundamental hypermultiplets. The gauge groups with ranks α_k, with $k = 1, \dots, P-1$ have U(1) flavour symmetries. The gauge group with rank α_P has U(2) flavour symmetry. The rest of gauge groups do not have attached any flavours.

checked that, as expected, the number of gauge nodes equals the total number of NS5-branes minus 1. In this quiver circles denote $\mathcal{N} = (4,\,4)$ vector multiplets and black lines $\mathcal{N} = (4,\,4)$ bifundamental hypermultiplets. It must be noticed that it has been rescaled so that the intervals have length $[0, 2\pi]$, as it is more standard in the literature. In this parametrisation we have that (6.64) reads

$$\alpha_k = \sum_{j=0}^{k-1} \beta_j \quad \text{for} \quad k = 1, \dots, P\,. \tag{6.79}$$

We propose that the QFTs defined by these quivers flow in the IR to the 2d CFTs dual to the subset of the class of solutions in (6.53) that satisfy h =constant and g given by (6.62). We provided a non-trivial check of this proposal, consisting on the matching between the field theory and holographic central charges. Nevertheless, before we can present it, we should recall that the Higgs and Coulomb branches of 2d $\mathcal{N} = (4,\,4)$ theories are given by different CFTs. The different branches have different R-symmetries and usually different central charges [134, 135]. Which of these branches is described holographically by our class of solutions? In [135] it is argued that the scalars should be singlets under the SO(4) R-symmetry of the 2d CFT. In our case this symmetry arises as the isometry group of the 3-sphere in the internal space. The scalars in the Higgs branch are singlets under this group and, therefore, the Higgs branch flows to a CFT with R-symmetry coming from this SO(4). On the other hand, the scalars in the Coulomb branch transform in the $(\mathbf{2},\,\mathbf{2})$ representation of SO(4). This implies that said branch must flow to a 2d CFT with R-symmetry coming from the SU(2) associated to the S^2 within the $\mathbb{T}^3$. This symmetry should be enhanced to SO(4) at strong coupling, as we will explore in more detail in the next subsection. This argument points towards our solutions being holographically dual to the Higgs branch 2d CFT. Accordingly, the holographic central charge must match the central charge of the Higgs branch.

As our theories display $\mathcal{N} = (4,\,4)$ supersymmetry, we can use the expression for the central charge of the left- or right-moving SU(2) of R-symmetries to compute the central charge of the Higgs branch via equation (5.4), $c = 6(n_{hyp} - n_{vec})$, where n_{hyp} stands for the number of $\mathcal{N} = (0,\,4)$ hypermultiplets and n_{vec} for the number of $\mathcal{N} = (0,\,4)$ vector multiplets. We observe that they can be substituted, respectively, by the number of $\mathcal{N} = (4,\,4)$ hypermultiplets and $\mathcal{N} = (4,\,4)$ vector multiplets. This is more useful for our purposes, as the $\mathcal{N} = (0,\,4)$ Fermi multiplets and $\mathcal{N} = (0,\,4)$ adjoint twisted hypermultiplets do not contribute to the R-symmetry anomaly. For our quivers of Figure 6.6 we have

$$n_{hyp} = 2 \sum_{k=1}^{\alpha_P - 1} k(k+1) + q \sum_{k=1}^{P-1} \alpha_k + 2q\,\alpha_P \quad \text{and} \quad n_{vec} = 2 \sum_{k=1}^{\alpha_P - 1} k^2 + \alpha_P^2\,, \tag{6.80}$$

yielding a central charge of

$$c = 6\,q \sum_{k=1}^{P} \alpha_k. \tag{6.81}$$

We want to compare this result with the holographic central charge in (6.66). Taking $h = 1$ and g as defined by (6.62) it turns out to be

$$c_{\text{hol}} = 6\,q \sum_{k=1}^{P} \alpha_k \tag{6.82}$$

after using (6.63) and (6.64), finding exact agreement with the field theory calculation.

An example of interest in our class of solutions is the scenario where the ρ interval is periodically identified, rendering g constant. In this case the quantised charges read

$$Q_{\text{D2}} = qg\,, \quad Q_{\text{D4}} = 1 \quad \text{for} \quad \rho \in \left[0, \frac{2\pi}{q}\right] \tag{6.83}$$

or alternatively

$$Q_{\text{D2}} = g\,, \quad Q_{\text{D4}} = q \quad \text{for} \quad \rho \in [0, 2\pi]\,. \tag{6.84}$$

This solution is T-dual to the D1-D5 system in the particular case where the CY_2 is a $\mathbb{T}^4$, and the T-duality takes place along one of the circles of the $\mathbb{T}^4$. The D5-branes become D4-branes smeared on the T-duality direction and the quiver collapses to the one describing the D1-D5 system, depicted in Figure 6.7 (for $\rho \in [0, 2\pi]$). Equation (5.4) gives the well-known result $c = 6\,Q_{\text{D2}}Q_{\text{D4}}$ for the central charge and agrees with the holographic result.

6.3.3. Realisation in M-theory

We now shift our gaze towards the M-theory regime of the brane intersection in Table 6.5, which we studied in the previous subsection. At strong coupling the D4-branes become

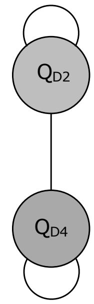


Figure 6.7: Quiver associated to the solution with $g =$ constant, corresponding to the T-dual of the D1-D5 system.

M5-branes, its extra worldvolume direction being the 11th one, while the NS5-branes give rise to M5′-branes transverse to it. Consequently, from this viewpoint the Hanany-Witten set-up consists on M2-branes stretched between M5′-branes with M5-branes orthogonal to them. As the M5- and M5′-branes are clearly equally non-perturbative, one could alternatively consider the configuration in which the M2-branes are stretched between the M5-branes with the M5′-branes orthogonal to them. In order to derive the field content associated to this alternative configuration in weakly coupled string theory we need to reduce to ten dimensions in a direction in which the M5-branes become NS5-branes. According to Table 6.5 this can be achieved by reducing along the Hopf-fibre direction within the S^3, which is transverse to the M5-branes. This halves the number of supersymmetries and creates a D6-brane. Furthermore, in the reduction the $\mathbb{T}^3$ combines with the S^1 that was the 11th direction (denoted by ψ) to produce a $\mathbb{T}^4$. The resulting brane set-up is summarised in Table 6.6, which is the one underlying the AdS$_3 \times S^2 \times \mathbb{T}^4 \times I$ solutions first presented in [38], restricted to the massless case. In the brane intersection

branes	x^0	x^1	z_1	z_2	z_3	ψ	ρ	ζ	θ^1	θ^2
D2	×	×	–	–	–	–	×	–	–	–
NS5	×	×	×	×	×	×	–	–	–	–
D4	×	×	–	–	–	–	–	×	×	×
D6	×	×	×	×	×	×	×	–	–	–

Table 6.6: $\frac{1}{8}$-BPS brane intersection associated to the solutions in [38]. (x^0, x^1) are the directions where the 2d dual CFT lives. (z_1, z_2, z_3, ψ) span the $\mathbb{T}^4$, on which the NS5- and D6-branes are wrapped. The coordinates $(\zeta, \theta^1, \theta)$ are the transverse directions realising the SO(3)-symmetry of the S^2.

underlying our solutions there are α_j D2-branes[16] and a D6-brane wrapped around the $\mathbb{T}^4$ stretched between NS5-branes, which play the role of colour branes. Nevertheless, the number of D6-branes should be large in order to have a consistent Type IIA supergravity background. This means that S^3 has to be modded by $\mathbb{Z}_k$ in the 11-dimensional solution,

[16] The α_j defined as in (6.79).

which gives rise to k D6-branes upon reduction. At each interval, we also have $(\beta_{j-1} - \beta_j)$ orthogonal D4-branes, which play the role of flavour branes. The holographic central charge can be particularised from the formula in [94], where this quantity was computed for the general class of solutions in [38]. It can be checked that for our configuration it coincides with the holographic central charge in (6.82), multiplied by k due to the $\mathbb{Z}_k$ orbifolding of the S^3. The field theory living in the brane intersection can also be alternatively obtained from the general study in [94][17]. The result is the gauge theory whose quiver can be found in Figure 6.8. In this figure circles represent $\mathcal{N} = (0, 4)$ vector multiplets, blue

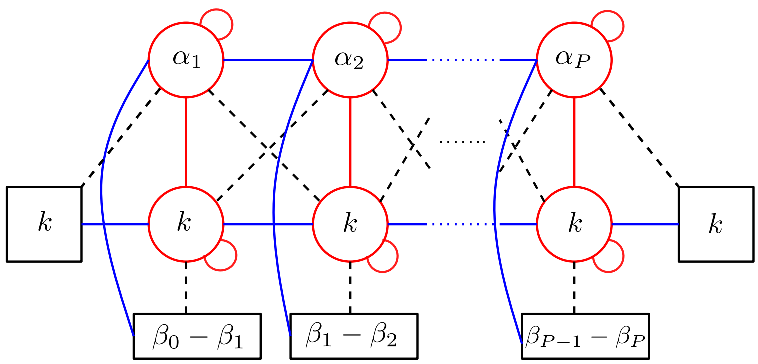


Figure 6.8: 2d quiver associated to the $\mathrm{AdS}_3 \times \mathrm{S}^2 \times \mathbb{T}^4 \times I$ solutions with α_k D2-branes and k D6-branes wrapped on the $\mathbb{T}^4$. Circles denote $\mathcal{N} = (0, 4)$ vector multiplets, blue lines $\mathcal{N} = (4, 4)$ twisted hypermultiplets, red lines $\mathcal{N} = (0, 4)$ hypermultiplets and dashed lines $\mathcal{N} = (0, 2)$ Fermi multiplets.

lines $\mathcal{N} = (4, 4)$ twisted hypermultiplets, red lines $\mathcal{N} = (0, 4)$ hypermultiplets and dashed lines $\mathcal{N} = (0, 2)$ Fermi multiplets. We remind that 2d $\mathcal{N} = (0, 4)$ theories do not have a Coulomb branch, as $\mathcal{N} = (0, 4)$ vector multiplets contain no scalars. Besides, for the Higgs branch one can use (5.4) to compute[18]

$$c_R = 6\, q\, k \sum_{j=1}^{P} \alpha_j \,. \tag{6.85}$$

As the considered theory is $\mathcal{N} = (0, 4)$ supersymmetric, expression (6.51) can be used to see that $c_L = c_R = c$, due to the condition of anomaly cancellation. This expression agrees with the central charge of (the Higgs branch of) the quiver in Figure 6.6, given by expression (6.81) times k (due to the orbifolding of the S^3 by $\mathbb{Z}_k$). This result shows that the different light multiplets appearing in the quivers depicted in Figures 6.6 and 6.8, both of which coming from precise derivations of perturbative string theory, lead to the same central charge. Clearly the underlying reason for this agreement is the common origin in M-theory of both classes of solutions. From the field theory perspective, we found a realisation of the mirror symmetry of the dual CFT, in the precise sense discussed below.

[17]See also [49], where some corrections to the analysis in [94] were pointed out.

[18]The factor of q appears because the quiver needs to be rescaled by q in order to be consistent with Figure 6.6.

6.3.4. Realisation in Type IIB

The common M-theory origin of both classes of Type IIA supergravity solutions implies that they are related by a chain of T-S-T dualities, as explained in subsection 1.4.3. This supports the idea of them flowing to the same CFT in the IR. Which deformation of the field theory is more convenient to use away from the critical point depends on the concrete value of the gauge coupling. Besides, both classes of Type IIB supergravity solutions are $\mathcal{N} = (0, 4)$ supersymmetric. The reason behind this is that T-duality on the Type IIA solutions in (6.53) takes place along the Hopf-fibre of the S^3, halving the supersymmetries to give $\mathcal{N} = (0, 4)$[19]. The resulting Type IIB solutions are interesting on their own, since they provide explicit holographic duals to D3-brane boxes constructions [149], realising in this case small $\mathcal{N} = (0, 4)$ supersymmetry[20]. In the remainder of the subsection we explore this class of Type IIB backgrounds, and show that they are related through an $\mathrm{SL}(2, \mathbb{R})$ transformation to the T-duals (along a circle within the $\mathbb{T}^4$) of the $\mathrm{AdS}_3 \times S^2 \times \mathbb{T}^4 \times I$ solutions of massless Type IIA constructed in [38].

The brane set-up underlying the T-dual of the $\mathrm{AdS}_3 \times S^3 \times \mathbb{T}^3 \times I$ solutions studied in section 6.3 is summarised in Table 6.7, while that of the T-dual of the $\mathrm{AdS}_3 \times S^2 \times \mathbb{T}^4 \times I$ solutions presented in [38] appears in Table 6.8. These brane set-ups can be proven to be S-dual to each other. It can be seen that S-duality interchanges the $\mathcal{N} = (0, 4)$

branes	x^0	x^1	z_1	z_2	z_3	ρ	ζ	ψ	θ^1	θ^2
D3	×	×	−	−	−	×	−	×	−	−
D5	×	×	×	×	×	−	−	×	−	−
NS5	×	×	−	−	−	−	×	×	×	×
NS5′	×	×	×	×	×	×	−	−	−	−

Table 6.7: $\frac{1}{8}$-BPS brane intersection T-dual to that of Table 6.5, realising a D3-brane box model. (x^0, x^1) parametrise the space where the field theory lives. (z_1, z_2, z_3) span the $\mathbb{T}^3$. ρ is the direction where the NS5-branes are located. ζ and θ^i are respectively the radial coordinate of AdS_3 and the angles that parametrise the S^2. ψ describes the S^1 generated upon the dualisation, where the NS5′-branes are located. (ρ, ψ) are thus the two directions of the brane box.

hypermultiplets and $\mathcal{N} = (0, 4)$ twisted hypermultiplets associated to the massless string modes living in the studied brane intersections. This turns out to be the 2d manifestation of the mirror symmetry. It is known that in 3d gauge theories [108, 151] mirror symmetry interchanges the scalars in the hypermultiplets and vector multiplets, and therefore the Higgs and Coulomb branches. Since $\mathcal{N} = (0, 4)$ vector multiplets contain no scalars, 2d $\mathcal{N} = (0, 4)$ field theories lack a Coulomb branch. Therefore, 2d mirror symmetry cannot result in the interchanging of both branches. Remarkably, mirror symmetry is realised in

[19]The 2d dual CFT is the same, independently on the number of supersymmetries appearing in the UV.

[20]The D3-brane boxes constructed in [149] have $\mathrm{SO}(4)_R$ R-symmetry and consequently should be dual to AdS_3 solutions with *large* $\mathcal{N} = (0, 4)$ supersymmetry instead.

branes	x^0	x^1	z_1	z_2	z_3	ψ	ρ	ζ	θ^1	θ^2
D3	×	×	–	–	–	×	×	–	–	–
NS5	×	×	×	×	×	×	–	–	–	–
D5	×	×	–	–	–	×	–	×	×	×
D5′	×	×	×	×	×	–	×	–	–	–

Table 6.8: $\frac{1}{8}$-BPS brane intersection T-dual to the brane intersection depicted in Table 6.6. (x^0, x^1) are the directions where the field theory lives. (z_1, z_2, z_3) span a $\mathbb{T}^3$. ψ is the T-duality circle and ρ is the field theory direction. This set-up is clearly S-dual to the configuration in Table 6.7.

the set-ups of Tables 6.7 and 6.8 as the interchange between the scalars transforming under the $\mathrm{SU}(2)_R$ R-symmetry, i.e those belonging to the twisted hypermultiplets, with those that are singlets under the $\mathrm{SU}(2)_R$, i.e the ones belonging to the untwisted hypermultiplets. This extends very naturally the mirror symmetry present in 3d gauge theories to these 2d ones and parallels the interchange between chiral and twisted chiral superfields inherent to mirror symmetry in supersymmetric sigma models.

Solutions of Type IIB supergravity

We now complement the above holographic discussion with the explicit construction of the Type IIB supergravity solutions. To attain this purpose, we start by presenting the T-dual of the solutions studied in section 6.3. Namely, we T-dualise along the Hopf fibre of the S^3 of the $\mathrm{AdS}_3 \times S^3 \times \mathbb{T}^3$ solutions in (6.53), thus arriving at the following class of Type IIB backgrounds,

$$
\begin{aligned}
ds^2 =& q\, h^{-1/2} \left[ds^2(\mathrm{AdS}_3) + 4^{-1}\, ds^2(\mathrm{S}^2)\right] + q^{-1}\, h^{1/2}\, d\psi^2 + \\
&+ g\, \left[h^{-1/2} d\rho^2 + h^{1/2}\, ds^2(\mathbb{T}^3)\right] , \\
e^{-\Phi} =& (q\, h)^{1/2}\, g^{-1/2} , \quad H_3 = \partial_\rho\, (hg)\, \mathrm{vol}(\mathbb{T}^3) - 2^{-1}\, \mathrm{vol}(S^2) \wedge d\psi, \\
F_1 =& g^{-1}\, \partial_\rho\, h\, d\psi\, , \quad F_3 = -2^{-1} q\, \mathrm{vol}(S^2) \wedge d\rho, \\
F_5 =& 2\, q\, \mathrm{vol}(\mathrm{AdS}_3) \wedge d\rho \wedge d\psi + 2^{-1}\, q\, g\, h\, \mathrm{vol}(\mathbb{T}^3) \wedge \mathrm{vol}(S^2)\, ,
\end{aligned}
\tag{6.86}
$$

where ψ parametrises the T-duality circle[21]. In order to provide the local representation of the brane set-ups of Tables 6.7 and 6.8 we need to focus on the particular situation

$$
h = \text{constant}\, , \quad g'' = 0\, , \tag{6.87}
$$

which corresponds to the massless solutions in Type IIA. In this case the metric exhibits the characteristic behaviour of NS5-branes wrapped on an $\mathrm{AdS}_3 \times S^2 \times S^1$ geometry. Indeed, it can be verified that these solutions arise as the near-horizon limit of the brane

[21] This solution is a particular example contained within the class of solutions presented in [51] section 3.1: one should identify (h, g) and (P, G) there, restrict $u' = 0$ and impose that ∂_{z_i} are all isometries.

solution associated to Table 6.7. More concretely, this happens in the particular case where the D3-D5-NS5′ branes have been fully localised within the worldvolume of the NS5-branes, as it was done for the $H_{\rm D2}(\zeta)$ and $H_{\rm D4}(\zeta)$ harmonic functions in subsection 6.1.2. We will restrict to this subclass of solutions, characterised by a vanishing axion, in the rest of this subsection.

Let us now perform an SL$(2,\mathbb{R})$ rotation of this subclass of solutions as described in subsection 1.4.2. The result is a family of solutions parametrised by the angle $\xi \in \left[0, \frac{\pi}{2}\right]$,

$$\begin{aligned}
ds^2 =&\Delta^{1/2}\left[q\, h^{-1/2}\left[ds^2(\mathrm{AdS}_3) + 4^{-1}\, ds^2(\mathrm{S}^2)\right] + q^{-1}\, h^{1/2}\, d\psi^2 + \right.\\
&\left. +g\,\left[h^{-1/2} d\rho^2 + h^{1/2}\, ds^2(\mathbb{T}^3)\right]\right]\,,\\
\Delta =&c^2 + q\, h\, g^{-1}\, s^2\,, \quad e^{-\Phi} = \Delta^{-1}(h\, q)^{1/2}\, g^{-1/2}\,, \quad C_0 = sc\,\Delta^{-1}\left(h\, q\, g^{-1} - 1\right)\,,\\
H_3 =&c\, h\, \partial_\rho\, g\, \mathrm{vol}(\mathbb{T}^3) - 2^{-1}\, c\mathrm{vol}(\mathrm{S}^2)\wedge d\psi - 2^{-1}\, s\, q\, \mathrm{vol}(\mathrm{S}^2)\wedge d\rho,\\
F_3 =&-2^{-1}\, q\, c\, \Delta^{-1}\mathrm{vol}(\mathrm{S}^2)\wedge d\rho - s\, q\, h^2 g^{-1}\Delta^{-1}\partial_\rho\, g\, \mathrm{vol}(\mathbb{T}^3) + \\
&+ 2^{-1} s\, q\, h\, g^{-1}\, \Delta^{-1}\mathrm{vol}(\mathrm{S}^2)\wedge d\psi\,,\\
F_5 =&2\, q\mathrm{vol}(\mathrm{AdS}_3)\wedge d\rho\wedge d\psi + 2^{-1}\, q\, g\, h\, \mathrm{vol}(\mathbb{T}^3)\wedge \mathrm{vol}(\mathrm{S}^2)\,,
\end{aligned} \tag{6.88}$$

where $s = \sin\xi$ and $c = \cos\xi$[22]. In particular, the family of S-dual solutions is obtained by setting $\xi = \frac{\pi}{2}$ in (6.88). The result is displayed below,

$$\begin{aligned}
ds^2 =&q^{3/2} g^{-1/2}\left(ds^2(\mathrm{AdS}_3) + 4^{-1} ds^2(\mathrm{S}^2)\right) + q^{-1/2} h\, g^{-1/2} d\psi^2\\
&+ q^{1/2} g^{1/2} d\rho^2 + q^{1/2} g^{1/2}\, h\, ds^2(\mathbb{T}^3),\\
e^{-\Phi} =&(q\, h)^{-1/2} g^{1/2}\,, \qquad\qquad H_3 = -2^{-1} q\, \mathrm{vol}(\mathrm{S}^2)\wedge d\rho\\
F_3 =&-h\, \partial_\rho\, g\, \mathrm{vol}(\mathbb{T}^3) + 2^{-1}\mathrm{vol}(\mathrm{S}^2)\wedge d\psi,\\
F_5 =&2\, q\mathrm{vol}(\mathrm{AdS}_3)\wedge d\rho\wedge d\psi + 2^{-1}\, q\, g\, h\, \mathrm{vol}(\mathbb{T}^3)\wedge \mathrm{vol}(\mathrm{S}^2)\,.
\end{aligned} \tag{6.89}$$

As expected, the 5-branes exchanged their roles. In turn, the metric exhibits the characteristic behaviour of D5-branes wrapped on an AdS$_3 \times S^2 \times S^1$ geometry, originated by a D3-D5′-NS5 fully-backreacted intersection. These solutions can be proven to arise in the near-horizon limit of the brane intersection depicted in Table 6.8, in the particular case where the D3-D5′-NS5 branes are fully localised within the worldvolume of the D5-branes.

The class of solutions presented in this subsection can be related to the Type IIB $\mathcal{N} = (0,4)$ AdS$_3$ solutions constructed in [46], from slightly more general D3-D5-NS5-D5′-NS5′ brane set-ups. The easiest way to show this is by relating the solution with $\xi = \frac{\pi}{2}$ given in (6.89) with equation (2.7) in [46]. One needs to impose that $H_{\mathrm{NS5}'} = 1$, rename $H_{\mathrm{D5}'} = g$ and smear the solution in [46] in such a way that $H_{\mathrm{D5}'} = g$ is delocalised over the internal $\mathbb{R}^3$ and it can be replaced by a $\mathbb{T}^3$.

[22] This generalised solution is an example contained in the class of [51] section 3.2: again one should identify (h, g) with (P, G) there, restrict $u' = 0$ and impose that ∂_{z_i} are all isometries.

6.4. $\mathbf{AdS}_3 \times S^3 \times \mathbb{T}^3$ in Type I′

We go back to the solutions constructed in section 6.3, but we now explore the massive case $F_0 \neq 0$. We recall that said backgrounds display $\mathrm{AdS}_3 \times S^3 \times \mathbb{T}^3$ geometries fibred over an interval given by (6.53), with defining functions satisfying the Bianchi identities (6.54). In the massive case we can write (g, h) in terms of a certain positive real function u as below,

$$h = \sqrt{u}\,, \quad g = \frac{c}{\sqrt{u}}\,, \tag{6.90}$$

such that the Bianchi identities are satisfied with c constant and u a linear function. The solutions are accordingly rewritten as follows,

$$\begin{aligned} ds^2 =& \frac{q}{u^{\frac{1}{4}}}\Big[ds^2(\mathrm{AdS}_3) + ds^2(\mathrm{S}^3)\Big] + \frac{c}{u^{\frac{1}{4}}}\Big[ds^2(\mathbb{T}^3) + \frac{1}{\sqrt{u}}d\rho^2\Big]\,, \quad e^{-\Phi} = \frac{u^{\frac{5}{8}}}{\sqrt{c}}\,,\\ F_0 =& \frac{u'}{2c}\,, \quad F_4 = 2\,q\Big(\mathrm{vol}(\mathrm{AdS}_3) + \mathrm{vol}(\mathrm{S}^3)\Big)\wedge d\rho\,,\\ F_6 =& 2\,q\,c\mathrm{vol}(\mathbb{T}^3)\wedge(\mathrm{vol}(\mathrm{S}^3) + \mathrm{vol}(\mathrm{AdS}_3))\,. \end{aligned} \tag{6.91}$$

The underlying brane set-up is the one in Table 6.9. As mentioned above, u has to be a

	x^0	x^1	z_1	z_2	z_3	ρ	ζ	θ^1	θ^2	θ^3
D2	×	×	–	–	–	×	–	–	–	–
D4	×	×	×	×	×	–	–	–	–	–
D8	×	×	×	×	×	–	×	×	×	×

Table 6.9: $\frac{1}{8}$-BPS brane intersection underlying the geometry (6.91). (x^0, x^1) are the directions where the 2d dual CFT lives. (z_1, z_2, z_3) span the $\mathbb{T}^3$, around which the D4s and the D8s are wrapped. ρ is the field theory direction, where the D2 branes are stretched and θ^i parametrise the S^3.

linear function in order to satisfy the Bianchi identities. We will take it to be piecewise linear so that D8-branes can be inserted at the different discontinuities of its derivative, according to the expression for F_0 in (6.91). We take ρ to parametrise an interval which begins at $\rho = 0$ and ends at ρ_P, where u vanishes. At the zeros of u the solutions behave as

$$ds^2 = \frac{q}{\sqrt{x}}\Big[ds^2(\mathrm{AdS}_3) + ds^2(\mathrm{S}^3) + c\,ds^2(\mathbb{T}^3)\Big] + 4c\sqrt{x}dx^2\,, \quad e^{-\Phi} = \frac{x^{\frac{5}{4}}}{\sqrt{c}}\,, \tag{6.92}$$

where $\rho = x^2$, which is the behaviour of a localised D8/O8 system on $\mathrm{AdS}_3 \times S^3 \times \mathbb{T}^3$. We will then define the solutions globally by embedding them into Type I' string theory[23].

[23]Type I′ string theory is the T-dual of Type I along a circle direction. It can also be seen as the orientifold of Type IIA compactified on a circle that results after gauging the worldsheet parity symmetry and the reflection symmetry of the circle. These projections identify left- and right-moving modes on the worldsheet and the opposite points on the circle, respectively. For further information, one can read [152].

This is achieved by introducing O8 orientifold fixed points at both ends of the space and 16 D8-branes (together with their mirrors under $\mathbb{Z}_2$) at arbitrary positions in ρ. Taking $\rho_P = \rho_{17} = \pi$ and the 16 D8-branes located at arbitrary points $\rho_1, \ldots, \rho_{16}$ between $\rho = 0$ and $\rho_{17} = \pi$, $u(\rho)$ turns out to be

$$u(\rho) = \begin{cases} -\frac{16c}{2\pi}\rho\,, & 0 \leq \rho_1\,, \\ \alpha_1 - \frac{14c}{2\pi}(\rho - \rho_1)\,, & \rho_1 \leq \rho \leq \rho_2\,, \\ \vdots & \\ \alpha_k + \frac{2c(k-8)}{2\pi}(\rho - \rho_k)\,, & \rho_k \leq \rho \leq \rho_{k+1}\,, \\ \vdots & \\ \alpha_{15} + \frac{14c}{2\pi}(\rho - \rho_{15})\,, & \rho_{15} \leq \rho \leq \rho_{16}\,, \\ \alpha_{16} + \frac{16c}{2\pi}(\rho - \pi)\,, & \rho_{16} \leq \rho \leq \pi\,, \end{cases} \tag{6.93}$$

where for continuity the α_k must satisfy

$$\alpha_k = \alpha_{k-1} - \frac{2c}{2\pi}(9-k)(\rho_k - \rho_{k-1}) \quad \text{for} \quad k = 1, \ldots, 16\,. \tag{6.94}$$

Furthermore, the condition $u(\pi) = 0$ imposes that the positions of the D8-branes must satisfy

$$\sum_{k=1}^{17}(9-k)(\rho_k - \rho_{k-1}) = 0\,. \tag{6.95}$$

Note that this is trivially met when $\rho_{17-k} = \pi - \rho_k$ for $k = 1, \ldots, 8$, i.e. when the D8-branes are symmetrically distributed along the interval, and also when the D8-branes are equally spaced, such that $\rho_k - \rho_{k-1} = \pi/16$ for all k.

Besides the D8-brane charge increasing in one unit at the position of each D8-brane, we have the following quantised charges

$$Q_{D2}^{(k)} = \frac{1}{(2\pi)^5}\int_{\mathbb{T}^3, \mathrm{S}^3} f_6 = c\,q\,, \quad Q_{D4}^{(k)} = \frac{1}{(2\pi)^3}\int_{I_\rho, \mathrm{S}^3} F_4 = \frac{q}{2\pi}(\rho_{k+1} - \rho_k)\,. \tag{6.96}$$

The number of D2-branes must thus be the same in all intervals, with $c = Q_{D2}/q$, while the jumps in the D4-brane charge are given by the second expression in (6.96).

Now one can take into account the previous results in order to build the quiver gauge theories that flow in the IR to the CFTs dual to the solutions in (6.91). In order to infer the different massless fields that appear in the quivers, we study the quantisation of the open strings stretched between the different branes in the set-up of Table 6.9. Following [153][24] we find:

- D2-D2 strings: Open strings with both ends on the same stack of D2-branes give rise to $\mathcal{N} = (0,4)$ SO(Q_{D2}) vector multiplets and $\mathcal{N} = (0,4)$ hypermultiplets in the symmetric representation of SO(Q_{D2}).

[24]In this reference the projection induced by the orientifold fixed points was carefully analysed for the Type I D1-D5 system, T-dual to our D2-D4-D8 brane set-up.

- D2-D4 strings: Open strings stretched between D2- and D4-branes give rise to $\mathcal{N} = (0,4)$ hypermultiplets in the bifundamental representation of SO(Q_{D2})×Sp($2Q_{D4}$).
- D2-D8 strings: Open strings stretched between D2- and D8-branes give rise to $\mathcal{N} = (0,2)$ Fermi multiplets in the bifundamental representation of SO(Q_{D2})×SO(Q_{D8}).

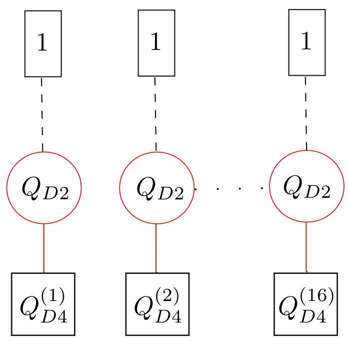


Figure 6.9: Quiver associated to the AdS$_3$ × S^3 × $\mathbb{T}^4$ solutions in Type I'. In it the circles correspond to $\mathcal{N} = (0,4)$ vector multiplets plus hypermultiplets in the symmetric representation of SO(Q_{D2}). The dashed lines denote $\mathcal{N} = (0,4)$ hypermultiplets in the bifundamental representation of SO(Q_{D2})×Sp($2Q_{D4}$) and the red lines are $\mathcal{N} = (0,2)$ Fermi multiplets in the bifundamental representation of SO(Q_{D2})×SO(Q_{D8}).

These massless modes give rise to the $\mathcal{N} = (0,4)$ disconnected quivers depicted in Figure 6.9. In these quivers anomaly cancellation imposes that

$$2Q_{D4}^{(k)} = \Delta Q_{D8}^{(k)} = 1\,, \tag{6.97}$$

as explained below equation (6.44). Given that D4-branes in Type I′ carry 1/2 units of charge [154], in order to obtain a consistent CFT in the IR the D4-branes must be located in exactly the same positions in ρ as the D8-branes. This fixes the total number of D4-branes to 16[25]. This condition needs to be imposed on the supergravity solution in order to describe a proper Type I' background with a well-defined 2d dual CFT. Finally, substituting (6.90) in (6.66), the holographic central charge for this class of solutions is obtained,

$$c_{\text{hol}} = 48\,Q_{D2}\,. \tag{6.98}$$

We observe that this matches exactly the field theory result, obtained from (5.4), which gives in this case

$$c_R = c_L = 6\sum_{k=1}^{16} Q_{D2}Q_{D4}^{(k)} = 48\,Q_{D2}\,. \tag{6.99}$$

[25]Note that it is possible to consider the situation in which some of the ρ_k coincide, such that a group of D8-D4 branes are located at that position.

Chapter 7

AdS$_2$/SCQM in Type II

This chapter is devoted to presenting the study of the AdS$_2$/SCQM correspondence performed in [65, 67, 68]. In these works we built new AdS$_2$ Type II backgrounds. As explained in section 5.1, this was achieved by either adding defect branes to a well-known brane set-up or performing a duality transformation on a known (brane or AdS) solution. In particular, we explored non-Abelian T-duality (NATD) as a solution generating technique that relates inequivalent backgrounds. This is quite relevant, as NATD is not fully understood yet. On the other hand, whenever we knew the brane intersection underlying a new supergravity solution, we built the quiver quantum mechanics living in said brane set-up. We also addressed the problem of computing the central charge of a SCQM. As we explained in section 5.3, it has been conjectured that the central charge of a 1d field theory dual to an AdS$_2$ Type II supergravity solution can always be computed by using the conventions and formulae of two-dimensional field theory. This hypothesis was tested with our solutions by comparing the central charge so obtained with the one derived from the dual AdS$_2$ solution in the IR.

In sections 7.1-7.3 we explore new AdS$_2 \times S^2$ Type IIA/B backgrounds, the last section also containing the field theory interpretation. Section 7.4 is devoted to presenting a new class of Type IIA solutions obtained via non-Abelian T-duality. Its field theory interpretation as defects is then performed in section 7.5. Finally, in sections 7.6-7.8 we revisit another new class of Type IIB brane solutions also obtained via NATD.

7.1. The F1-D2-D4′-NS5′-D4-NS5 brane system

In this section we present a new class of solutions to Type IIA supergravity that can be described by a set-up consisting of F1-D2-D4′-NS5′ branes ending on a D4-NS5 bound state. In the near-horizon limit, these solutions are characterised by $\mathcal{N}=4$ AdS$_2 \times S^2 \times \tilde{S}^2 \times \mathbb{R}^2 \times S^1$ geometries foliated over a line. We show that an appropriate distribution of charges of the D4-NS5 branes produces a non-compact solution within this class that asymptotes locally to an AdS$_5$ vacuum. Moreover, these AdS$_5$ solutions are associated to the D4-NS5 brane intersection and belong to the class of so-called *Gaiotto-Maldacena* *geometries* introduced in [143]. This can be used to resolve the divergences associated to the non-compactness of the internal space within the AdS$_5$ geometry, and to interpret the solution as describing a line defect CFT within the $\mathcal{N} = 2$ 4d CFT dual to the aforementioned AdS$_5$ vacuum.

7.1.1. The brane solutions to Type IIA

We first consider the brane set-up depicted in Table 7.1. This is a BPS/8 brane intersection that can be regarded as a F1-D2-D4′-NS5′ brane intersection ending on a BPS/4 bound state of D4-NS5 branes. We assume the F1-D2-D4′-NS5′ system of defect branes

branes	t	ρ	φ^1	φ^2	y	z	ψ	r	θ^1	θ^2
D4	×	×	×	×	–	–	×	–	–	–
NS5	×	×	×	×	×	×	–	–	–	–
F1	×	–	–	–	–	×	–	–	–	–
D2	×	–	–	–	×	–	×	–	–	–
D4′	×	–	–	–	×	–	–	×	×	×
NS5′	×	–	–	–	–	×	×	×	×	×

Table 7.1: $\frac{1}{8}$-BPS brane intersection describing F1-D2-D4′-NS5′ branes ending on a D4-NS5 bound state. The field theory living in this brane set-up can be regarded as an $\mathcal{N} = 4$ line defect SCQM within the $\mathcal{N} = 2$ 4d CFT living in the D4-NS5 branes.

to be completely localised within the four dimensional worldvolume of the orthogonal D4-NS5 background. This condition makes the equations describing both sets of branes to decouple from one another. This prescription leads to the condition that their associated warp functions H_{F1}, H_{D2}, $H_{\text{D4}'}$ and $H_{\text{NS5}'}$ must depend only on ρ. Moreover, we assume that the D4- and NS5-branes are completely localised in their respective transverse spaces, except for the latter ones being smeared along the circle direction ψ. This requirement implies that we must have $H_{\text{D4}} = H_{\text{D4}}(y, z, r)$ and $H_{\text{NS5}} = H_{\text{NS5}}(r)$.

Under these restrictions, the fields generated by this brane set-up display the following form,

$$
\begin{aligned}
ds^2_{10} =& H_{\text{D4}}^{-1/2}\left[-H_{\text{F1}}^{-1}H_{\text{D2}}^{-1/2}H_{\text{D4}'}^{-1/2}\,dt^2 + H_{\text{D2}}^{1/2}H_{\text{D4}'}^{1/2}H_{\text{NS5}'}\big(d\rho^2+\rho^2 ds^2_{S^2}\big)\right]\\
&+ H_{\text{D4}}^{1/2}\left[H_{\text{D2}}^{-1/2}H_{\text{D4}'}^{-1/2}H_{\text{NS5}'}\,dy^2 + H_{\text{F1}}^{-1}H_{\text{D2}}^{1/2}H_{\text{D4}'}^{1/2}\,dz^2\right]\\
&+ H_{\text{NS5}}H_{\text{D4}}^{-1/2}H_{\text{D2}}^{-1/2}H_{\text{D4}'}^{1/2}\,d\psi^2 + H_{\text{NS5}}H_{\text{D4}}^{1/2}H_{\text{D2}}^{1/2}H_{\text{D4}'}^{-1/2}\big(dr^2+r^2 ds^2_{\tilde{S}^2}\big)\,,\\
e^{\Phi} =& H_{\text{NS5}}^{1/2}H_{\text{D4}}^{-1/4}H_{\text{F1}}^{-1/2}H_{\text{D2}}^{1/4}H_{\text{D4}'}^{-1/4}H_{\text{NS5}'}^{1/2}\,,\\
H_{(3)} =& -\partial_\rho H_{\text{F1}}^{-1}dt\wedge d\rho\wedge dz + \partial_\rho H_{\text{NS5}'}\,\rho^2\,\text{vol}_{S^2}\wedge dy + \partial_r H_{\text{NS5}}\,r^2\,d\psi\wedge\text{vol}_{\tilde{S}^2}\,,\\
F_{(4)} =& \partial_\rho H_{\text{D2}}^{-1}dt\wedge d\rho\wedge dy\wedge d\psi + \partial_\rho H_{\text{D4}'}\,\rho^2\,\text{vol}_{S^2}\wedge dz\wedge d\psi +\\
&+ \partial_r H_{\text{D4}}\,r^2\,dy\wedge dz\wedge\text{vol}_{\tilde{S}^2} + H_{\text{D2}}H_{\text{NS5}'}^{-1}H_{\text{NS5}}\partial_y H_{\text{D4}}\,r^2\,dz\wedge dr\wedge\text{vol}_{\tilde{S}^2}\\
&- H_{\text{F1}}H_{\text{D4}'}^{-1}H_{\text{NS5}}\partial_z H_{\text{D4}}\,r^2\,dy\wedge dr\wedge\text{vol}_{\tilde{S}^2}\,.
\end{aligned}
\tag{7.1}
$$

Here S^2 is the 2-sphere spanned by the coordinates (φ^1, φ^2) in Table 7.1, and $\tilde{S}^2$ the one parametrised by (θ^1, θ^2). It can be seen that, as mentioned above, the equations of motion and Bianchi identities for (7.1) decouple into two groups. The equations for F1-D2-D4′-NS5′ defect branes are equivalent to the PDEs below,

$$
\begin{aligned}
&\nabla^2_{\mathbb{R}^3_\rho} H_{\mathrm{D2}} = 0 \quad \text{with} \quad H_{\mathrm{NS5'}} = H_{\mathrm{D2}}\,, \\
&\nabla^2_{\mathbb{R}^3_\rho} H_{\mathrm{F1}} = 0 \quad \text{with} \quad H_{\mathrm{D4'}} = H_{\mathrm{F1}}\,,
\end{aligned}
\tag{7.2}
$$

while the D4-NS5 system satisfies the following equations,

$$
\nabla^2_{\mathbb{R}^3_r} H_{\mathrm{D4}} + H_{\mathrm{NS5}} \nabla^2_{\mathbb{R}^2_{(y,z)}} H_{\mathrm{D4}} = 0 \quad \text{and} \quad \nabla^2_{\mathbb{R}^3_r} H_{\mathrm{NS5}} = 0\,. \tag{7.3}
$$

We observe that the PDEs in (7.2) do not impose any warping within the 2d subspace $\mathbb{R}^2_{(y,z)}$ parametrised by y and z. This is a direct consequence of the defect branes being completely smeared within this subspace. Besides $H_{\mathrm{D4}}(y, z, r)$ and $H_{\mathrm{NS5}}(r)$ describe a D4-NS5 bound state localised in the subspace $\mathbb{R}^2_{(y,z)} \times \mathbb{R}^3_r$ with $\mathbb{R}^3_r$ parametrised by r and $\tilde{S}^2$.

In order to derive the background with AdS_2 geometry we first choose the particular solutions below,

$$
H_{\mathrm{D2}} = 1 + \frac{q_{\mathrm{D2}}}{\rho} \quad \text{and} \quad H_{\mathrm{F1}} = 1 + \frac{q_{\mathrm{F1}}}{\rho}\,, \tag{7.4}
$$

where q_{D2} and q_{F1} are integration constants related to the quantised charges of the defect branes. Taking the near-horizon limit $\rho \to 0$ we arrive at the desired solution[1],

$$
\begin{aligned}
ds^2_{10} =& q_{\mathrm{D2}}^{3/2} q_{\mathrm{F1}}^{1/2} H_{\mathrm{D4}}^{-1/2} \left(ds^2_{\mathrm{AdS}_2} + ds^2_{S^2}\right) + q_{\mathrm{D2}}^{1/2} q_{\mathrm{F1}}^{-1/2} H_{\mathrm{D4}}^{1/2} \left(dy^2 + dz^2\right) \\
&+ q_{\mathrm{D2}}^{-1/2} q_{\mathrm{F1}}^{1/2} H_{\mathrm{NS5}} H_{\mathrm{D4}}^{-1/2}\, d\psi^2 + H_{\mathrm{NS5}} H_{\mathrm{D4}}^{1/2} q_{\mathrm{D2}}^{1/2} q_{\mathrm{F1}}^{-1/2} \left(dr^2 + r^2 ds^2_{\tilde{S}^2}\right), \\
e^{\Phi} =& q_{\mathrm{D2}}^{3/4} q_{\mathrm{F1}}^{-3/4} H_{\mathrm{NS5}}^{1/2} H_{\mathrm{D4}}^{-1/4}\,, \\
H_{(3)} =& -q_{\mathrm{D2}} (\mathrm{vol}_{\mathrm{AdS}_2} \wedge dz + \mathrm{vol}_{S^2} \wedge dy) + \partial_r H_{\mathrm{NS5}}\, r^2\, d\psi \wedge \mathrm{vol}_{\tilde{S}^2}\,, \\
F_{(4)} =& q_{\mathrm{F1}} (\mathrm{vol}_{\mathrm{AdS}_2} \wedge dy - \mathrm{vol}_{S^2} \wedge dz) \wedge d\psi + \\
&+ r^2 \left[\partial_r H_{\mathrm{D4}}\, dy \wedge dz + H_{\mathrm{NS5}} \partial_y H_{\mathrm{D4}}\, dz \wedge dr - H_{\mathrm{NS5}} \partial_z H_{\mathrm{D4}}\, dy \wedge dr\right] \wedge \mathrm{vol}_{\tilde{S}^2}\,.
\end{aligned}
\tag{7.5}
$$

This is a new class of backgrounds characterised by a geometry $\mathrm{AdS}_2 \times S^2 \times \tilde{S}^2 \times \mathbb{R}^2_{(y,z)} \times S^1_\psi$, fibred over an interval parametrised by r. We have that $H_{\mathrm{D4}}(y, z, r)$ and $H_{\mathrm{NS5}}(r)$ must satisfy the equations (7.3). These new backgrounds constitute a vast class of $\mathcal{N} = 4$ solutions to Type IIA string theory, determined by the charge distribution of the D4-NS5 system.

In the next subsection we will explore a solution in the class (7.5) associated to a particular choice of H_{D4} and H_{NS5} in which the $\mathrm{AdS}_2 \times S^2$ geometry reproduces asymptotically locally an AdS_5 vacuum related by T-duality to $\mathrm{AdS}_5 \times S^5/\mathbb{Z}_n$.

[1] In order to reproduce unitary AdS_2 at the horizon we rescaled the time as $t \to q_{\mathrm{D2}} q_{\mathrm{F1}} t$.

7.1.2. F1-D2-D4′-NS5′ line defects within AdS$_5$

As we have mentioned several times before, a key property of the brane system depicted in Table 7.1 is the possibility of decoupling the dynamics of the F1-D2-D4′-NS5′ defect branes from that of the D4-NS5 bound system. This is manifest at the level of the equations of motion, with the PDEs in (7.2) describing the F1-D2-D4′-NS5′ subsystem and those in (7.3), the D4-NS5 bound state. At the level of the solution (7.1), this property can be exploited by taking the $\rho \to +\infty$ limit in (7.4), thus "zooming out" the F1-D2-D4′-NS5′ defect branes and recovering the background associated to the D4-NS5 branes. This D4-NS5 brane system was explored in [155], where an explicit AdS$_5$ solution was derived by applying an appropriate change of coordinates and taking a concrete limit. This AdS$_5$ solution falls within the class of so-called *Gaiotto-Maldacena geometries*.

The Gaiotto-Maldacena backgrounds are a class of Type IIA supergravity solutions first introduced in [143][2]. They preserve $\mathcal{N} = 2$ supersymmetry, display an AdS$_5$ factor and can be parametrised by a potential $V(\sigma, \eta)$. Denoting $V' = \partial_\eta V$ and $\dot{V} = \sigma\partial_\sigma V$, a generic Gaiotto-Maldacena solution can be written as follows [146, 147],

$$\begin{aligned}
ds^2 =&\alpha' \left(\frac{2\dot{V} - \ddot{V}}{V''}\right)^{1/2} \Bigg[4ds^2_{\mathrm{AdS}_5} + \mu^2 \frac{2V''\dot{V}}{\Delta} ds^2_{S^2} + \\
&+\mu^2 \frac{2V''}{\dot{V}}(d\sigma^2 + d\eta^2) + \mu^2 \frac{4V''\sigma^2}{2\dot{V} - \ddot{V}} d\beta^2\Bigg] , \\
e^{4\Phi} =&4\frac{(2\dot{V} - \ddot{V})^3}{\mu^4 V''\dot{V}^2\Delta^2} , \quad B_2 = 2\mu^2\alpha' \left(\frac{\dot{V}\dot{V}'}{\Delta} - \eta\right) \mathrm{vol}_{S^2} , \\
C_1 =&2\mu^4\sqrt{\alpha'}\frac{2\dot{V}\dot{V}'}{2\dot{V} - \ddot{V}} d\beta , \quad C_3 = -4\mu^4\alpha'^{3/2}\frac{\dot{V}^2V''}{\Delta} d\beta \wedge \mathrm{vol}_{S^2} , \\
\Delta =&(2\dot{V} - \ddot{V})V'' + (\dot{V}')^2 ,
\end{aligned} \tag{7.6}$$

where the radius of the space is $\mu^2\alpha' = L^2$. The problem of writing a solution in this class then boils down to finding a function V that solves a Laplace equation with charge density $\lambda(\eta)$,

$$\partial_\sigma[\sigma\partial_\sigma V] + \sigma\partial^2_\eta V = 0 , \quad \lambda(\sigma) = \sigma\partial_\sigma V|_{\sigma=0} . \tag{7.7}$$

We remark that the metric, dilaton and fluxes depend on $\dot{V}$, $\dot{V}'$, $\ddot{V}$ and $V'' = -\sigma^{-2}\ddot{V}$. Hence, a choice of $\dot{V}$ determines a solution in the class of Gaiotto-Maldacena backgrounds. Regarding the boundary conditions, Gaiotto and Maldacena found them by imposing a correct charge quantisation and the smooth-shrinking of some submanifolds. They can be summarised as below [146, 147]:

- $\dot{V}(\sigma = 0, \eta) = \lambda(\eta)$ must vanish at $\eta = 0$.
- $\lambda(\eta)$ must be a piecewise linear continuous function $\lambda = a_i\eta + q_i$ with a_i an integer.

[2] Reference [93] was consulted for the following description of these solutions.

- The change in slope between consecutive intervals must be a negative integer, i.e. $a_i - a_{i-1} < 0$. A kink in which the slope changes by k units is associated with the presence of D6-branes.

- The positions of the kinks must be integers in the η-axis.

- Some solutions satisfy $\lambda(N_*) = 0$. In that case, η is bounded in $[0, N_*]$. The associated electrostatic problem can then be understood as a line charge density $\lambda(\eta)$ bounded by two 'conductive plates' located at $\eta = 0$ and $\eta = N_*$.

Let us now come back to the more general situation in which we have F1-D2-D4′-NS5′ defect branes ending on the D4-NS5 system. In particular, we are interested in the $\rho \to 0$ limit of (7.4) and, therefore, in the $\mathcal{N} = 4$ backgrounds defined by (7.5). We remark that these solutions display the crucial property: the backreaction of the F1-D2-D4′-NS5′ branes on the D4-NS5 system modifies only the 4d worldvolume space of the D4-NS5 solution, keeping intact its $\mathbb{R}^2_{(y,z)} \times \mathbb{R}^3_r$ transverse space. This follows from the fact that the equations of motion associated to the F1-D2-D4′-NS5′ branes, given by (7.2), and those of the D4-NS5 system, given by (7.3), are completely independent. This implies, among other things, that the PDEs for the D4-NS5 intersection must coincide with the equations in (7.3). Consequently, we can still consider the semi-localised warp factors

$$H_{\text{D4}} = 1 + \frac{4\pi q_{\text{D4}} q_{\text{NS5}}}{(y^2 + z^2 + 2q_{\text{NS5}} r)^2} \qquad \text{and} \quad H_{\text{NS5}} = \frac{q_{\text{NS5}}}{2r} \,, \tag{7.8}$$

which were introduced for the D4-NS5 bound state in [155][3]. We then introduce the following new coordinates (μ, α, ϕ) [155],

$$y = \mu \sin\alpha \cos\phi \,, \qquad z = \mu \sin\alpha \sin\phi \quad \text{and} \quad r = 2^{-1}\, q_{\text{NS5}}^{-1}\, \mu^2 \cos^2\alpha \,. \tag{7.9}$$

Nevertheless, the presence of the defect branes breaks the isometries of the 4d worldvolume of the D4-NS5 intersection, giving rise to an $\text{AdS}_2 \times S^2$ backreacted geometry. It is useful to change from the (y, z, r) coordinates to (μ, α, ϕ) as in (7.9). This brings to light an asymptotically locally AdS_5 geometry when $\mu \to 0$. The fields in said coordinates and

[3]We remark that there are some precedents of similar semi-localised warp factors in [156–158].

limit are the ones below,

$$
\begin{aligned}
ds_{10}^2 =& q_{\text{D2}}^{1/2} q_{\text{F1}}^{-1/2} (4\pi q_{\text{NS5}} q_{\text{D4}})^{1/2} \overbrace{\left[(4\pi q_{\text{NS5}} q_{\text{D4}})^{-1} q_{\text{D2}}\, q_{\text{F1}} \mu^2 \left(ds^2_{\text{AdS}_2} + ds^2_{S^2}\right) + \frac{d\mu^2}{\mu^2}\right]}^{\text{locally AdS}_5 \text{ geometry}} + \\
&+ q_{\text{D2}}^{1/2} q_{\text{F1}}^{-1/2} (4\pi q_{\text{NS5}} q_{\text{D4}})^{1/2} \left[d\alpha^2 + s^2 d\phi^2 + q_{\text{D2}}^{-1} q_{\text{F1}} q_{\text{NS5}} (4\pi q_{\text{D4}})^{-1} c^{-2} d\psi^2 + \right. \\
&\left. + 4^{-1} c^2 ds^2_{\tilde{S}^2}\right] , \\
e^{\Phi} =& q_{\text{D2}}^{3/4} q_{\text{F1}}^{-3/4} q_{\text{NS5}}^{3/4} (4\pi q_{\text{D4}})^{-1/4} c^{-1} , \\
H_{(3)} =& - q_{\text{D2}} \left(\tilde{s}\, \text{vol}_{\text{AdS}_2} + \tilde{c}\, \text{vol}_{S^2}\right) \wedge (s\, d\mu + \mu c\, d\alpha) - q_{\text{D2}} \mu s \left(\tilde{c}\, \text{vol}_{\text{AdS}_2} - \tilde{s}\, \text{vol}_{S^2}\right) \wedge d\phi \\
& - 2^{-1} q_{\text{NS5}} d\psi \wedge \text{vol}_{\tilde{S}^2} , \\
F_{(4)} =& q_{\text{F1}} \left(\tilde{c}\, \text{vol}_{\text{AdS}_2} - \tilde{s}\, \text{vol}_{S^2}\right) \wedge (s\, d\mu + \mu c\, d\alpha) \wedge d\psi \\
& - q_{\text{F1}} \mu s \left(\tilde{s}\, \text{vol}_{\text{AdS}_2} + \tilde{c}\, \text{vol}_{S^2}\right) \wedge d\phi \wedge d\psi + 4\pi q_{\text{D4}}\, c^3 s\, d\phi \wedge d\alpha \wedge \text{vol}_{\tilde{S}^2} , \wedge d\mu
\end{aligned}
\tag{7.10}
$$

with $s = \sin\alpha$, $c = \cos\alpha$, $\tilde{s} = \sin\phi$ and $\tilde{c} = \cos\phi$. We remark that the internal space of the above solution is the same one of the AdS$_5$ vacuum that arises as the near-horizon limit of the D4-NS5 bound system studied in [155].

Summing up, the previous analysis has brought to the fore that the general class of brane solutions in (7.1) with the particular profiles (7.4) for the F1-D2-D4′-NS5′ branes and the semi-localised profile (7.8) for the D4-NS5 system gives rise to two interesting regimes. The first remarkable behaviour appears in the $\rho \to 0$ limit. In this case the defect branes are resolved, producing a fully backreacted AdS$_2 \times S^2$ geometry within the 4d worldvolume of the D4-NS5 system, displaying explicitly the breaking of its isometries. The second regime becomes manifest in the system of coordinates introduced in (7.9) when, after taking the $\rho \to 0$, one also sends $\mu \to 0$, thus approaching the origin of the $\mathbb{R}^2_{(y,z)}$ plane. In this regime the metric is split into a 5d "external" part, asymptotically reproducing a locally AdS$_5$ geometry, and a 5d internal manifold, which coincides exactly with that of the pure AdS$_5$ vacuum geometry associated to the D4-NS5 brane system studied in [155]. The isometries of the AdS$_5$ vacuum are however broken by the background fluxes, as shown by their expressions in (7.10). The extra terms show that a 5d observer located at $\mu \to 0$ feels the effect of the global charges of the defect branes. They backreact warping the 5d geometry to give a curved domain wall with AdS$_2 \times S^2$ slicings, which is only asymptotically locally AdS$_5$. It is clear that the presence of the extra terms in the fluxes prevents the supersymmetry to enjoy an enhancement to the (four-dimensional) $\mathcal{N} = 2$ supersymmetry displayed by the AdS$_5$ solution found in [155]. These two limits of the solution (7.1), underlying the F1-D2-D4′-NS5′-D4-NS5 brane intersection of Table 7.1, along with the $\rho \to +\infty$ regime, are collected in Figure 7.1

We observe that our class of solutions (7.5) realises in Type IIA supergravity a conformal line defect within the 4d $\mathcal{N} = 2$ SCFT that results by orbifolding the 4d $\mathcal{N} = 4$ SYM CFT by $\mathbb{Z}_n$. This is the 4d field theory dual to the AdS$_5$ solution in [155]. The defect is described by a superconformal quantum mechanics that is holographically dual to an AdS$_2$ geometry with $\mathcal{N} = 4$ supersymmetries (in one dimension). In the $\mu \to 0$

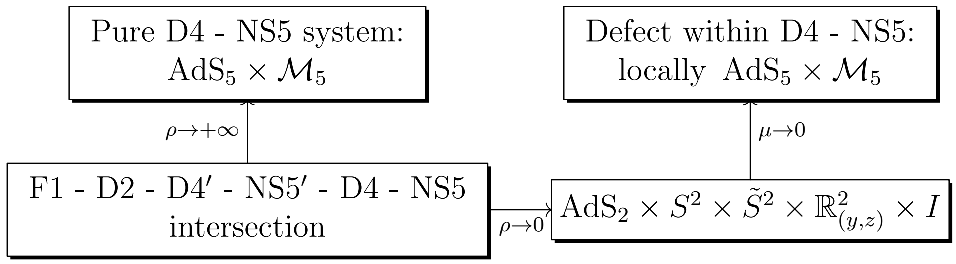


Figure 7.1: The $\rho \to +\infty$ and $\rho \to 0$ limits of the F1-D2-D4′-NS5′-D4-NS5 brane configuration depicted in Table 7.1, along with its defect structure. The $\rho \to +\infty$ limit zooms out the F1-D2-D4′-NS5′ branes, leaving behind the $\mathrm{AdS}_5 \times \mathcal{M}_5$ solution associated to the D4-NS5 bound system. The $\rho \to 0$ limit, on the contrary, zooms in the defect branes, giving rise to an AdS_2 geometry in the near-horizon. In turn, this geometry asymptotically approaches $\mathrm{AdS}_5 \times \mathcal{M}_5$ in the $\mu \to 0$ limit, allowing one to interpret the F1-D2-D4′-NS5′ brane intersection as describing a defect within the 4d SCFT associated to the D4-NS5 system.

limit, where this class of solution asymptotes to AdS_5, the defect is perceived by a 5d observer as an angular wedge located at the conformal boundary of AdS_2. This can be made explicit by rewriting the asymptotically locally AdS_5 part of the metric in (7.10) as below,

$$ds_5^2 \sim f^{-2}\left(-dt^2 + d\tilde{\rho}^2 + \tilde{\rho}^2 ds^2_{S^2} + \tilde{\rho}^2 d\lambda^2\right) , \tag{7.11}$$

where $f^{-2} = \mu^2 \tilde{\rho}^{-2}$, $d\lambda = \mu^{-2} d\mu$ and $\tilde{\rho}$ parametrises the radial direction of the AdS_2 in Poincaré coordinates. From the above expression one can see that the metric in the $(\tilde{\rho}, \lambda)$ plane develops a conical defect at $\tilde{\rho} = 0$. This fixes the locus of the defect and allows one to interpret μ as an angular coordinate parametrising the wedge in which a 5d observer probes the defect geometry.

7.2. The D1-F1-D5-NS5-D3-KK brane system

This section is dedicated to the Type IIB realisation of the constructions in the previous one, where the main features already discussed become more transparent. For this purpose, we built a new class of AdS_2 solutions to Type IIB supergravity with $\mathcal{N} = 4$ supersymmetry, and showed that such solutions find an interesting line defect interpretation within $\mathrm{AdS}_5 \times S^5/\mathbb{Z}_n$. It is critical to notice that the AdS_5 solution that arises far away from the Type IIA defects is the T-dual of the Type IIB $\mathrm{AdS}_5 \times S^5/\mathbb{Z}_n$ background. This latter solution arises in the near-horizon of a semi-localised system containing D3-branes and KK-monopoles. The appropriate change of coordinates in which the $\mathrm{AdS}_5 \times S^5/\mathbb{Z}_n$ geometry becomes manifest appears in [159] and we recommend the reader to check said reference if they are not familiar with the solution. In our case, the T-duality of the D4-NS5 brane intersection should be performed along the circular ψ direction, transforming the D4-branes into D3-branes and the NS5-branes into KK-monopoles. This latter objects produce a foliation of the circle and the emergence of the Lens space $S^5/\mathbb{Z}_n$. When the

KK charge is one, the round S^5 is recovered and the D3-branes become isotropic.

We start in subsection 7.2.1 with the thorough analysis of the T-dual realisation of the F1-D2-D4′-NS5′-D4-NS5 intersection discussed in subsection 7.1.1. This becomes a bound state of F1-D1-D5-NS5 branes ending on a D3-KK intersection, of which we provide both the full brane solution and its near-horizon limit. Said limit gives rise to a new class of $\mathcal{N}=4$ Type IIB backgrounds characterised by $\text{AdS}_2 \times S^2 \times S^2 \times \mathbb{R}^2 \times S^1$ geometries foliated over a line. Then in subsection 7.2.2, we show that a suitable prescription for the distribution of charges of the D3-KK system yields a solution within this class that asymptotes locally to the $\text{AdS}_5 \times S^5/\mathbb{Z}_n$ vacuum. This allows us to interpret this solution as holographically dual to a line defect CFT within $\mathcal{N}=4$ SYM modded by $\mathbb{Z}_n$. In the absence of KK-monopoles the solution describes a line defect CFT within 4d $\mathcal{N}=4$ SYM, which preserves 1/4 of the supersymmetries.

7.2.1. The brane solution to Type IIB

Let us consider the Type IIA brane set-up of Table 7.1. By T-dualising it along the ψ circular direction, the F1-D2-D4′-NS5′ defect branes become a D1-F1-D5-NS5 brane system localised within the common worldvolume of the D3-KK branes. The background

branes	t	ρ	φ^1	φ^2	y	z	ψ	r	θ^1	θ^2
D3	×	×	×	×	–	–	–	–	–	–
KK	×	×	×	×	×	×	ISO	–	–	–
D1	×	–	–	–	×	–	–	–	–	–
F1	×	–	–	–	–	×	–	–	–	–
D5	×	–	–	–	×	–	×	×	×	×
NS5	×	–	–	–	–	×	×	×	×	×

Table 7.2: BPS/8 intersection describing D1-F1-D5-NS5 branes ending on the D3-KK system.

fields generated by these brane configuration are the ones below,

$$
\begin{aligned}
ds^2_{10} =& H_{\mathrm{D3}}^{-1/2}\left[-H_{\mathrm{D1}}^{-1/2}H_{\mathrm{F1}}^{-1}H_{\mathrm{D5}}^{-1/2}dt^2+H_{\mathrm{D1}}^{1/2}H_{\mathrm{D5}}^{1/2}H_{\mathrm{NS5}}\left(d\rho^2+\rho^2ds^2_{S^2}\right)\right]+\\
&+H_{\mathrm{D3}}^{1/2}\left[H_{\mathrm{D1}}^{-1/2}H_{\mathrm{D5}}^{-1/2}H_{\mathrm{NS5}}dy^2+H_{\mathrm{D1}}^{1/2}H_{\mathrm{F1}}^{-1}H_{\mathrm{D5}}^{1/2}dz^2\right]+\\
&+H_{\mathrm{D3}}^{1/2}H_{\mathrm{D1}}^{1/2}H_{\mathrm{D5}}^{-1/2}\left[H_{\mathrm{KK}}^{-1}\left(d\psi+2^{-1}q_{\mathrm{KK}}\omega\right)^2+H_{\mathrm{KK}}\left(dr^2+r^2ds^2_{\tilde{S}^2}\right)\right],\\
e^{\Phi} =& H_{\mathrm{NS5}}^{1/2}H_{\mathrm{D5}}^{-1/2}H_{\mathrm{F1}}^{-1/2}H_{\mathrm{D1}}^{1/2},\\
H_{(3)} =& -\partial_\rho H_{\mathrm{F1}}^{-1}dt\wedge d\rho\wedge dz+\partial_\rho H_{\mathrm{NS5}}\,\rho^2\,\mathrm{vol}_{S^2}\wedge dy,\\
F_{(3)} =& -\partial_\rho H_{\mathrm{D1}}^{-1}dt\wedge d\rho\wedge dy-\partial_\rho H_{\mathrm{D5}}\,\rho^2\,\mathrm{vol}_{S^2}\wedge dz,\\
F_{(5)} =& H_{\mathrm{D5}}H_{\mathrm{NS5}}\rho^2\,dt\wedge d\rho\wedge\mathrm{vol}_{S^2}\wedge(\partial_r H_{\mathrm{D3}}^{-1}\,dr+\partial_y H_{\mathrm{D3}}^{-1}\,dy+\partial_z H_{\mathrm{D3}}^{-1}\,dz)+\\
&-r^2\partial_r H_{\mathrm{D3}}dy\wedge dz\wedge d\psi\wedge\mathrm{vol}_{\tilde{S}^2}+\\
&-H_{\mathrm{F1}}H_{\mathrm{D5}}^{-1}H_{\mathrm{KK}}r^2\partial_z H_{\mathrm{D3}}dy\wedge d\psi\wedge dr\wedge\mathrm{vol}_{\tilde{S}^2}+\\
&+H_{\mathrm{D1}}H_{\mathrm{NS5}}^{-1}H_{\mathrm{KK}}r^2\partial_y H_{\mathrm{D3}}dz\wedge d\psi\wedge dr\wedge\mathrm{vol}_{\tilde{S}^2},
\end{aligned}
\tag{7.12}
$$

where $d\omega=\mathrm{vol}_{\tilde{S}^2}$ and q_{KK} is the KK-monopole charge. The warp factors of the background system are taken to be $H_{\mathrm{KK}}=H_{\mathrm{KK}}(r)$ and $H_{\mathrm{D3}}=H_{\mathrm{D3}}(y,z,r)$. Besides, the defect branes are taken to have charge distributions localised within the worldvolume of the D3 branes. In other words, they only depend on ρ and read $H_{\mathrm{D1}}(\rho)$, $H_{\mathrm{F1}}(\rho)$, $H_{\mathrm{NS5}}(\rho)$ and $H_{\mathrm{D5}}(\rho)$.

As usual, the equations of motion and Bianchi identities decouple into two groups of PDEs. Those for the D1-F1-D5-NS5 defect branes are given by

$$
\begin{aligned}
\nabla^2_{\mathbb{R}^3_\rho}H_{\mathrm{D1}}=0 \quad \text{with} \quad H_{\mathrm{NS5}}=H_{\mathrm{D1}},\\
\nabla^2_{\mathbb{R}^3_\rho}H_{\mathrm{F1}}=0 \quad \text{with} \quad H_{\mathrm{D5}}=H_{\mathrm{F1}},
\end{aligned}
\tag{7.13}
$$

while for the D3-KK bound system we have

$$
\nabla^2_{\mathbb{R}^3_r}H_{\mathrm{D3}}+H_{\mathrm{KK}}\nabla^2_{\mathbb{R}^2_{(y,z)}}H_{\mathrm{D3}}=0 \quad \text{with} \quad H_{\mathrm{KK}}=\frac{q_{\mathrm{KK}}}{2r}. \tag{7.14}
$$

Mirroring what we did in section 7.1, we are interested in studying the near-horizon limit of the solution at hand. In order to achieve this, we consider the particular solution for the defect branes,

$$
H_{\mathrm{D1}}=1+\frac{q_{\mathrm{D1}}}{\rho} \quad \text{and} \quad H_{\mathrm{F1}}=1+\frac{q_{\mathrm{F1}}}{\rho}, \tag{7.15}
$$

where q_{D1} and q_{F1} are integration constants related to the quantised charges of the respective branes. As in the Type IIA case, the $\rho\to+\infty$ limit corresponds the situation in which the defect branes are taken far away from D3-KK, recovering the background associated to this latter subsystem. In turn, the $\rho\to 0$ limit gives rise to a new class of

$\mathcal{N}=4$ AdS$_2$ backgrounds[4] of Type IIB string theory,

$$\begin{aligned}
ds^2_{10} =& q_{\rm D1}^{3/2}q_{\rm F1}^{1/2}H_{\rm D3}^{-1/2}\left(ds^2_{\rm AdS_2}+ds^2_{S^2}\right)+q_{\rm D1}^{1/2}q_{\rm F1}^{-1/2}H_{\rm D3}^{1/2}\left(dy^2+dz^2\right)\\
&+q_{\rm D1}^{1/2}q_{\rm F1}^{-1/2}H_{\rm D3}^{1/2}\left(H_{\rm KK}^{-1}\left(d\psi+2^{-1}q_{\rm KK}\omega\right)^2+H_{\rm KK}\left(dr^2+r^2ds^2_{\tilde{S}^2}\right)\right)\,,\\
e^{\Phi} =& q_{\rm D1}q_{\rm F1}^{-1}\,,\qquad H_{(3)}=-q_{\rm D1}{\rm vol}_{\rm AdS_2}\wedge dz-q_{\rm D1}{\rm vol}_{S^2}\wedge dy\,,\\
F_{(3)} =& -q_{\rm F1}{\rm vol}_{\rm AdS_2}\wedge dy+q_{\rm F1}{\rm vol}_{S^2}\wedge dz\,,\\
F_{(5)} =& q_{\rm D1}^2q_{\rm F1}^2\partial_rH_{\rm D3}^{-1}\,{\rm vol}_{\rm AdS_2}\wedge{\rm vol}_{S^2}\wedge dr+q_{\rm D1}^2q_{\rm F1}^2\partial_yH_{\rm D3}^{-1}\,{\rm vol}_{\rm AdS_2}\wedge{\rm vol}_{S^2}\wedge dy\\
&+q_{\rm D1}^2q_{\rm F1}^2\partial_zH_{\rm D3}^{-1}\,{\rm vol}_{\rm AdS_2}\wedge{\rm vol}_{S^2}\wedge dz-r^2\partial_rH_{\rm D3}dy\wedge dz\wedge d\psi\wedge{\rm vol}_{\tilde{S}^2}\\
&-H_{\rm KK}r^2\partial_zH_{\rm D3}dy\wedge d\psi\wedge dr\wedge{\rm vol}_{\tilde{S}^2}+H_{\rm KK}r^2\partial_yH_{\rm D3}dz\wedge d\psi\wedge dr\wedge{\rm vol}_{\tilde{S}^2}\,,
\end{aligned}\tag{7.16}$$

where $H_{\rm D3}$ solves the master equation (7.14). These geometries represent a new class of solutions to Type IIB supergravity, displaying AdS$_2\times S^2\times\tilde{S}^2\times\mathbb{R}^2\times S^1$ foliations over a line. A simple check shows that these backgrounds are related by T-duality along the ψ direction to the AdS$_2$ one in (7.5), which describes the near-horizon of the F1-D2-D4′-NS5′-D4-NS5 brane intersections discussed in section 7.1.

7.2.2. Defects within 4d SCFTs in Type IIB

In this subsection we draw inspiration from subsection 7.1.2 to provide a defect interpretation for the background (7.16). As in the aforementioned subsection, the key feature that allows to find such an interpretation is the decoupling between the dynamics of the D1-F1-D5-NS5 defect branes and that of the D3-KK system. In other words, the equations of motion and Bianchi identities of the two groups of branes, given by (7.13) and (7.14), respectively, are completely independent. Since we are searching for a possible completion within an AdS$_5$ vacuum, we choose the semi-localised solution for the D3-KK system [157],

$$H_{\rm D3}=1+\frac{4\pi q_{\rm D3}q_{\rm KK}}{(y^2+z^2+2q_{\rm KK}r)^2}\qquad\text{and}\qquad H_{\rm KK}=\frac{q_{\rm KK}}{2r}\,.\tag{7.17}$$

and introduce the new coordinates (μ,α,ϕ) [155, 159],

$$y=\mu\sin\alpha\cos\phi\,,\qquad z=\mu\sin\alpha\sin\phi\qquad\text{and}\qquad r=2^{-1}\,q_{\rm KK}^{-1}\,\mu^2\cos^2\alpha\,.\tag{7.18}$$

With these considerations in mind, the backgrounds (7.16) take the form of a stack of D3-branes wrapping the AdS$_2\times S^2$ backreacted geometry. The D3-branes are intersected with n KK-monopoles, orbifolding the S^5 transverse space into $S^5/\mathbb{Z}_n$,

$$\begin{aligned}
ds^2_{10} =& q_{\rm D1}^{3/2}q_{\rm F1}^{1/2}H_{\rm D3}^{-1/2}\left(ds^2_{\rm AdS_2}+ds^2_{S^2}\right)+q_{\rm D1}^{1/2}q_{\rm F1}^{-1/2}H_{\rm D3}^{1/2}\left(d\mu^2+\mu^2ds^2_{S^5/\mathbb{Z}_n}\right)\,,\\
ds^2_{S^5/\mathbb{Z}_n} =& d\alpha^2+s^2d\phi^2+c^2ds^2_{S^3/\mathbb{Z}_n}\,,\qquad H_{\rm D3}=1+\frac{4\pi q_{\rm KK}q_{\rm D3}}{\mu^4}\,.
\end{aligned}\tag{7.19}$$

[4] In order to obtain a unitary AdS$_2$ at the horizon we rescaled the time as $t\to q_{\rm D1}q_{\rm F1}t$.

Analogously to what happened in the Type IIA case, the defect branes do not modify the space transverse to the D3-branes. Thus, the defect branes are only manifest by curving the worldvolume of the D3-branes in the fully-backreacted $\mathrm{AdS}_2 \times S^2$ geometry. Interestingly, in these coordinates the geometry of this background locally asymptotes to $\mathrm{AdS}_5 \times S^5/\mathbb{Z}_n$ in the $\mu \to 0$ limit,

$$
\begin{aligned}
ds_{10}^2 =& (4\pi q_{\mathrm{KK}} q_{\mathrm{D3}})^{1/2} q_{\mathrm{D1}}^{1/2} q_{\mathrm{F1}}^{-1/2} \overbrace{\left[(4\pi q_{\mathrm{KK}} q_{\mathrm{D3}})^{-1} q_{\mathrm{D1}}\, q_{\mathrm{F1}} \mu^2 \left(ds^2_{\mathrm{AdS}_2} + ds^2_{S^2}\right) + \frac{d\mu^2}{\mu^2}\right]}^{\text{locally AdS}_5\text{ geometry}} \\
&+ (4\pi q_{\mathrm{KK}} q_{\mathrm{D3}})^{1/2} q_{\mathrm{D1}}^{1/2} q_{\mathrm{F1}}^{-1/2} \left[d\alpha^2 + s^2 d\phi^2 + c^2 ds^2_{S^3/\mathbb{Z}_k}\right], \\
e^{\Phi} =& q_{\mathrm{D1}} q_{\mathrm{F1}}^{-1}, \\
H_{(3)} =& -q_{\mathrm{D1}} \left(\tilde{s}\mathrm{vol}_{\mathrm{AdS}_2} + \tilde{c}\mathrm{vol}_{S^2}\right) \wedge (s d\mu + \mu c d\alpha) - q_{\mathrm{D1}} \mu s \left(\tilde{c}\mathrm{vol}_{\mathrm{AdS}_2} - \tilde{s}\mathrm{vol}_{S^2}\right) \wedge d\phi, \\
F_{(3)} =& q_{\mathrm{F1}} \left(-\tilde{c}\mathrm{vol}_{\mathrm{AdS}_2} + \tilde{s}\mathrm{vol}_{S^2}\right) \wedge (s d\mu + \mu c d\alpha) + q_{\mathrm{F1}} \mu s \left(\tilde{s}\mathrm{vol}_{\mathrm{AdS}_2} + \tilde{c}\mathrm{vol}_{S^2}\right) \wedge d\phi, \\
F_{(5)} =& 4 q_{\mathrm{F1}}^2 q_{\mathrm{D1}}^2 (4\pi q_{\mathrm{KK}} q_{\mathrm{D3}})^{-1} \mu^3 \mathrm{vol}_{\mathrm{AdS}_2} \wedge \mathrm{vol}_{S^2} \wedge d\mu \\
&- 4\pi q_{\mathrm{D3}} c^3 s d\phi \wedge d\alpha \wedge d\psi \wedge \mathrm{vol}_{\tilde{S}^2},
\end{aligned}
\tag{7.20}
$$

where $s = \sin\alpha$, $c = \cos\alpha$ and $\tilde{s} = \sin\phi$, $\tilde{c} = \cos\phi$.

In complete analogy with the Type IIA analysis of section 7.1, the constructions in this section show that starting with the general brane intersection specified by the solutions (7.12) and taking the particular profiles (7.15) for the D1-F1-D5-NS5 branes and the semi-localised profile (7.17) for the D3-KK system, two interesting regimes emerge. The first regime appears in the $\rho \to 0$ limit. In this case the defect branes are resolved into a fully backreacted $\mathrm{AdS}_2 \times S^2$ geometry within the 4d worldvolume of the D3-KK bound state, making manifest the breaking of its isometries. The second regime becomes manifest in the system of coordinates introduced in (7.18), when apart from taking the $\rho \to 0$ limit one also sends $\mu \to 0$, thus approaching the origin of the $\mathbb{R}^2_{(y,z)}$ plane. In this regime the metric is split into a 5d "external" part, reproducing locally an AdS_5 geometry, and a 5d internal manifold, which coincides with that of the pure $\mathrm{AdS}_5 \times S^5/\mathbb{Z}_n$ vacuum [159]. The isometries of the AdS_5 vacuum are however broken by the background fluxes in (7.20). The extra terms show that a 5d observer placed at $\mu \to 0$ feels the global charges of the defect branes. They backreact into a geometry described by a 5d curved domain wall with $\mathrm{AdS}_2 \times S^2$ slicings that is only asymptotically locally AdS_5. We observe that the presence of the extra terms in the fluxes forbids any supersymmetric enhancement to the (four-dimensional) $\mathcal{N} = 2$ supersymmetry of the AdS_5 solution.

Similarly to what we obtained in subsection 7.1.2, our construction realises a conformal line defect in 4d $\mathcal{N} = 4$ SYM modded by $\mathbb{Z}_n$, this time in terms of D1-F1-D5-NS5 branes. Nevertheless, the Type IIB realisation allows one to study the interesting case in which these defect branes are introduced within 4d $\mathcal{N} = 4$ SYM, breaking the supersymmetries to 1/4 BPS. In this case it should be possible to interpret the D5- and the F1-branes as realising the baryon vertex of 4d $\mathcal{N} = 4$ SYM, and the AdS_2 solutions as describing the corresponding backreacted geometries. In the IR the gauge symmetry on the D3-branes would become global, turning them into flavour branes, with the D5-branes becoming the new colour branes. However, we highlight that in the backreacted geometry there are as well D1 colour branes. These should find an interpretation in terms of instantons within the worldvolume of the D5-branes. The possibility of such an interpretation will become clearer after our field theory analysis in the next section.

7.3. The F1-D2-D4′-NS5′-D4-NS5-D6 brane system

In this section we generalise the F1-D2-D4′-NS5′-D4-NS5 brane intersection studied in section 7.1 to include D6-branes localised within the $\mathbb{R}^2_{(y,z)}$ plane. We will see that adding D6-branes challenges the construction of a solution with AdS$_5$ asymptotics. However, by taking a simplified ansatz, we will see that it is possible to construct an explicit quiver quantum mechanics that can be interpreted as describing D2-D4′ baryon vertices within the 4d SCFT living in the D4-NS5-D6 brane intersection.

7.3.1. Adding D6-branes to the brane set-up

The brane set-up thoroughly explored in subsection 7.1.1 can be extended by including D6-branes localised within the $\mathbb{R}^2_{(y,z)}$ plane. The result is the extended brane set-up depicted in Table 7.3. This generalisation does not imply any further breaking of supersymmetries. However, as we mentioned above, finding an AdS$_5$ completion is a more difficult task in this case.

branes	t	ρ	φ^1	φ^2	y	z	ψ	r	θ^1	θ^2
D6	×	×	×	×	–	–	–	×	×	×
D4	×	×	×	×	–	–	×	–	–	–
NS5	×	×	×	×	×	×	–	–	–	–
D2	×	–	–	–	×	–	×	–	–	–
F1	×	–	–	–	–	×	–	–	–	–
D4′	×	–	–	–	×	–	–	×	×	×
NS5′	×	–	–	–	–	×	×	×	×	×

Table 7.3: BPS/8 intersection describing F1-D2-D4′-NS5′ branes ending on a D4-NS5-D6 bound state.

For simplicity, we consider that the D6-branes are smeared along the ψ direction. Consequently, the associated warp factor is $H_{\mathrm{D6}} = H_{\mathrm{D6}}(y, z)$. The resulting background is given by the following fields,

$$
\begin{aligned}
ds^2_{10} =& H_{\mathrm{D6}}^{-1/2} H_{\mathrm{D4}}^{-1/2}\left[-H_{\mathrm{F1}}^{-1} H_{\mathrm{D2}}^{-1/2} H_{\mathrm{D4'}}^{-1/2}\, dt^2 + H_{\mathrm{D2}}^{1/2} H_{\mathrm{D4'}}^{1/2} H_{\mathrm{NS5'}}\big(d\rho^2+\rho^2 ds^2_{S^2}\big)\right]+\\
&+ H_{\mathrm{D6}}^{1/2} H_{\mathrm{D4}}^{1/2}\left[H_{\mathrm{D2}}^{-1/2} H_{\mathrm{D4'}}^{-1/2} H_{\mathrm{NS5'}}\, dy^2 + H_{\mathrm{F1}}^{-1} H_{\mathrm{D2}}^{1/2} H_{\mathrm{D4'}}^{1/2}\, dz^2\right]+\\
&+ H_{\mathrm{D6}}^{1/2} H_{\mathrm{NS5}} H_{\mathrm{D4}}^{-1/2} H_{\mathrm{D2}}^{-1/2} H_{\mathrm{D4'}}^{1/2}\, d\psi^2+\\
&+ H_{\mathrm{D6}}^{-1/2} H_{\mathrm{NS5}} H_{\mathrm{D4}}^{1/2} H_{\mathrm{D2}}^{1/2} H_{\mathrm{D4'}}^{-1/2}\big(dr^2+r^2 ds^2_{\tilde{S}^2}\big)\,,\\
e^{\Phi} =& H_{\mathrm{D6}}^{-3/4} H_{\mathrm{NS5}}^{1/2} H_{\mathrm{D4}}^{-1/4} H_{\mathrm{F1}}^{-1/2} H_{\mathrm{D2}}^{1/4} H_{\mathrm{D4'}}^{-1/4} H_{\mathrm{NS5'}}^{1/2}\,,\\
H_{(3)} =& -\partial_\rho H_{\mathrm{F1}}^{-1} dt\wedge d\rho\wedge dz + \partial_\rho H_{\mathrm{NS5'}}\,\rho^2\,\mathrm{vol}_{S^2}\wedge dy + \partial_r H_{\mathrm{NS5}}\, r^2\, d\psi\wedge \mathrm{vol}_{\tilde{S}^2}\,,\\
F_{(2)} =& -H_{\mathrm{F1}} H_{\mathrm{D2}}^{-1}\partial_z H_{\mathrm{D6}} dy\wedge d\psi + H_{\mathrm{D4'}} H_{\mathrm{NS5'}}^{-1}\partial_y H_{\mathrm{D6}} dz\wedge d\psi+\,,\\
F_{(4)} =& H_{\mathrm{D6}}\partial_\rho H_{\mathrm{D2}}^{-1} dt\wedge d\rho\wedge dy\wedge d\psi + H_{\mathrm{D6}}\partial_\rho H_{\mathrm{D4'}}\,\rho^2\,\mathrm{vol}_{S^2}\wedge dz\wedge d\psi+\\
&+ H_{\mathrm{D2}} H_{\mathrm{NS5'}}^{-1} H_{\mathrm{NS5}}\partial_y H_{\mathrm{D4}}\, r^2\, dz\wedge dr\wedge \mathrm{vol}_{\tilde{S}^2}\\
&- H_{\mathrm{F1}} H_{\mathrm{D4'}}^{-1} H_{\mathrm{NS5}}\partial_z H_{\mathrm{D4}}\, r^2\, dy\wedge dr\wedge \mathrm{vol}_{\tilde{S}^2} + H_{\mathrm{D6}}\partial_r H_{\mathrm{D4}}\, r^2\, dy\wedge dz\wedge \mathrm{vol}_{\tilde{S}^2}\,,\\
F_{(6)} =& H_{\mathrm{D4'}} H_{\mathrm{NS5'}}\rho^2\Big(\partial_y H_{\mathrm{D4}}^{-1} dy + \partial_z H_{\mathrm{D4}}^{-1} dz + \partial_r H_{\mathrm{D4}}^{-1} dr\Big)\wedge dt\wedge d\rho\wedge \mathrm{vol}_{S^2}\wedge d\psi+\\
&- H_{\mathrm{NS5}} H_{\mathrm{D4}}\partial_\rho H_{\mathrm{D2}} r^2\rho^2 \mathrm{vol}_{S^2}\wedge dz\wedge dr\wedge \mathrm{vol}_{\tilde{S}^2}+\\
&- H_{\mathrm{D4}} H_{\mathrm{NS5}} r^2\partial_\rho H_{\mathrm{D4'}}^{-1} dt\wedge d\rho\wedge dy\wedge dr\wedge \mathrm{vol}_{\tilde{S}^2}\,.
\end{aligned}
\tag{7.21}
$$

As in the absence of D6-branes, the equations of motion and Bianchi identities decouple into two groups of PDEs, one associated to the F1-D2-D4′-NS5′ defect branes,

$$
\nabla^2_{\mathbb{R}^3_\rho} H_{\mathrm{D2}} = 0 \quad \text{with} \quad H_{\mathrm{D4'}} = H_{\mathrm{NS5'}} = H_{\mathrm{F1}} = H_{\mathrm{D2}}\,, \tag{7.22}
$$

and an independent one describing the D4-NS5-D6 brane system,

$$
H_{\mathrm{D6}}\nabla^2_{\mathbb{R}^3_r} H_{\mathrm{D4}} + H_{\mathrm{NS5}}\nabla^2_{\mathbb{R}^2_{(y,z)}} H_{\mathrm{D4}} = 0\,, \quad \nabla^2_{\mathbb{R}^3_r} H_{\mathrm{NS5}} = 0 \quad \text{and} \quad \nabla^2_{\mathbb{R}^2_{(y,z)}} H_{\mathrm{D6}} = 0\,. \tag{7.23}
$$

In order to extract the AdS_2 near-horizon geometry we make the following choice for the warp factor describing the defect branes,

$$
H_{\mathrm{D2}} = 1 + \frac{q_{\mathrm{D2}}}{\rho}\,, \tag{7.24}
$$

where q_{D2} is an integration constant related to the quantised charges of the defect branes.

We then obtain the near-horizon limit of the considered background by sending $\rho \to 0$[5],

$$
\begin{aligned}
ds^2_{10} =& q^2_{\text{D2}} H_{\text{D6}}^{-1/2} H_{\text{D4}}^{-1/2} \left(ds^2_{\text{AdS}_2} + ds^2_{S^2}\right) + H_{\text{D6}}^{1/2} H_{\text{D4}}^{1/2} \left(dy^2 + dz^2\right) \\
& + H_{\text{D6}}^{1/2} H_{\text{NS5}} H_{\text{D4}}^{-1/2} d\psi^2 + H_{\text{D6}}^{-1/2} H_{\text{NS5}} H_{\text{D4}}^{1/2} \left(dr^2 + r^2 ds^2_{\tilde{S}^2}\right) , \\
e^{\Phi} =& H_{\text{D6}}^{-3/4} H_{\text{NS5}}^{1/2} H_{\text{D4}}^{-1/4} , \\
H_{(3)} =& - q_{\text{D2}} \text{vol}_{\text{AdS}_2} \wedge dz - q_{\text{D2}} \text{vol}_{S^2} \wedge dy + \partial_r H_{\text{NS5}} \, r^2 \, d\psi \wedge \text{vol}_{\tilde{S}^2} , \\
F_{(2)} =& - \partial_z H_{\text{D6}} dy \wedge d\psi + \partial_y H_{\text{D6}} dz \wedge d\psi , \\
F_{(4)} =& q_{\text{D2}} H_{\text{D6}} (\text{vol}_{\text{AdS}_2} \wedge dy - \text{vol}_{S^2} \wedge dz) \wedge d\psi + H_{\text{D6}} \partial_r H_{\text{D4}} \, r^2 \, dy \wedge dz \wedge \text{vol}_{\tilde{S}^2} \\
& + H_{\text{NS5}} \partial_y H_{\text{D4}} \, r^2 \, dz \wedge dr \wedge \text{vol}_{\tilde{S}^2} + H_{\text{NS5}} \partial_z H_{\text{D4}} \, r^2 \, dy \wedge dr \wedge \text{vol}_{\tilde{S}^2} , \\
F_{(6)} =& q^4_{\text{D2}} \Big(\partial_y H_{\text{D4}}^{-1} dy + \partial_z H_{\text{D4}}^{-1} dz + \partial_r H_{\text{D4}}^{-1} dr \Big) \wedge \text{vol}_{\text{AdS}_2} \wedge \text{vol}_{S^2} \wedge d\psi \\
& + q_{\text{D2}} H_{\text{D4}} H_{\text{NS5}} r^2 \Big(-\text{vol}_{\text{AdS}_2} \wedge dy + \text{vol}_{S^2} \wedge dz \Big) \wedge dr \wedge \text{vol}_{\tilde{S}^2} .
\end{aligned}
\tag{7.25}
$$

These fields define a new class of $\mathcal{N} = 4$ $\text{AdS}_2 \times S^2 \times \tilde{S}^2 \times \mathbb{R}^2 \times S^1$ geometries fibred over a line, parametrised by r. The functions $H_{\text{D4}}(y, z, r)$, $H_{\text{NS5}}(r)$ and $H_{\text{D6}}(y, z)$ are solutions to the equations in (7.23), and describe a D4-NS5-D6 bound state localised in the subspace $\mathbb{R}^2_{(y,z)} \times \mathbb{R}^3_r$, where $\mathbb{R}^3_r$ is spanned by I_r and $\tilde{S}^2$. A solution in the class given by (7.25) is thus specified by the D4-NS5-D6 charge distributions that solve these equations. The explicit study of the solutions becomes more involved than in section 7.1, due to the logarithmic behaviour of H_{D6}. Moreover, the defect interpretation found in that section in terms of semi-localised D4-NS5 branes is lost in the presence of the D6-branes, making challenging the construction of an explicit solution that asymptotes to an AdS_5 vacuum.

In order to proceed further with an explicit analysis of the solutions we take the y direction inside the $\mathbb{R}^2_{(y,z)}$ plane as a circular direction, and the D6- and the D4-branes smeared along it. Under this assumption the background (7.25) turns out to be driven by the functions $H_{\text{D4}}(z)$, $H_{\text{NS5}}(r)$ and $H_{\text{D6}}(z)$, and their master PDEs take the following simpler form,

$$
\partial_z^2 H_{\text{D4}} = 0 \, , \quad \partial_z^2 H_{\text{D6}} = 0 \quad \text{with} \quad H_{\text{NS5}} = \frac{q_{\text{NS5}}}{r} \, . \tag{7.26}
$$

Even if in this case we could not construct a solution with AdS_5 asymptotics, we show in the next subsections that it is possible to interpret a wide subclass of these solutions in terms of D2-D4′ baryon vertices within the 4d CFT living in D4-NS5-D6 branes.

7.3.2. Quantised charges

The most general solution to the equations of motion defined by (7.26) consists on H_{D4} and H_{D6} being piecewise linear functions of z. This allows one to introduce D4 and D6 source branes in the geometry. This is compatible with the quantised charges obtained from the Page fluxes, as we show below.

[5] In order to reproduce unitary AdS_2 at the horizon we rescaled the time as $t \to q^2_{\text{D2}} t$.

We start by computing the F1-strings charge. The F1-strings are electrically charged with respect to the NSNS 3-form. Their quantised charges in units with $\alpha' = g_s = 1$ take the following form[6],

$$Q^e_{F1} = \frac{1}{(2\pi)^2}\int_{\mathrm{AdS}_2\times I_z} H_{(3)}\,. \tag{7.27}$$

Regularising the volume of AdS_2 as $\mathrm{Vol}_{\mathrm{AdS}_2} = 4\pi$[7] we find that

$$Q^e_{F1} = \frac{1}{\pi} q_{\mathrm{D2}}(z_f - z_i)\,, \tag{7.28}$$

where this must be computed at both ends of the I_z interval. Therefore, there are $k\,q_{\mathrm{D2}}$ F1-strings in the $z\in[0,k\pi]$ interval. We set $q_{\mathrm{D2}}=1$ for simplicity, so that one F1-string is created as we move in z-intervals of length π. This is equivalent to imposing that $B_{(2)}$ lies in the fundamental region when it is integrated over AdS_2 (see [43]),

$$\frac{1}{4\pi^2}\left|\int_{\mathrm{AdS}_2} B_{(2)}\right| \in [0,1]\,, \tag{7.29}$$

therefore we must take

$$B^e_{(2)} = -(z-k\pi)\mathrm{vol}_{\mathrm{AdS}_2} \quad \text{with} \quad z\in[k\pi,(k+1)\pi] \tag{7.30}$$

for the electric part of $B_{(2)}$. The large gauge transformation parameter k affects the electric components of the RR Page fluxes, $\hat{F} = F\wedge e^{-B_{(2)}}$, as follows,

$$\hat{F}_{(p)} \to \hat{F}_{(p)} - k\pi F_{(p-2)}\wedge \mathrm{vol}_{\mathrm{AdS}_2} \quad \text{for} \quad z\in[k\pi,(k+1)\pi]\,. \tag{7.31}$$

In particular, we have that

$$\begin{aligned}\hat{F}^e_{(4)} &= \Big(H_{\mathrm{D6}} - (z-k\pi)\partial_z H_{\mathrm{D6}}\Big)\mathrm{vol}_{\mathrm{AdS}_2}\wedge dy\wedge d\psi\,,\\ \hat{F}^e_{(6)} &= -H_{\mathrm{NS5}} r^2\Big(H_{\mathrm{D4}} - (z-k\pi)\partial_z H_{\mathrm{D4}}\Big)\mathrm{vol}_{\mathrm{AdS}_2}\wedge dr\wedge \mathrm{vol}_{\tilde{S}^2}\end{aligned} \tag{7.32}$$

for $z\in[k\pi,(k+1)\pi]$. These electric fluxes give rise to D2 and D4$'$ electric quantised charges, respectively. In units with $\alpha' = g_s = 1$, they can be computed by applying

$$Q^e_{\mathrm{D}p} = \frac{1}{(2\pi)^{p+1}}\int_{\mathrm{AdS}_2\times\Sigma_p} \hat{F}^e_{(p+2)}, \tag{7.33}$$

For the D4- and D6-branes it will be more convenient to compute their magnetic charges, associated to the magnetic components of $\hat{F}_{(4)}$ and $\hat{F}_{(2)}$, respectively, which are given by

$$\hat{F}^m_{(4)} = -H_{\mathrm{NS5}}\partial_z H_{\mathrm{D4}} r^2 dy\wedge dr\wedge \mathrm{vol}_{\tilde{S}^2}\,, \quad \hat{F}^m_{(2)} = -\partial_z H_{\mathrm{D6}} dy\wedge d\psi\,. \tag{7.34}$$

[6] We use the superscript e to explicitly indicate that this is an electric charge.

[7] This regularisation prescription is based on the analytical continuation that relates the AdS_2 space with an S^2.

In the presence of sources the Bianchi identities in equation (7.26) are modified so that

$$d\hat{F}_{(4)} = \partial_z^2 H_{\rm D4} dz \wedge dy \wedge dr \wedge \text{vol}_{\tilde{S}^2}\,, \quad d\hat{F}_{(2)} = \partial_z^2 H_{\rm D6} dz \wedge dy \wedge d\psi\,. \tag{7.35}$$

Therefore, the D4- and D6-branes provide sources localised in the z direction. They are thus flavour branes. They span, respectively, the $\text{AdS}_2 \times S^2 \times S^1_\psi$ and $\text{AdS}_2 \times S^2 \times I_r \times \tilde{S}^2$ subspaces of the solution (see below).

We need to specify now the linear functions $H_{\rm D4}$ and $H_{\rm D6}$. We take both to be piecewise linear in the different $z \in [k\pi, (k+1)\pi]$ intervals, with the space starting and ending at $z = 0$ and $z = \pi(P+1)$, where both $H_{\rm D4}$ and $H_{\rm D6}$ are assumed to vanish. This parallels the analysis performed in [42, 43] for the AdS$_2$ solutions presented in said works, which, in turn, was based on the field theory interpretation in [94, 160] of the AdS$_3$ solutions constructed in [38][8]. In this way the singularity structure at both ends of the space is given by

$$ds^2 \sim x^{-1}(ds^2_{\text{AdS}_2} + ds^2_{S^2}) + x(dy^2 + dx^2) + d\psi^2 + dr^2 + r^2 ds^2_{\tilde{S}^2}\,, \quad e^\Phi \sim x^{-1}\,, \tag{7.36}$$

where $x = z$ close to $z = 0$ and $x = \pi(P+1) - z$ close to $z = \pi(P+1)$. It corresponds to a superposition of D4-branes wrapped on $\text{AdS}_2 \times S^2 \times S^1_\psi$ and smeared on $(y, r, \tilde{S}^2)$, and D6 branes wrapped on $\text{AdS}_2 \times S^2 \times I_r \times \tilde{S}^2$ and smeared on (ψ, y)[9]. The $H_{\rm D4}$ and $H_{\rm D6}$ functions then read

$$H_{\rm D4}(z) = \begin{cases} \frac{\beta_0}{\pi} z & \text{with} \quad 0 \leq z \leq \pi\,, \\ \alpha_k + \frac{\beta_k}{\pi}(z - \pi k) & \text{with} \quad \pi k \leq z \leq \pi(k+1)\,, \quad k = 1, .., P-1\,, \\ \alpha_P - \frac{\alpha_P}{\pi}(z - \pi P) & \text{with} \quad \pi P \leq z \leq \pi(P+1), \end{cases} \tag{7.37}$$

$$H_{\rm D6}(z) = \begin{cases} \frac{\nu_0}{\pi} z & \text{with} \quad 0 \leq z \leq \pi\,, \\ \mu_k + \frac{\nu_k}{\pi}(z - \pi k) & \text{with} \quad \pi k \leq z \leq \pi(k+1)\,, \quad k = 1, ..., P-1\,, \\ \mu_P - \frac{\mu_P}{\pi}(z - \pi P) & \text{with} \quad \pi P \leq z \leq \pi(P+1)\,, \end{cases} \tag{7.38}$$

where α_k, β_k, μ_k and ν_k are integration constants. They must satisfy the following condition (see [43]),

$$\alpha_k = \sum_{j=0}^{k-1} \beta_j, \quad \mu_k = \sum_{j=0}^{k-1} \nu_j \quad \text{with} \quad k = 1, \ldots, P \tag{7.39}$$

which comes from the requirement of continuity of $H_{\rm D4}$ and $H_{\rm D6}$. Their continuity ensures, in turn, that of the metric and dilaton. However, the fluxes can have discontinuities, which give away the presence of branes.

Note that in order to find well-defined quantised charges from the electric and magnetic fluxes in (7.32) and (7.34), we need to globally define the r-direction. This is done

[8]One can check [161], a review article which summarises these developments.

[9]Note that the same behaviour is obtained from a superposition of O4 and O6 orientifold fixed planes.

by taking the $I_r \times \tilde{S}^2$ space to span a 3-torus T^3. The resulting quantised charges are given by

$$Q_{\mathrm{D2}}^{e(k)} = \mu_k\,, \quad Q_{\mathrm{D4}'}^{e(k)} = \alpha_k\,, \quad Q_{\mathrm{F1}}^{e(k)} = 1\,, \quad Q_{\mathrm{D4}}^{m(k)} = \beta_k\,, \quad Q_{\mathrm{D6}}^{m(k)} = \nu_k \tag{7.40}$$

in the different $z \in [k\pi, (k+1)\pi]$ intervals. Here we have also used that $y \in [0, \pi]$[10]. The charges in (7.40) show that the integration constants α_k, β_k, μ_k and ν_k must be integer numbers. Moreover, the conditions (7.39) imply that the D2 and D4′ charges at each $z \in [k\pi, (k+1)\pi]$ interval equal the sum of the D6 and D4 charges, respectively, in the previous $[0, k\pi]$ intervals. We will find an explanation for this fact when we give a field theory interpretation to these solutions in the next subsections. The number of D4 and that of D6 source branes present at each interval are given, respectively, by

$$F_k = \beta_{k-1} - \beta_k\,, \quad \tilde{F}_k = \nu_{k-1} - \nu_k, \tag{7.41}$$

consistently with the derivatives

$$\partial_z^2 H_{\mathrm{D4}} = \frac{1}{\pi}\sum_{k=1}^{P}(\beta_{k-1} - \beta_k)\delta(z - \pi k)\,, \qquad \partial_z^2 H_{\mathrm{D6}} = \frac{1}{\pi}\sum_{k=1}^{P}(\nu_{k-1} - \nu_k)\delta(z - \pi k)\,, \tag{7.42}$$

which follow from (7.37) and (7.38).

Finally, we would like to briefly discuss the role played by the NS5- and NS5′-branes in the brane set-up. As we have discussed, the D2- and D4′-branes play the role of colour branes of the configuration. The D2-branes are wrapped on the two circular directions (y, ψ). Furthermore, they are stretched in the y direction between NS5′-branes, which are located at $y = 0, \pi$ and are periodically identified, and between two NS5-branes[11] in the ψ direction, located at $\psi = 0, 2\pi$ and periodically identified. The field theory that lives in them is therefore one-dimensional at low energies. In turn, the D4′-branes are wrapped around y and the T^3. In the y direction, they extend between NS5′-branes. The field theory that lives in them is, therefore, also one-dimensional at low energies. Our conjecture is that the $\mathcal{N} = 4$ quantum mechanics that lives in the D2-D4′ branes flows to a super conformal quantum mechanics in the IR that is dual to the backgrounds defined by the functions (7.37), (7.38). We turn to this analysis in subsection 7.3.4, after discussing the holographic central charge in the next subsection.

7.3.3. Holographic central charge

Although, as mentioned before, the definition of a central charge for a conformal quantum mechanics is a subtle issue, there is a notion of holographic "central charge" that can be computed by applying the formula that we introduced in (3.20) to the supergravity

[10]The reason for this particular periodicity will become clear in the conclusions when we discuss the relation between these geometries and the double analytical continuation of the $\mathrm{AdS}_3 \times S^2$ geometries studied in [46].

[11]We take $q_{\mathrm{NS5}} = 1$ for simplicity.

solution. In the case of a Type IIA/B supergravity solution with two-dimensional external space this formula boils down to

$$c_{\text{hol}} = \frac{3}{4\pi G_N} \int d\vec{\theta} \sqrt{e^{-4\Phi} \det(g_{ij})}\,, \tag{7.43}$$

where G_N is the ten-dimensional Newton's constant, $G_N = 8\pi^6$, g_{ij} is the metric of the inner space and $\vec{\theta}$ are coordinates defined over it. For our class of backgrounds we find

$$c_{\text{hol}} = \frac{6}{\pi} \int_0^{\pi(P+1)} H_{\text{D4}} H_{\text{D6}}\, dz\,. \tag{7.44}$$

Substituting here the expressions for H_{D4} and H_{D6} given by (7.37) and (7.38) we arrive at the final result,

$$c_{\text{hol}} = 6 \sum_{k=0}^{P} \left(\alpha_k \mu_k + \frac{1}{3}\beta_k \nu_k + \frac{1}{2}(\alpha_k \nu_k + \beta_k \mu_k) \right)\,. \tag{7.45}$$

We will compare this value with the "central charge" of the superconformal quantum mechanics built in subsection 7.3.5.

7.3.4. Dual quiver quantum mechanics

In this subsection we propose quiver quantum mechanics supported by our solutions. The dynamics of the quivers is described in terms of the matter fields associated to the open strings that connect the different branes. We will follow closely the detailed description of the matter fields given in [43] (see Appendix B therein), since our brane system is related by two T-dualities (along the y and ψ directions) to the D0-D4-F1-D4′-D8 brane intersections studied there. As in that reference we will use 2d $\mathcal{N} = (0, 4)$ notation for the 1d $\mathcal{N} = 4$ matter fields.

As all D-branes are localised in the z-direction, strings stretched between branes in adjacent $[\pi k, \pi(k + 1)]$ intervals are long, thus describing massive states. Therefore, they will not contribute to the quiver quantum mechanics. We will discuss their role in the field theory in subsection 7.3.6. Therefore, the full Hilbert space of the system is given by the sum of the individual Hilbert spaces of the D2-D4′-D4-D6 branes in each $[\pi k, \pi(k+1)]$ interval, whose degrees of freedom we summarise next:

- D2-D2: Given that the D2 branes are wrapped on the y and ψ directions they are effectively point like. They contribute with a $\mathcal{N} = (4, 4)$ vector and a $\mathcal{N} = (4, 4)$ hypermultiplet in the adjoint.

- D4′-D4′: Given that the D4′ branes are wrapped on y and on the T^3 they are also effectively point like. They contribute as well with a $\mathcal{N} = (4, 4)$ vector and a $\mathcal{N} = (4, 4)$ hypermultiplet in the adjoint.

- D2-D4′: The D2-D4′ subsystem is related by two T-dualities to the D0-D4 system in [43]. They contribute with a $\mathcal{N} = (4, 4)$ hypermultiplet in the bifundamental representation of the two gauge groups.

- D2-D4: This subsystem is related by two T-dualities to the D0-D4′ system in [43]. They contribute with a $\mathcal{N}=(4,4)$ bifundamental twisted hypermultiplet.
- D2-D6: This is related by two T-dualities to D0-D8. They contribute with a $\mathcal{N}=(0,2)$ bifundamental Fermi multiplet.
- D4′-D4: This is related by two T-dualities to D4-D4′. They contribute with a $\mathcal{N}=(0,2)$ bifundamental Fermi multiplet.
- D4′-D6: This is related by two T-dualities to D4-D8. They contribute with a $\mathcal{N}=(4,4)$ bifundamental twisted hypermultiplet.

These multiplets were summarised in Table 7.4. Considering this table, along with the

String	Multiplet	Representation
D2-D2	$\mathcal{N}=(4,4)$ vector and $\mathcal{N}=(4,4)$ hyper	Adjoint
D4′-D4′	$\mathcal{N}=(4,4)$ vector and $\mathcal{N}=(4,4)$ hyper	Adjoint
D2-D4′	$\mathcal{N}=(4,4)$ hyper	Bifundamental
D2-D4	$\mathcal{N}=(4,4)$ twisted hyper	Bifundamental
D2-D6	$\mathcal{N}=(0,2)$ Fermi	Bifundamental
D4′-D4	$\mathcal{N}=(0,2)$ Fermi	Bifundamental
D4′-D6	$\mathcal{N}=(4,4)$ twisted hyper	Bifundamental

Table 7.4: Summary of the multiplets associated to the brane intersection in Table 7.3. The considered open strings end on the specified branes, which lie on the same $z \in [\pi k, \pi(k+1)]$ interval.

ranks of the gauge and flavour groups associated to the D2-D4′-D4-D6 branes in a given $z \in [\pi k, \pi(k+1)]$ interval, which are given by μ_k, α_k, $\beta_{k-1}-\beta_k$ and $\nu_{k-1}-\nu_k$, respectively (see equations (7.40) and (7.41)), we get the field content depicted in Figure 7.2. In this figure circles represent $\mathcal{N}=(4,4)$ vector multiplets, black lines $\mathcal{N}=(4,4)$ twisted hypermultiplets, grey lines $\mathcal{N}=(4,4)$ hypermultiplets and dashed lines $\mathcal{N}=(0,2)$ Fermi multiplets. We remark that this is the same quiver quantum mechanics discussed in [43] (see section 3.3 therein), now associated to a different brane system. As in said paper, the quantum mechanics will find an interpretation as Wilson lines (more specifically, baryon vertices) within a higher dimensional QFT, once we introduce the massive F1-strings stretched between the branes in the z-direction. Before turning to this description in subsection 7.3.6, we briefly address the computation of the quantum mechanical central charge.

7.3.5. Quantum mechanical central charge

In this subsection we address the computation of the central charge, following closely [43]. The usual caveats involved in the definition of a superconformal quantum mechan-

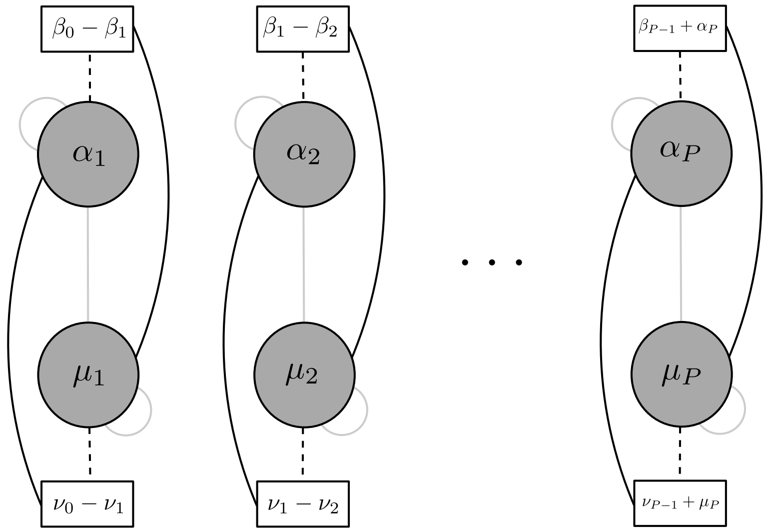


Figure 7.2: Disconnected quivers describing the SCQMs dual to the class of solutions (6.14).

ical central charge are present in our current set-up. The central charge should then be interpreted as counting the degeneracy of ground states of the system.

Following the reasoning presented in section 5.3, we arrived at the conclusion that expression (5.5) could be used to compute the dimension of the Higgs branch of the quivers depicted in Figure 7.2, as they are Kronecker. It is straightforward to check that for these quivers the dimension of the Higgs branch so obtained agrees with the central charge computed using (5.4), up to a factor of 1[12] and the global normalisation. The quantum mechanical central charge computed from (5.4) was shown to agree to leading order with the corresponding holographic expression in a number of examples [43].

We will thus use (5.4) to compute the central charge of our quantum mechanics described by the quivers in Figure 7.2. In this computation, as remarked in [49], n_{hyp} counts the number of ordinary (as opposed to twisted) $\mathcal{N}=4$ hypermultiplets, since the $\mathrm{U}(1)_R$-charge of the fermions in twisted hypermultiplets vanishes. For the case at hand, we have that

$$n_{hyp} = \sum_{k=1}^{P} (\alpha_k \mu_k + \alpha_k^2 + \mu_k^2), \qquad n_{vec} = \sum_{k=1}^{P} (\alpha_k^2 + \mu_k^2), \tag{7.46}$$

and, therefore,

$$c = 6 \sum_{k=1}^{P} \alpha_k \mu_k, \tag{7.47}$$

identically. Keeping in mind the definitions of α_k, μ_k, given by (7.39), we find that this expression agrees in the large number of nodes limit, i.e. $P \to \infty$, with the holographic expression (7.45). Moreover, the agreement is exact in the absence of any of the two types

[12]This factor of 1 is irrelevant in the holographic limit, but we are lacking a precise understanding of the origin of this discrepancy.

of flavour branes. It would be interesting to have a more precise understanding of this exact agreement.

7.3.6. Baryon vertex interpretation

In this subsection we explore the interpretation of the massive F1-strings. The discussion will again follow very closely the field theory interpretation given to the AdS_2 solutions constructed in [43]. The key point is to realise that the orientation between the D4- and the D4′-branes is such that one can consider F1-strings stretched between said branes, which is also the case for the D6- and D2-branes. These strings have as their lowest energy excitation a fermionic field, which upon integration leads to a Wilson loop.

In [113, 162] it was shown that a half-BPS Wilson loop in a $\mathrm{U}(N)$ antisymmetric representation of 4d $\mathcal{N}=4$ SYM can be described by an array of M D5-branes with fundamental strings dissolved in their worldvolumes. This is the realisation in the near-horizon limit of a configuration of M stacks of D5-branes separated a distance L from N D3-branes, with $(m_1, m_2, \ldots m_M)$ F1-strings stretched between the stacks. The brane set-up is depicted in Table 7.5. It can easily checked that this is precisely the relative

branes	t	x^1	x^2	x^3	x^4	x^5	x^6	x^7	x^8	x^9
D3	×	×	×	×	–	–	–	–	–	–
D5	×	–	–	–	–	×	×	×	×	×
F1	×	–	–	–	×	–	–	–	–	–

Table 7.5: Brane set-up associated to the D3-D5-F1 brane configuration that describes Wilson loops in antisymmetric representations of 4d $\mathrm{U}(N)$ $\mathcal{N}=4$ SYM.

orientation between the D4-, the F1- and the D4′-branes in Table 7.3 and the D6, the F1 and the D2 branes. Indeed, the couplings that describe Wilson lines in the worldvolumes of the D4′ and D2 colour branes are, respectively,

$$S_{\mathrm{D4}'} = T_4 \int \hat{F}_{(4)} \wedge A_t, \quad S_{\mathrm{D2}} = T_2 \int \hat{F}_{(2)} \wedge A_t\,. \tag{7.48}$$

In the first expression the D4′-branes are wrapped on y and the T^3, therefore they capture the $\hat{F}^m_{(4)}$ magnetic flux given in (7.34). In turn, the D2-branes are wrapped on y and ψ, so they capture the $\hat{F}^m_{(2)}$ magnetic flux. Substituting these fluxes in the $[\pi k, \pi(k+1)]$ z-interval we arrive at

$$S_{\mathrm{D4}'} = \beta_k T_{F1} \int dt A_t\,, \quad S_{\mathrm{D2}} = \nu_k T_{F1} \int dt A_t\,. \tag{7.49}$$

These expressions describe, respectively, β_k and ν_k Wilson lines. If we add now the contributions of the F1-strings stretched between the D4′-branes in the k-th interval and the D4-branes in all previous intervals, and the same for the D2-branes and the D6-branes, as

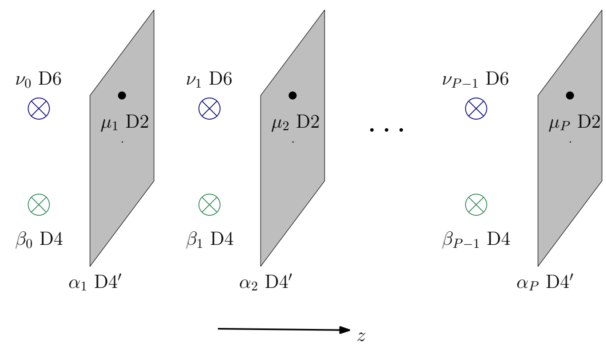


Figure 7.3: Hanany-Witten brane set-up associated to the quantised charges of the solutions.

depicted in Figure 7.3, we find Wilson lines in the $(\beta_0, \beta_1, \dots, \beta_{k-1})$ and $(\nu_0, \nu_1, \dots, \nu_{k-1})$ antisymmetric representations of the U(α_k) and U(μ_k) gauge groups. This is precisely the realisation of the baryon vertices associated to these gauge groups.

Indeed, the brane set-up depicted in Figure 7.3 can be related after the combination of a T-duality, an S-duality, successive Hanany-Witten moves and a further T-duality to the brane set-up depicted in Figure 7.4. This relation is carefully explained in [43]. For

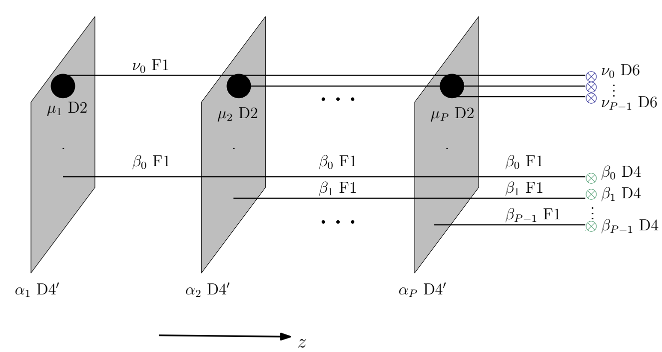


Figure 7.4: Hanany-Witten brane set-up equivalent to the brane configuration in Figure 7.3.

the D4′-F1-D4 brane subsystem it follows directly from the analysis of the D4′-F1-D4 brane system in [43][13], while for the D2-F1-D6 subsystem it follows from the analysis of the D0-F1-D8 subsystem therein after two T-dualities. The reader can find more details about this description in that reference.

Our previous description is consistent with an interpretation of the AdS$_2$ solutions given by (7.25), with the profiles specified by (7.37) and (7.38), as describing backreacted baryon vertices within the 4d $\mathcal{N} = 2$ CFT living in the D4-NS5-D6 branes. In this interpretation the SCQM arises in the very low energy limit of a D4-NS5-D6 brane system in which one-dimensional defects are introduced. The one-dimensional defects consist on D4′ baryon vertices, connected to the D4-branes with F1-strings, and D2-brane baryon vertices, connected to the D6 by F1-strings. In the IR the gauge symmetry on the D4-branes becomes global, turning them from colour to flavour branes. In turn, the D4′

[13]Note that the D4′- and the D4-branes are interchanged in that reference.

and D2 defect branes become the new colour branes of the backreacted geometry. This interpretation goes in parallel with the proposed defect interpretation for the classes of AdS_2 solutions found in [43] and [45]. Interestingly, for the first class of geometries the AdS_6 solution of Brandhuber-Oz [140] was shown to arise locally far away from the defect [34]. In our case we should be able to find the AdS_5 geometry dual to the D4-NS5-D6 brane intersection far away from the defect. This deservers further research.

7.4. A new class of $\mathrm{AdS}_2 \times S^2 \times S^2$ solutions to Type IIA via non-Abelian T-duality

In this section we take as our starting point the class of $\mathrm{AdS}_2 \times S^3 \times S^2$ solutions to Type IIB supergravity we presented in (7.16)[14]. In the previous subsection we explained how some evidence was gathered supporting the proposal that the line defect operators these solutions are dual to are baryon vertices in $\mathcal{N} = 4$ SYM. These operators would be realised in string theory as (p, q) strings stretched between stacks of (q, p) 5-branes and D3 colour branes, generalising the constructions in [113,114] by acting with $\mathrm{SL}(2, \mathbb{Z})$. We also showed in subsection 7.2.1 that these solutions are mapped through Abelian T-duality to $\mathrm{AdS}_2 \times S^2 \times S^2$ solutions to Type IIA supergravity arising in the near-horizon limit of F1-D2-D4′-NS5′ defect branes embedded in the Type IIA realisation of 4d $\mathcal{N} = 4$ SYM, namely the semi-localised D4-NS5 brane intersection studied in [93, 155–158, 163].

Now we want to construct more general AdS_2 geometries in Type IIA supergravity, admitting a similar holographic interpretation as line defects within 4d $\mathcal{N} = 2$ SCFTs dual to Gaiotto-Maldacena geometries. We obtain these new backgrounds by acting with non-Abelian T-duality on the class of $\mathrm{AdS}_2 \times S^3 \times S^2$ solutions to Type IIB supergravity constructed in (7.16). The new backgrounds are fibrations of $\mathrm{AdS}_2 \times S^2 \times S^2$ over four intervals and contain a particular solution that asymptotes locally to the Gaiotto-Maldacena geometry introduced in [164]. We will show that it is possible to give an explicit interpretation for this AdS_2 solution as a baryon vertex defect embedded in the 4d $\mathcal{N} = 2$ SCFT dual to the Gaiotto-Maldacena geometry studied in [93].

Let us go back to our results. We start by rewriting the class of solutions (7.16) in the absence of KK-monopoles as below[15],

$$
\begin{aligned}
ds^2_{10} =& q_{\mathrm{D1}}^{3/2} q_{\mathrm{F1}}^{1/2} H_{\mathrm{D3}}^{-1/2} \left(ds^2_{\mathrm{AdS}_2} + ds^2_{S^2} \right) + q_{\mathrm{D1}}^{1/2} q_{\mathrm{F1}}^{-1/2} H_{\mathrm{D3}}^{1/2} \left(dy^2 + dz^2 + dr^2 + r^2 ds^2_{S^3} \right) , \\
e^{\Phi} =& q_{\mathrm{D1}} q_{\mathrm{F1}}^{-1} , \quad H_3 = -q_{\mathrm{D1}} \mathrm{vol}_{\mathrm{AdS}_2} \wedge dz - q_{\mathrm{D1}} \mathrm{vol}_{S^2} \wedge dy , \\
F_3 =& - q_{\mathrm{F1}} \mathrm{vol}_{\mathrm{AdS}_2} \wedge dy + q_{\mathrm{F1}} \mathrm{vol}_{S^2} \wedge dz , \\
F_5 =& d[q_{\mathrm{D1}}^2 q_{\mathrm{F1}}^2 H_{\mathrm{D3}}^{-1} \mathrm{vol}_{\mathrm{AdS}_2} \wedge \mathrm{vol}_{S^2}] + \\
& + r^3 [(\partial_z H_{\mathrm{D3}} dy - \partial_y H_{\mathrm{D3}} dz) \wedge dr - \partial_r H_{\mathrm{D3}} dy \wedge dz] \wedge \mathrm{vol}_{S^3} ,
\end{aligned}
\tag{7.50}
$$

[14] In the next sections we restrict ourselves to the $n = 1$ case. This does not affect the number of preserved supersymmetries.

[15] We have substituted the r coordinate in (7.16) as $r \to (2q_{\mathrm{KK}})^{-1} r^2$ for convenience and then set $q_{\mathrm{KK}} = 1$ in order to obtain the solution at hand.

with $H_{\rm D3}$ satisfying the master equation

$$\nabla^2_{\mathbb{R}^4_r} H_{\rm D3} + \nabla^2_{\mathbb{R}^2_{(y,z)}} H_{\rm D3} = 0\,. \tag{7.51}$$

These solutions arise in the near-horizon limit of the brane intersection depicted in Table 7.6, and preserve small $\mathcal{N}=4$ supersymmetry, with the SU(2) R-symmetry group realised on the S^2.

branes	t	x^1	x^2	x^3	z	y	x^6	x^7	x^8	x^9
D3	×	×	×	×	−	−	−	−	−	−
D1	×	−	−	−	−	×	−	−	−	−
F1	×	−	−	−	×	−	−	−	−	−
D5	×	−	−	−	−	×	×	×	×	×
NS5	×	−	−	−	×	−	×	×	×	×

Table 7.6: BPS/8 intersection describing D1-F1-D5-NS5 branes ending on D3 branes. x^1, x^2, x^3 are the coordinates realising the SO(3) R-symmetry.

We now perform a non-Abelian T-duality transformation (as explained in subsection 1.4.1) along the S^3, that we parametrise as

$$ds^2_{S^3} = \left(d\psi + \frac{\omega}{2}\right)^2 + \frac{1}{4} ds^2_{\tilde{S}_2}. \tag{7.52}$$

After this transformation the S^3 group manifold is replaced by an open subset of $\mathbb{R}^3$. The new class of solutions to Type IIA supergravity reads,

$$\begin{aligned}
ds^2_{10} =& q_{\rm D1}^{3/2} q_{\rm F1}^{1/2} H_{\rm D3}^{-1/2}\left(ds^2_{\rm AdS_2} + ds^2_{S^2}\right) + q_{\rm D1}^{1/2} q_{\rm F1}^{-1/2} H_{\rm D3}^{1/2}\left(dy^2 + dz^2 + dr^2\right)\\
&+ 4 q_{\rm D1}^{-1/2} q_{\rm F1}^{1/2} H_{\rm D3}^{-1/2} r^{-2}(d\rho^2 + H\rho^2 ds^2_{\tilde{S}^2})\,,\\
e^{\Phi} =& 8 q_{\rm D1}^{1/4} q_{\rm F1}^{-1/4} H_{\rm D3}^{-3/4} H^{1/2} r^{-3}\,,\\
B_2 =& q_{\rm D1}(z\,{\rm vol}_{\rm AdS_2} + y\,{\rm vol}_{S^2}) + \frac{16 q_{\rm F1}\rho^3}{16 q_{\rm F1}\rho^2 + q_{\rm D1} H_{\rm D3} r^4}\,{\rm vol}_{\tilde{S}^2}\,,\\
F_2 =& -8^{-1} r^3[(\partial_z H_{\rm D3}\,dy - \partial_y H_{\rm D3}\,dz)\wedge dr - \partial_r H_{\rm D3}\,dy\wedge dz]\,,\\
F_4 =& ({\rm vol}_{\rm AdS_2}\wedge dy - {\rm vol}_{S^2}\wedge dz)\wedge(q_{\rm F1}\rho\,d\rho - 8^{-1} q_{\rm D1} H_{\rm D3} r^3\,dr) +\\
&+ \frac{16 q_{\rm F1}\rho^3}{16 q_{\rm F1}\rho^2 + q_{\rm D1} H_{\rm D3} r^2}\,{\rm vol}_{\tilde{S}^2}\wedge F_2\,,\\
F_6 =& d[q_{\rm D1}^2 q_{\rm F1}^2 \rho H_{\rm D3}^{-1}\,{\rm vol}_{\rm AdS_2}\wedge{\rm vol}_{S^2}\wedge d\rho] +\\
&+ \rho^2 H r^{-2}\, d[q_{\rm F1}\rho r^2({\rm vol}_{\rm AdS_2}\wedge dy - {\rm vol}_{S^2}\wedge dz)\wedge{\rm vol}_{\tilde{S}^2}]\,,
\end{aligned} \tag{7.53}$$

where we have defined

$$H = \frac{q_{\rm D1} H_{\rm D3} r^4}{16 q_{\rm F1}\rho^2 + q_{\rm D1} H_{\rm D3} r^4}\,. \tag{7.54}$$

$H_{\rm D3}$ satisfies (7.51) and $(\rho, \tilde{S}^2)$ parametrise the open subset of $\mathbb{R}^3$ that arises after the non-Abelian T-duality transformation. Note that, as it is common after non-Abelian T-duality, the brane intersection from where the AdS geometry arises in the near-horizon limit cannot be easily identified[16]. The obvious candidate as brane intersection underlying the solutions (7.53) would be the non-Abelian T-dual of the brane intersection underlying the AdS$_2$ solutions (7.50). We will see that the brane intersection depicted in Table 7.7 is fully consistent with the quantised charges associated to the solutions (7.53), ergo we will take it as the starting point of our quiver constructions. Note that the solutions described by (7.53) preserve the same amount of supersymmetry as the original class of solutions, since the S^2 is left untouched after the non-Abelian T-duality transformation.

branes	t	x^1	x^2	x^3	z	y	ρ	x^7	x^8	x^9
D4	×	×	×	×	–	–	×	–	–	–
D6	×	×	×	×	–	–	–	×	×	×
NS5	×	×	×	×	×	×	–	–	–	–
D2	×	–	–	–	–	×	×	–	–	–
F1	×	–	–	–	×	–	–	–	–	–
D4′	×	–	–	–	–	×	–	×	×	×
NS5′	×	–	–	–	×	–	×	×	×	×

Table 7.7: BPS/8 intersection describing D2-F1-D4′-NS5′− branes ending on D4-D6-NS5 branes, associated to the class of solutions (7.53). As before, x^1, x^2, x^3 parametrise the directions realising the SO(3) R-symmetry.

In the next subsection we turn to the defect interpretation of the solutions.

7.4.1. F1-D2-D4′-NS5 line defects within AdS$_5$

Analogously to what we did in subsection 7.2.2, we consider the semi-localised profile defined by [157]

$$H_{\rm D3} = 1 + \frac{4\pi q_{\rm D3}}{(y^2 + z^2 + r^2)^2} \,, \tag{7.55}$$

and perform the change of coordinates

$$y = \mu \sin\alpha \cos\phi \,, \quad z = \mu \sin\alpha \sin\phi \,, \quad r = \mu \cos\alpha \tag{7.56}$$

[16] The reader is referred to [165], where this is discussed for the AdS$_5$ geometry constructed in [164], by performing non-Abelian T-duality on the AdS$_5 \times S^5$ background. Even in this simpler example the non-Abelian T-dual of the solution associated to N D3-branes cannot be easily interpreted as a D4-NS5-D6 brane intersection.

in order to obtain a solution that asymptotes locally to an AdS$_5$ geometry in the $\mu \to 0$ limit. This solution reads[17]

$$
\begin{aligned}
ds_{10}^2 =& \overbrace{\mu^2\left(ds^2_{\mathrm{AdS}_2}+ds^2_{S^2}\right)+\frac{d\mu^2}{\mu^2}}^{\text{locally AdS}_5\text{ geometry}} + d\alpha^2 + s^2 d\phi^2 + 4c^{-2}\left(d\rho^2+\frac{\rho^2 c^4}{16\,\rho^2+c^4}ds^2_{\tilde{S}^2}\right)\,,\\
e^{\Phi} =& 2\pi^{-1}q_{\mathrm{D3}}^{-1}c^{-1}(16\,\rho^2+c^4)^{-1/2}\,,\\
B_2 =& \mu s(\tilde{s}\mathrm{vol}_{\mathrm{AdS}_2}+\tilde{c}\mathrm{vol}_{S^2})+\frac{16\,\rho^3}{16\,\rho^2+c^4}\,\mathrm{vol}_{\tilde{S}^2}\,,\\
F_2 =& -2\pi q_{\mathrm{D3}} s c^3 d\alpha\wedge d\phi\,,\\
F_4 =& d[4\pi q_{\mathrm{D3}}\rho\mu s(\tilde{c}\mathrm{vol}_{\mathrm{AdS}_2}-\tilde{s}\mathrm{vol}_{S^2})\wedge d\rho]+\frac{16\,\rho^3}{16\,\rho^2+c^4}\,\mathrm{vol}_{\tilde{S}^2}\wedge F_2+\\
&+2^{-1}\pi q_{\mathrm{D3}}c^3[(\tilde{c}\mathrm{vol}_{\mathrm{AdS}_2}-\tilde{s}\mathrm{vol}_{S^2})\wedge d\mu\wedge d\alpha+\\
&+s(\tilde{s}\mathrm{vol}_{\mathrm{AdS}_2}+\tilde{c}\mathrm{vol}_{S^2})\wedge d(\mu c\, d\phi)]\,,\\
F_6 =& -\frac{4\pi q_{\mathrm{D3}}\rho^3 c^4}{16\,\rho^2+c^4}\,d[\mu s(\tilde{c}\,\mathrm{vol}_{\mathrm{AdS}_2}-\tilde{s}\,\mathrm{vol}_{S^2})]\wedge d[\log\left(\rho\mu^2c^2\right)\mathrm{vol}_{\tilde{S}^2}]+\\
&+16\pi q_{\mathrm{D3}}\rho\mu^3\,\mathrm{vol}_{\mathrm{AdS}_2}\wedge\mathrm{vol}_{S^2}\wedge d\mu\wedge d\rho\,.
\end{aligned}
\tag{7.57}
$$

Note that in order to obtain this solution we could have alternatively non-Abelian T-dualised the solution (7.16) that asymptotes to AdS$_5\times S^5$.

The geometry defined by the metric in (7.57) asymptotes locally to the Gaiotto-Maldacena geometry constructed in [164], by acting with non-Abelian T-duality on AdS$_5\times S^5$. The fluxes associated to said solution are the ones below,

$$
\begin{aligned}
F_2 =& -2\pi q_{\mathrm{D3}} s c^3 d\alpha\wedge d\phi\,,\\
F_4 =& -2^5\pi q_{\mathrm{D3}} s c^{-1}\rho^3 H d\alpha\wedge d\phi\wedge\mathrm{vol}_{\tilde{S}^2}\,,\\
H_3 =& d\left(\frac{16\,\rho^3}{16\,\rho^2+c^4}\right)\wedge\mathrm{vol}_{\tilde{S}^2}\,.
\end{aligned}
\tag{7.58}
$$

However, the isometries of the AdS$_5$ solution are broken by the presence of the extra contributions to the fluxes in (7.57), which are subleading in μ. These terms give rise to new global charges associated to the defect branes. These branes, in turn, backreact into a geometry described by a 5d curved domain wall with AdS$_2\times S^2$ slicings that is only locally AdS$_5$. The presence of the extra fluxes also forbids any supersymmetry enhancement. Note that, as in the examples discussed in [46, 58, 66], the R-symmetry of the 4d $\mathcal{N}=2$ AdS$_5$ solution is realised on the internal space (in this case on the $\tilde{S}^2\times S^1_\phi$ subspace), while the R-symmetry of the AdS$_2$ solution becomes part of the superconformal group of the higher dimensional theory.

Let us now proceed with a detailed analysis of the solution described by (7.57).

[17] We have fixed the constants such that the AdS$_5$ subspace has radius one.

7.5. Line defects within 4d $\mathcal{N} = 2$ SCFTs and brane boxes

In this section we construct the brane set-up underlying the solution (7.57) and show that it consists on D2 colour branes stretched between perpendicular NS5-branes. This realises a one-dimensional brane box model from which the 1d quiver QFT can be read. We show that this theory can be embedded within the 4d $\mathcal{N} = 2$ SCFT dual to the Gaiotto-Maldacena geometry arising in the asymptotics, described by a linear quiver with gauge groups of increasing ranks [93]. Furthermore, we discuss in detail the interpretation of the F1-strings present in the brane intersection, and show that together with the D2 and one of the families of D4-branes describe baryon vertices in the 4d SCFT.

7.5.1. The 4d Superconformal Background Theory

As a Gaiotto-Maldacena geometry, the AdS_5 solution constructed in [164] is dual to a 4d $\mathcal{N} = 2$ superconformal field theory living in a D4-NS5-D6 brane intersection. This CFT was thoroughly studied in [93], to which the reader is referred for more details. The D4-NS5-D6 intersection is described by the first three lines in Table 7.7. We start by computing the quantised charges associated to the NS5-branes, following [93]. Integrating

$$B_2^{\tilde{S}^2} = \frac{16\,\rho^3}{16\,\rho^2 + c^4}\text{vol}_{\tilde{S}^2} \tag{7.59}$$

on the cycle defined by the $\tilde{S}^2$ positioned at $\alpha = \pi/2$, we have,

$$\frac{1}{(2\pi)^2}\int_{\Sigma_2} B_2^{\tilde{S}^2} = \frac{\rho}{\pi}. \tag{7.60}$$

Since this quantity has to take values between 0 and 1 in order to have a well-defined partition function, the ρ direction must be divided in intervals of length π, such that when $\rho \in [n\pi, (n+1)\pi]$ a large gauge transformation of parameter n must be performed for (7.60) to be satisfied. $B_2^{\tilde{S}^2}$ must thus be modified as

$$B_2^{\tilde{S}^2} = \Big(\frac{16\,\rho^3}{16\,\rho^2 + c^4} - n\pi\Big)\text{vol}_{\tilde{S}^2} \quad \text{for} \quad \rho \in [n\pi, (n+1)\pi]\,. \tag{7.61}$$

One effect of this split into intervals is that upon crossing $\rho = n\pi$ a NS5-brane is created, generating a Hanany-Witten brane creation effect. Indeed, we have in each interval

$$Q_{\text{NS5}} = \frac{1}{(2\pi)^2}\int_{\Sigma_3} H_3 = \frac{1}{(2\pi)^2}\int_{n\pi}^{(n+1)\pi}\int_{\Sigma_2} H_3 = 1 \tag{7.62}$$

with Σ_2 the $\tilde{S}^2$ located at $\alpha = \pi/2$. Moreover, the n term in (7.61) contributes to the 4-form Page flux with $\tilde{S}^2$ component such that

$$\hat{F}_4^{\tilde{S}^2} = n\pi\,\text{vol}_{\tilde{S}^2} \wedge F_2\,, \tag{7.63}$$

which, knowing that in units with $g_s = \alpha' = 1$,

$$Q_{\mathrm{D}p} = \frac{1}{(2\pi)^{7-p}} \int_{\Sigma_{8-p}} \hat{F}_{8-p} \,, \tag{7.64}$$

gives

$$Q_{\mathrm{D4}} = n\, Q_{\mathrm{D6}} \quad \text{with} \quad \rho \in [n\pi, (n+1)\pi] \,, \tag{7.65}$$

where[18]

$$Q_{\mathrm{D6}} = \frac{\pi}{2} q_{\mathrm{D3}}. \tag{7.66}$$

These quantised charges give rise to the Hanany-Witten brane set-up depicted in Figure 7.5, where the D4-branes play the role of colour branes and there are no D6 flavour branes, as the D6-brane charge remains constant across intervals. Note that this brane

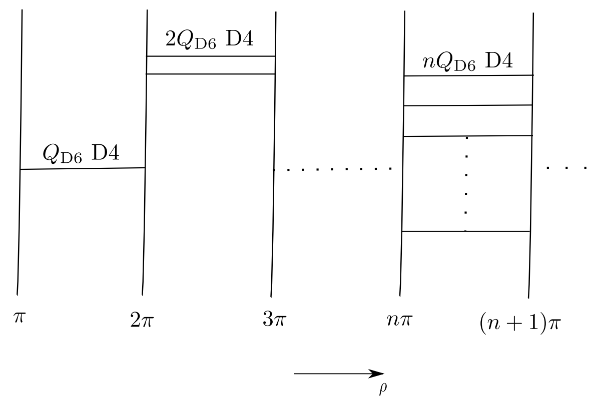


Figure 7.5: Brane set-up associated to the 4d background theory. NS5-branes are located at $\rho_n = n\pi$ and nQ_{D6} D4-branes are stretched between them in each $[\rho_n, \rho_{n+1}]$ interval.

set-up extends infinitely in the ρ-direction, due to the non-compact character of the ρ-coordinate. This happens because after the non-Abelian T-duality transformation the S^3 of the original solution is replaced by an open subset of $\mathbb{R}^3$, and due to our lack of knowledge of how non-Abelian T-duality extends beyond spherical worldsheets it is not possible to infer its global properties [166]. In view of this in [93] different ways of terminating the brane set-up were discussed. Here we will choose the simplest scenario, namely, we will terminate the brane set-up at $\rho_P = P\pi$ by adding a set of PQ_{D6} D6-branes (or semi-infinite D4-branes). The resulting quiver is the one depicted in Figure 7.6. One can check that at each node of the quiver the condition on the ranks of the gauge groups, l_i,

$$2l_n = l_{n+1} + l_{n-1}, \tag{7.67}$$

required for the β-function to vanish [109], is satisfied. Moreover, the field theory and holographic central charges can be shown to agree to leading order in P, i.e. the holographic limit in these quiver constructions [93].

[18]Note that as usual the quantised charges need to be renormalised after a non-Abelian T-duality transformation. This can be done through a redefinition of Newton's constant. This will affect our normalisation of the holographic central charge in subsection 7.5.4 (see for instance [93]).

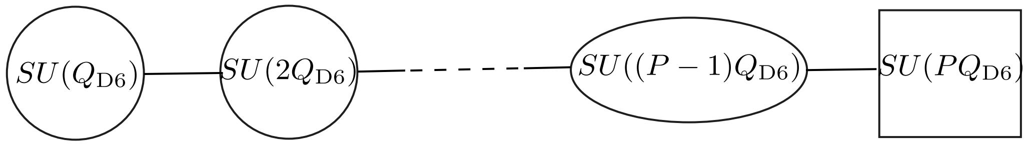


Figure 7.6: Quiver describing the 4d $\mathcal{N}=2$ SCFT dual to the background geometry.

7.5.2. The line defect theory

We proceed now to the description of the D2-F1-D4′-NS5′ defect branes whose back-reaction in the AdS$_5$ geometry generates the AdS$_2$ solution. We start focusing on the F1- and NS5′-branes. For this purpose, it is useful to define

$$y = \mu \sin\alpha \cos\phi\,, \quad z = \mu \sin\alpha \sin\phi\,, \tag{7.68}$$

as in equation (7.56). The B_2 field then reads

$$B_2 = z\,\mathrm{vol}_{\mathrm{AdS}_2} + y\,\mathrm{vol}_{S^2} + \frac{16\,\rho^3}{16\,\rho^2 + c^4}\mathrm{vol}_{\tilde{S}^2}\,. \tag{7.69}$$

The component along the $\tilde{S}^2$ is associated to the NS5-branes of the 4d background theory, as we saw in the previous subsection, so we will no longer discuss it. In turn, the first and second components are associated to the F1 and NS5′ defect branes. Let us start by looking at the NS5′-branes. A very similar analysis to the one performed in the previous subsection for the NS5-branes of the background theory shows that we must divide the y-direction in $[m\pi, (m+1)\pi]$ intervals and perform a large gauge transformation of gauge parameter m in each one of them to satisfy that B_2 lies in the fundamental region. This fixes

$$B_2^{S^2} = (y - m\pi)\mathrm{vol}_{S^2} \quad \text{for} \quad y \in [m\pi, (m+1)\pi] \tag{7.70}$$

and creates NS5′-branes along the y direction at each $y = m\pi$. Let us now turn our attention to the electric component of B_2. The F1-strings are electrically charged with respect to the NSNS 3-form. Therefore, their charges are computed according to

$$Q^e_{\mathrm{F1}} = \frac{1}{(2\pi)^2}\int H_3 = \frac{1}{(2\pi)^2}\int_{\mathrm{AdS}_2} B^e_2\,, \tag{7.71}$$

where we use the superscript e to denote that we are referring to electric as opposed to magnetic charges. Regularising the volume of the AdS$_2$ space such that $\mathrm{Vol}_{\mathrm{AdS}_2} = 4\pi$[19] and dividing the z direction into intervals of length $[k\pi, (k+1)\pi]$, a single F1-string lies in each such interval. This is also implied by the condition that the integral of B_2 lies in the fundamental region, as previously discussed (see [43]) . In this case a large gauge transformation of gauge parameter k must be performed for $z \in [k\pi, (k+1)\pi]$, such that

$$B^e_2 = (z - k\pi)\mathrm{vol}_{\mathrm{AdS}_2} \tag{7.72}$$

[19]This regularisation prescription is based on the analytical continuation that relates the AdS$_2$ space with an S^2.

in this interval. We will come back to the physical interpretation of this condition after we discuss the quiver quantum mechanics associated to the solution.

For this purpose, let us now look at the D2 and D4′ defect branes. The large gauge transformations of parameters m and k modify the S^2 and AdS$_2$ components of the Page fluxes, according to

$$\begin{aligned}\hat{F}_4 &\to \hat{F}_4 + m\pi F_2 \wedge \text{vol}_{S^2} + k\pi F_2 \wedge \text{vol}_{\text{AdS}_2}\,,\\ \hat{F}_6 &\to \hat{F}_6 + mn\pi^2 F_2 \wedge \text{vol}_{S^2} \wedge \text{vol}_{\tilde{S}^2} + kn\pi^2 F_2 \wedge \text{vol}_{\text{AdS}_2} \wedge \text{vol}_{\tilde{S}^2}\,,\end{aligned} \tag{7.73}$$

generating a Hanany-Witten brane creation effect. This affects the (α, ϕ) components of the Page fluxes, where F_2 lies. Since, in the absence of large gauge transformations, we have for these components that

$$F_4 = F_2 \wedge B_2 \quad \text{and} \quad F_6 = F_4 \wedge B_2 - \frac{1}{2} F_2 \wedge B_2 \wedge B_2\,, \tag{7.74}$$

we find that

$$\begin{aligned}\hat{F}_4 =& k\pi F_2 \wedge \text{vol}_{\text{AdS}_2} + m\pi F_2 \wedge \text{vol}_{S^2} + n\pi F_2 \wedge \text{vol}_{\tilde{S}^2}\,,\\ \hat{F}_6 =& mn\pi^2 F_2 \wedge \text{vol}_{S^2} \wedge \text{vol}_{\tilde{S}^2} + kn\pi^2 F_2 \wedge \text{vol}_{\text{AdS}_2} \wedge \text{vol}_{\tilde{S}^2} +\\ &+ km\pi^2 F_2 \wedge \text{vol}_{\text{AdS}_2} \wedge \text{vol}_{S^2}\,.\end{aligned} \tag{7.75}$$

Let us focus on the magnetic components. Equations (7.75) imply that D4′-branes are created across NS5′-branes as we move in the y-direction, and D2-branes are created both across NS5′-branes as we move in the y-direction and across NS5-branes as we move in the ρ-direction. To this we have to add the D4-branes that were already created across NS5-branes in the ρ-direction in the background theory. The corresponding quantised charges in the $y \in [m\pi, (m+1)\pi]$, $\rho \in [n\pi, (n+1)\pi]$ intervals are given by

$$Q_{\text{D4}'} = m\, Q_{\text{D6}}\,, \quad Q_{\text{D2}} = mn\, Q_{\text{D6}}\,, \quad Q_{\text{D4}} = n\, Q_{\text{D6}}\,. \tag{7.76}$$

We thus find a brane scenario in which two directions, y and ρ, play the role of field theory directions. The D2-branes, stretched between both types of NS5- and NS5′-branes in these directions, are interpreted as the colour branes where a 1d supersymmetric field theory lives. In turn, the charges carried by the D4′- and D4-branes are induced by the D2-branes they end on, with which they share the y and ρ field theory directions, respectively (see Table 7.7). The brane set-up in the (ρ, y) plane is then the one depicted in Figure 7.7. Once we have identified the brane set-up we can proceed to construct the quiver that describes the field theory living in the D2-branes. In order to achieve that goal, we must look at the quantisation of the open strings that stretch between the branes in the different boxes. As it is customary for 1d $\mathcal{N} = 4$ multiplets, we will use 2d $\mathcal{N} = (0, 4)$ notation. We will follow closely [149], where the quantisation of open strings in D3-brane box models realising 2d $\mathcal{N} = (0, 4)$ field theories was studied in detail. Our brane set-up is simply related to the D3-NS5-D5-NS5'-D5' brane intersection studied in [149] by Abelian T-duality along the x^1 direction therein, and thus realises a 1d $\mathcal{N} = 4$ instead of a 2d $\mathcal{N} = (0, 4)$ field theory. Except for this subtlety, the analysis is completely analogous. There are four types of D2-D2 strings to consider:

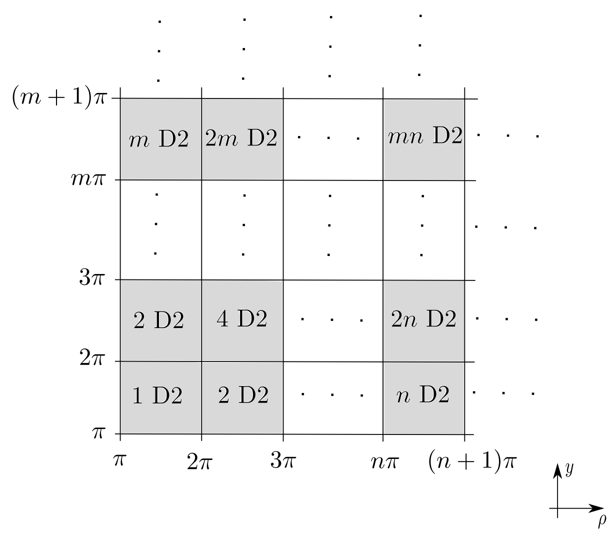


Figure 7.7: Brane set-up associated to the AdS_2 solution (7.57) (in units of $Q_{\mathrm{D6}} = 1$).

- When the end-points of the string lie on the same stack of D2-branes, the projections induced by both the NS5 and the NS5′ branes leave behind an $\mathcal{N} = (0, 4)$ vector multiplet, since the D2-branes cannot move in any of the transverse directions.

- When the end-points of the string lie on two different stacks of D2-branes separated by an NS5-brane, the degrees of freedom along the (x^7, x^8, x^9) directions are fixed, leaving behind the scalars associated to the (x^1, x^2, x^3) directions. Together with the A_y component of the gauge field, they combine into an $\mathcal{N} = (0, 4)$ twisted hypermultiplet in the bifundamental representation, since the scalars are charged under the R-symmetry.

- When the end-points of the string lie on two different stacks of D2-branes separated by an NS5′-brane, the degrees of freedom along the (x^1, x^2, x^3) directions are fixed, leaving behind the scalars associated to the (x^7, x^8, x^9) directions. The unconstrained coordinates, together with the A_ρ component of the gauge field, give rise to an $\mathcal{N} = (0, 4)$ hypermultiplet in the bifundamental representation, since the scalars are uncharged under the R-symmetry.

- Finally, when the end-points of the string lie on two different stacks of D2-branes separated by both an NS5- and an NS5′-brane, all the scalars are fixed, leaving behind the fermionic mode associated to a bifundamental $\mathcal{N} = (0, 2)$ Fermi multiplet.

These multiplets give rise to the planar quiver in Figure 7.8. This quiver consists of two arrows of linear quivers, associated to the D2-branes stretched between NS5-branes in the ρ direction and NS5′-branes in the y direction, with mutual interactions consisting of $\mathcal{N} = (0, 2)$ Fermi multiplets. The quiver is terminated in both directions with two families of flavour groups, arising from nP' D6-branes (or semi-infinite D4′-branes) placed at $\rho = n\pi$, with $n = 1, 2, \ldots, P$, $y_{P'} = P'\pi$ and mP D6-branes (or semi-infinite D4-branes) placed at $\rho_P = P\pi$, $y = m\pi$ with $m = 1, 2, \ldots, P'$. This allows to construct a well-defined one dimensional quiver quantum mechanics, from which one can compute the degrees of freedom of the 1d SCQM that arises in the IR, as will be pursued in subsection 7.5.4.

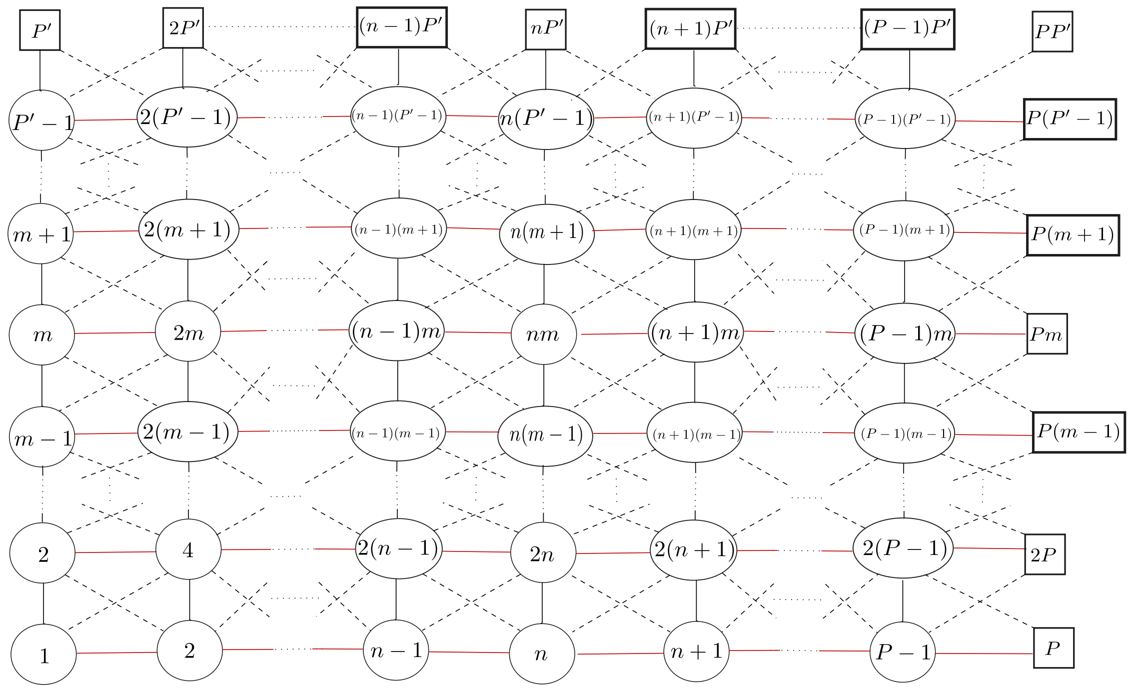


Figure 7.8: Quiver quantum mechanics associated to the AdS$_2$ solution given by (7.57). Circles denote $\mathcal{N} = (0, 4)$ vector multiplets, red lines $\mathcal{N} = (0, 4)$ bifundamental twisted hypermultiplets, black lines $\mathcal{N} = (0, 4)$ bifundamental hypermultiplets and dashed lines bifundamental $\mathcal{N} = (0, 2)$ Fermi multiplets. We have taken units in which $Q_{\text{D6}} = 1$.

Our proposal is that the quiver depicted in Figure 7.8 describes a 1d field theory that flows in the IR to the 1d SCQM dual to the AdS$_2$ solution. Note that, as a one-dimensional field theory, there is no condition for gauge anomaly cancellation. However, it is striking that the quiver mechanics satisfies the conditions for gauge anomaly cancellation of a 2d $\mathcal{N} = (0, 4)$ field theory. This can be checked right away in our 2d $\mathcal{N} = (0, 4)$ notation for the superfields. For this matter, we need to recall the contribution from the different fields to the U(N) gauge anomaly (see for instance [49, 94, 116, 149]):

- $\mathcal{N} = (0, 4)$ twisted or untwisted hypermultiplets contribute with a factor $2N$ if they are in the adjoint, or with a factor 1 if they are in the fundamental representation.
- $\mathcal{N} = (0, 4)$ vector multiplets contribute with a factor $-2N$.
- $\mathcal{N} = (0, 2)$ Fermi multiplets in the fundamental contribute with a factor -1/2.

With these contributions one can easily check that all nodes in the quiver in Figure 7.8 satisfy that the gauge anomaly vanishes. We will further discuss this property of the quiver when we interpret our result for the central charge in section 7.5.4.

Let us now explore another interesting property of the quiver in 7.8: it can be seen as the result of embedding the D2-D4′-NS5′-F1 defect branes in the 4d $\mathcal{N} = 2$ SCFT living in the D4-NS5-D6 brane intersection. From the point of view of the 4d theory the R-symmetry is realised on the x^7, x^8, x^9 directions, as mentioned in section 7.4. The 4d quiver depicted in Figure 7.6 is then decomposed in terms of 2d $\mathcal{N} = (0, 4)$ matter fields as shown in Figure 7.9.

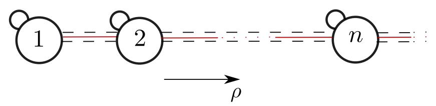


Figure 7.9: Field theory living in the D4-NS5-D6 subsystem (in units of $Q_{\rm D6} = 1$) in terms of 2d $\mathcal{N} = (0, 4)$ multiplets.

In this decomposition the 4d $\mathcal{N} = 2$ vector multiplet gives rise to a 2d $\mathcal{N} = (0, 4)$ vector multiplet with gauge field A_α, $\alpha = t, z$, plus a 2d $\mathcal{N} = (0, 4)$ adjoint hypermultiplet, arising from combining the reduction of the gauge field along the (x^1, x^2, x^3) directions and the fluctuations in the y-direction. In turn, the 4d $\mathcal{N} = 2$ bifundamental hypermultiplet gives rise to a 2d $\mathcal{N} = (4, 4)$ bifundamental twisted hypermultiplet, arising from combining the A_ρ component of the gauge field with the fluctuations in (x^7, x^8, x^9). These decompositions can be summarised as

$$\begin{aligned} &\text{4d } \mathcal{N} = 2 \text{ vector} \to \text{2d } \mathcal{N} = (0,4) \text{ vector} + \text{2d } \mathcal{N} = (0,4) \text{ adjoint hyper}\,, \\ &\text{4d } \mathcal{N} = 2 \text{ bifundamental hyper} \to \text{2d } \mathcal{N} = (4,4) \text{ bifundamental twisted hyper}\,. \end{aligned} \tag{7.77}$$

We can consider in an analogous way the D4′-NS5′-D6 brane subsystem of the brane set-up shown in Table 7.7. The 4d SCFT living in this brane subsystem is again the one in Figure 7.6, but this time y plays the role of field theory direction. However, in the decomposition into 2d $\mathcal{N} = (0, 4)$ matter multiplets, the 4d $\mathcal{N} = 2$ vector multiplet decomposes into a 2d $\mathcal{N} = (0, 4)$ vector multiplet plus a 2d $\mathcal{N} = (0, 4)$ adjoint twisted hypermultiplet, arising from combining the reduction of the gauge field along the (x^7, x^8, x^9) directions and the fluctuations in the ρ-direction. It is well-known that these multiplets combine into an $\mathcal{N} = (4, 4)$ vector multiplet. The difference with the decomposition of the gauge field living in the D4-branes is that the scalars are now charged with respect to the SU(2) R-symmetry. On the other hand, the 4d $\mathcal{N} = 2$ bifundamental hypermultiplet gives rise to an $\mathcal{N} = (4, 4)$ bifundamental hypermultiplet, arising from combining the A_y component of the gauge field with the fluctuations in (x^1, x^2, x^3). Again, the difference with the decomposition of the 4d hypermultiplet living on the D4-branes is that the scalars are now uncharged with respect to the SU(2) R-symmetry. These decompositions are collected below,

$$\begin{aligned} &\text{4d } \mathcal{N} = 2 \text{ vector} \to \text{2d } \mathcal{N} = (0,4) \text{ vector} + \text{2d } \mathcal{N} = (0,4) \text{ adjoint twisted hyper}\,, \\ &\text{4d } \mathcal{N} = 2 \text{ bifundamental hyper} \to \text{2d } \mathcal{N} = (4,4) \text{ bifundamental hyper}\,. \end{aligned} \tag{7.78}$$

The 4d quiver living in the D4′-NS5-D6 branes is represented in terms of 2d $\mathcal{N} = (0, 4)$ multiplets in Figure 7.10.

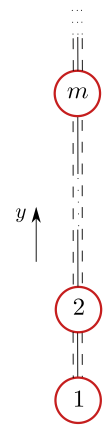


Figure 7.10: Field theory living in the D4′-NS5′-D6 subsystem (in units of $Q_{\text{D6}} = 1$) in terms of 2d $\mathcal{N} = (0, 4)$ multiplets. Red circles represent $\mathcal{N} = (4, 4)$ vector multiplets.

The quiver depicted in Figure 7.8 can now be seen as the result of assembling the two quivers represented in Figures 7.9 and 7.10. In this assembly the charges carried by the D4 and D4′-branes are now carried by D2-branes, that stretch in both the ρ and y field theory directions. Our proposal is that the 1d field theory described by this quiver flows in the IR to the SCQM dual to the AdS$_2$ solution (7.57). We would like to stress that the SCQM proposed in this section is far more elaborated than those previously constructed in the literature [42, 43, 45, 47, 65, 167, 168], since it involves the highly non-trivial brane box models constructed in [149], now realising an $\mathcal{N} = 4$ supersymmetric quantum mechanics. Moreover, we have at our disposal the explicit holographic dual, and therefore a well-controlled string theory realisation that allows to study these constructions geometrically. We will see that this is particularly useful when addressing the non-trivial issue of computing the central charge. Our construction provides, to our knowledge, the first example in which a brane box model has been described holographically[20]. We would like to emphasise the non-trivial role played by non-Abelian T-duality in making this possible.

In the next subsection we turn to a more precise interpretation of the massive F1-strings present in the defect sector of the theory. This discussion follows very closely the analysis carried out in subsection 7.3.6. We recall that, although in that case we were working with a class of AdS$_2$ solutions missing AdS$_5$ asymptotics, the field theory analysis allowed to interpret the D2-D4′-F1 branes as baryon vertices for the D4-D6-NS5 subsystem. This suggests that a defect interpretation should still be possible. We will show that the previous baryon vertex interpretation goes through, as expected, for our AdS$_2$/CFT$_1$ set-up, which finds in this way a defect interpretation from both the field theory and the geometrical sides.

[20]See [46] for AdS$_3$ solutions dual to D3-brane boxes with one circular direction.

7.5.3. Baryon vertex interpretation

In this subsection we turn to the interpretation of the F1-strings of the solution. We show that, together with the D2- and the D4′-branes, they find a baryon vertex interpretation within the 4d $\mathcal{N}=2$ background theory.

Let us start looking at the D2-branes by considering the below worldvolume coupling,

$$S_{\text{D2}} = T_2 \int F_2 \wedge A_t \,. \tag{7.79}$$

It shows that a D2-brane lying on (t, ϕ, α) behaves as a baryon vertex for the D6-branes, since it carries Q_{D6} units of F1-string charge. Analogously, the coupling

$$S_{\text{D4}'} = T_4 \int \hat{F}_4 \wedge A_t \tag{7.80}$$

in the worldvolume of a D4′-brane shows that a D4′-brane lying on $(t, \phi, \alpha, \tilde{S}^2)$ and located at a fixed position in $\rho \in [n\pi, (n+1)\pi]$ behaves as a baryon vertex for the D4-branes in this ρ-interval. This can be understood as the D4′-brane carrying $Q_{\text{D4}} = nQ_{\text{D6}}$ units of F1-string charge. Indeed, the relative orientation between the D4′- and the D4-branes in the brane set-up allows to create F1-strings stretched between them, as depicted in Figure 7.11. And the same can be stated about the D2- and D6-branes.

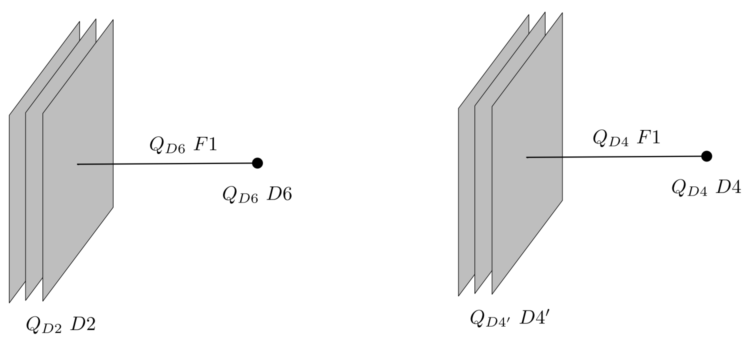


Figure 7.11: Wilson loop in the Q_{D6} (Q_{D4}) antisymmetric representation of $\text{U}(Q_{\text{D2}})$ ($\text{U}(Q_{\text{D4}'})$).

Coming back to our brane configuration, the relevant fluxes that give the quantised electric charges of the D2- and D4′-branes playing the role of baryon vertices are the $(\text{AdS}_2, \phi, \alpha)$ and $(\text{AdS}_2, \phi, \alpha, \tilde{S}^2)$ components of $\hat{F}_4$ and $\hat{F}_6$ found in (7.75), given by

$$\hat{F}_4^e = k\pi\, F_2 \wedge \text{vol}_{\text{AdS}_2} \,, \quad \hat{F}_6^e = nk\pi^2\, F_2 \wedge \text{vol}_{\text{AdS}_2} \wedge \text{vol}_{\tilde{S}^2} \,. \tag{7.81}$$

These fluxes give rise to the quantised charges below,

$$\begin{aligned} Q_{\text{D2}}^e =& k\, Q_{\text{D6}} \quad \text{for} \quad z \in [k\pi, (k+1)\pi] \,, \\ Q_{\text{D4}'}^e =& n\, Q_{\text{D2}}^e = nk\, Q_{\text{D6}} \quad \text{for} \quad z \in [k\pi, (k+1)\pi] \quad \text{and} \quad \rho \in [n\pi, (n+1)\pi] \,. \end{aligned} \tag{7.82}$$

From these charges we can read the brane set-up along the z direction for constant y and ρ, as depicted in Figure 7.12. In this set-up the stacks of D6-branes located at $\rho = n\pi$,

$y_{P'} = P'\pi$ that terminated the quiver quantum mechanics in the y direction allow us to also terminate the brane set-up in the z direction, if we locate them at $z_{P'} = P'\pi$ [21].

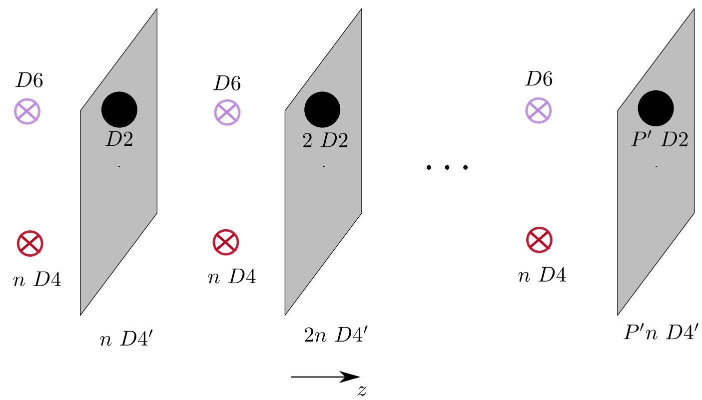


Figure 7.12: Brane set-up in the z-direction, for y and ρ constants. The numbers of D4- and D6-branes at each interval are given by their respective magnetic charges. Instead, for the numbers of D2- and D4′-branes we give their electric charges (7.82) as these are the ones that play a role in their interpretation as baryon vertices.

The brane set-up depicted in Figure 7.12 can now be related by a combination of a T-duality, an S-duality, successive Hanany-Witten moves and a further T-duality to the brane set-up depicted in Figure 7.13. This is carefully explained in [43] (see also [65]).

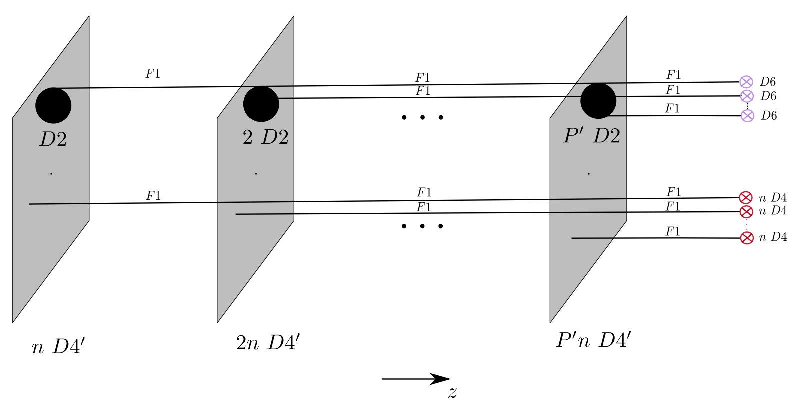


Figure 7.13: Hanany-Witten brane set-up equivalent to the brane configuration in Figure 7.12.

In this description of the system the relation with the constructions in [113, 114] becomes manifest. In our case the sum of the F1-strings stretched between each D2 (D4′) and the flavour D6 (D4) branes coincides with the rank of the gauge group of the D2 (D4′) branes. This implies that the Wilson lines are in the fundamental representation of the gauge groups. Therefore, the D2-D4′ branes describe baryon vertices for the D6-D4 branes of the 4d background theory. As we move in the ρ direction the NS5-branes

[21] Note that the relations (7.68) imply that z and y must reach the same maximum values.

located in the different positions in ρ allow to create D4-branes stretched between them in an increasing number in units of $Q_{\rm D6}$. Exactly the same phenomenon takes place for the D4′-branes, which are created, orthogonal to the D4-branes, as the NS5-branes are crossed, in an increasing number in units of $Q^e_{\rm D2}$, since they carry electric charge. In turn, as the number of D2-branes varies as we move in the z direction, the same happens with the D4′-branes. In this way one finds an analogous interpretation to that of the D2-branes for the D4′-branes, as baryon vertices for the D4-branes.

The conclusion of our analysis in this subsection is that the AdS$_2$ solution can be interpreted as describing backreacted baryon vertices within the 4d $\mathcal{N}=2$ CFT living in the D4-NS5-D6 branes. Consistently with this interpretation the AdS$_5$ solution associated to the D4-NS5-D6 intersection arises asymptotically locally far away from the defect. Even if the baryon vertices are described in terms of D2-D4′-F1 branes, NS5′-branes need also be introduced in the background so that a solution to Type IIA supergravity arises. The full brane set-up allows then for a field theory description of the AdS$_2$ solution in terms of D2-branes stretched between both the NS5-branes and the NS5′-branes in two perpendicular directions.

7.5.4. Computation of the central charge

In this subsection we show that the SCQM proposed in the previous sections provides one further example in which the central charge computed from the 2d expression is in agreement with the holographic calculation. If we now use the formula in (5.4) for our 1d quiver in Figure 7.8 we find[22],

$$n_{hyp}=\sum_{n=1}^{P-1}\sum_{m=1}^{P'-1}n^2m(m+1)Q_{\rm D6}^2\,,\quad n_{vec}=\sum_{n=1}^{P-1}\sum_{m=1}^{P'-1}n^2m^2Q_{\rm D6}^2\,. \tag{7.83}$$

We thus arrive at the following value for the central charge

$$c_R=\frac{1}{2}P(P-1)(2P-1)P'(P'-1)Q_{\rm D6}^2\,. \tag{7.84}$$

In the large number of nodes limit, i.e. to leading order in P and P', it behaves as

$$c_R\sim P^3P'^2Q_{\rm D6}^2\,. \tag{7.85}$$

In order to compare with the holographic calculation, we need to compute c_L as well, since

$$c_{\rm hol}=\frac{1}{2}(c_L+c_R)\,. \tag{7.86}$$

It can be obtained c_L from the relation below,

$$c_L=c_R+{\rm Tr}\big(\gamma^3\big)\,. \tag{7.87}$$

[22]Here we have reinstated $Q_{\rm D6}\neq 1$.

We can easily see that $\text{Tr}(\gamma^3) = 0$ for our quiver in Figure 7.8, therefore $c_L = c_R$.

Let us compute now the holographic central charge. We remark that we need to apply the proper normalisation of Newton's constant, as mentioned previously, arriving at the following,

$$c_{\text{hol}} = \frac{3}{2^5\pi^8} V_{int} = \frac{3}{2^5\pi^8} \int d\vec{\theta}\, e^{-2\Phi} \sqrt{\det(g_{ij})}\,, \tag{7.88}$$

where g_{ij} is the metric of the inner space and $\vec{\theta}$ are coordinates defined over it. Using this expression and integrating ρ between $[0, P\pi]$ and μ between $[0, P'\pi]$, we find that the holographic central charge reads

$$c_{\text{hol}} = P^3 P'^2 Q_{\text{D6}}^2\,, \tag{7.89}$$

in perfect agreement with the field theory calculation, in the large number of nodes limit.

7.6. The D1-F1-D3-D5-NS5-D7 brane set-up

In this section we construct a new family of AdS$_2$ solutions to Type IIB supergravity preserving $\mathcal{N} = 4$ supersymmetries. We obtain these solutions as near-horizon geometries of D1-F1-D3 branes ending on the D5-NS5-D7 brane system where the 5d Sp(N) gauge theory lives.

Such an intersection reproduces a class of AdS$_2 \times S^3 \times S^2 \times S^1$ geometries foliated over two intervals in the near-horizon. We show that a subset of non-compact backgrounds within this class flows asymptotically locally to the AdS$_6 \times S^2 \times \Sigma_2$ vacuum of Type IIB supergravity constructed in [159, 169]. This AdS$_6$ vacuum geometry was obtained by Abelian T-dualising the Brandhuber-Oz solution to massive Type IIA supergravity [140], and it is the only explicit solution within the general classification of AdS$_6 \times S^2 \times \Sigma_2$ solutions in [142, 170, 171] with Σ_2 an annulus (see [145]).

This asymptotic property of our AdS$_2$ solutions allows us to interpret them as holographic duals to line defects within the 5d Sp(N) fixed point theory. In favour of this duality we show that they are related by T-duality to the AdS$_2 \times S^3 \times \text{CY}_2 \times I$ solutions constructed in [43] (for CY$_2 = \mathbb{R}^4$), which were interpreted as line defects within the 5d Sp(N) CFT, as shown in [43, 58, 161].

Let us consider the brane picture summarised in Table 7.8, consisting on D1-F1-D3 branes ending on a D5-NS5-D7 system. Under certain conditions, this brane set-up gives rise to the aforementioned family of solutions to Type IIB supergravity, consisting on AdS$_2 \times S^3 \times S^2 \times S^1$ fibrations over a 2d Riemann surface.

Our first assumption consists on taking the D1-F1-D3 defect branes completely localised within the worldvolume of the orthogonal D5-NS5-D7 system of background branes, i.e. the $\mathbb{R}^4$ parametrised by ρ and the S^3. In other words, the associated warp factors H_{D1}, H_{F1} and H_{D3} are just functions of the radial coordinate ρ. As we saw before, this allows to decouple the field equations of the defect branes from those of the background ones.

branes	t	ρ	φ^1	φ^2	φ^3	z	ψ	r	θ^1	θ^2
D7	×	×	×	×	×	–	–	×	×	×
D5	×	×	×	×	×	–	×	–	–	–
NS5	×	×	×	×	×	×	–	–	–	–
D1	×	–	–	–	–	–	×	–	–	–
F1	×	–	–	–	–	×	–	–	–	–
D3	×	–	–	–	–	–	–	×	×	×

Table 7.8: $\frac{1}{8}$-BPS brane intersection of D5-NS5-D7 background branes with D1-F1-D3 branes ending on them.

The second assumption that we make is to take the D7 and NS5 charges smeared over a shared transverse direction ψ, which parametrises a circle. In terms of the warp factors this implies that $H_{\text{NS5}} = H_{\text{NS5}}(r)$ and $H_{\text{D7}} = H_{\text{D7}}(z)$[23]. This condition restricts one to the D5-NS5-D7 brane set-up where the 5d Sp(N) gauge theory lives, thus recovering the asymptotically locally $\text{AdS}_6 \times S^2 \times \Sigma_2$ solution of Type IIB dual to this SCFT, constructed in [159, 169].

Finally, we take completely localised D5-branes, i.e. its warp factor satisfies $H_{\text{D5}} = H_{\text{D5}}(z, r)$. Under these considerations, the brane set-up at hand gives rise to the fields below,

$$
\begin{aligned}
ds^2_{10} =& H_{\text{D7}}^{-1/2} H_{\text{D5}}^{-1/2} \Big[-H_{\text{D1}}^{-1/2} H_{\text{D3}}^{-1/2} H_{\text{F1}}^{-1}\, dt^2 + H_{\text{D1}}^{1/2} H_{\text{D3}}^{1/2} \big(d\rho^2 + \rho^2 ds^2_{S^3} \big) \Big] + \\
& + H_{\text{D7}}^{1/2} H_{\text{D5}}^{1/2} H_{\text{D1}}^{1/2} H_{\text{D3}}^{1/2} H_{\text{F1}}^{-1} dz^2 + H_{\text{D7}}^{1/2} H_{\text{D5}}^{-1/2} H_{\text{NS5}} H_{\text{D1}}^{-1/2} H_{\text{D3}}^{1/2} d\psi^2 + \\
& + H_{\text{D7}}^{-1/2} H_{\text{D5}}^{1/2} H_{\text{NS5}} H_{\text{D1}}^{1/2} H_{\text{D3}}^{-1/2} \big(dr^2 + r^2 ds^2_{S^2} \big) \,, \\
e^{\Phi} =& H_{\text{D7}}^{-1} H_{\text{D5}}^{-1/2} H_{\text{NS5}}^{1/2} H_{\text{D1}}^{1/2} H_{\text{F1}}^{-1/2} \,, \\
H_{(3)} =& -\partial_\rho H_{\text{F1}}^{-1} dt \wedge d\rho \wedge dz + \partial_r H_{\text{NS5}}\, r^2\, d\psi \wedge \text{vol}_{S^2} \,, \\
F_{(1)} =& H_{\text{D1}}^{-1} H_{\text{F1}}\, \partial_z H_{\text{D7}}\, d\psi \,, \\
F_{(3)} =& -H_{\text{D7}} \partial_\rho H_{\text{D1}}^{-1} dt \wedge d\rho \wedge d\psi - H_{\text{D7}}\, \partial_r H_{\text{D5}}\, r^2 dz \wedge \text{vol}_{S^2} + \\
& + H_{\text{F1}} H_{\text{D3}}^{-1} H_{\text{NS5}}\, r^2\, \partial_z H_{\text{D5}}\, dr \wedge \text{vol}_{S^2} \,, \\
F_{(5)} =& -H_{\text{D5}} H_{\text{NS5}}\, \partial_\rho H_{\text{D3}}^{-1}\, r^2\, dt \wedge d\rho \wedge dr \wedge \text{vol}_{S^2} + H_{\text{D7}} \partial_\rho H_{\text{D3}}\, \rho^3\, \text{vol}_{S^3} \wedge dz \wedge d\psi \,.
\end{aligned}
\tag{7.90}
$$

For this background, the equations of motion and Bianchi identities of Type IIB supergravity decouple in two groups. One group is associated to the D1-F1-D3 defect branes,

$$
\nabla^2_{\mathbb{R}^4_\rho} H_{\text{D1}} = 0 \quad \text{with} \quad H_{\text{D1}} = H_{\text{F1}} = H_{\text{D3}} \tag{7.91}
$$

[23]In the absence of D1-F1-D3 branes, one can T-dualise the D7-D5-NS5 subsystem in Table 7.8 along the ψ direction in order to obtain the D4-D8-KK system whose near-horizon geometry is the AdS_6 vacuum of massive IIA (orbifolded by $\mathbb{Z}_k$ [144]). The KK-monopoles arise from the dualisation of the NS5-branes. In the presence of D1-F1-D3 branes an extra D0-F1-D4′ bound state ending on the D4-D8-KK system is obtained. AdS_2 solutions associated to these brane intersections were constructed in [34, 43, 46], and interpreted as dual to D0-F1-D4′ baryon vertices within the 5d Sp(N) fixed point theory.

and the other to the D5-NS5-D7 background branes,

$$H_{\rm D7}\nabla^2_{\mathbb{R}^3_r}H_{\rm D5}+H_{\rm NS5}\,\partial_z^2H_{\rm D5}=0\,,\quad \nabla^2_{\mathbb{R}^3_r}H_{\rm NS5}=0\quad\text{and}\quad \partial_z^2H_{\rm D7}=0\,. \tag{7.92}$$

If we now pick the following particular solution to (7.91),

$$H_{\rm D1}=1+\frac{q_{\rm D1}}{\rho^2} \tag{7.93}$$

and we take the near-horizon limit by sending $\rho\to 0$, the following family of backgrounds arises[24],

$$\begin{aligned}
ds^2_{10}=&4^{-1}q_{\rm D1}H_{\rm D7}^{-1/2}H_{\rm D5}^{-1/2}\left[ds^2_{\rm AdS_2}+4ds^2_{S^3}\right]+H_{\rm D7}^{1/2}H_{\rm D5}^{1/2}dz^2+\\
&+H_{\rm D7}^{1/2}H_{\rm D5}^{-1/2}H_{\rm NS5}d\psi^2+H_{\rm D7}^{-1/2}H_{\rm D5}^{1/2}H_{\rm NS5}\left(dr^2+r^2ds^2_{S^2}\right)\,,\\
H_{(3)}=&-2^{-1}q_{\rm D1}^{1/2}{\rm vol}_{\rm AdS_2}\wedge dz+\partial_rH_{\rm NS5}\,r^2d\psi\wedge{\rm vol}_{S^2}\,,\quad e^{\Phi}=H_{\rm D7}^{-1}H_{\rm D5}^{-1/2}H_{\rm NS5}^{1/2}\,,\\
F_{(1)}=&\partial_zH_{\rm D7}\,d\psi\,,\\
F_{(3)}=&-2^{-1}q_{\rm D1}^{1/2}H_{\rm D7}{\rm vol}_{\rm AdS_2}\wedge d\psi-H_{\rm D7}\partial_rH_{\rm D5}\,r^2dz\wedge{\rm vol}_{S^2}+\\
&+H_{\rm NS5}\,r^2\,\partial_zH_{\rm D5}\,dr\wedge{\rm vol}_{S^2}\,,\\
F_{(5)}=&-2^{-1}q_{\rm D1}^{1/2}H_{\rm D5}H_{\rm NS5}\,r^2\,{\rm vol}_{\rm AdS_2}\wedge dr\wedge{\rm vol}_{S^2}-2q_{\rm D1}H_{\rm D7}\,{\rm vol}_{S^3}\wedge dz\wedge d\psi\,.
\end{aligned} \tag{7.94}$$

These backgrounds preserve $\mathcal{N}=4$ SUSY. The simplest way to infer this is to note that they are related to the $\mathcal{N}=(0,4)$ AdS$_3\times S^2$ solutions constructed in [46] through a double analytical continuation[25]. We thus obtained a class of $\mathcal{N}=4$ AdS$_2\times S^3\times S^2\times S^1\times I_z\times I_r$ geometries. Each solution of this class is characterised by a particular choice of the three functions $H_{\rm D7}(z)$, $H_{\rm D5}(z,r)$, $H_{\rm NS5}(r)$, which are solutions of the equations (7.92) and describe the dynamics of a D5-NS5-D7 bound state wrapping an AdS$_2\times S^3$ curved geometry.

7.6.1. Line defects within AdS$_6\times S^2\times\Sigma_2$ vacua

In our previous analysis we derived the supergravity solution describing D1-F1-D3 branes ending on a D5-NS5-D7 system. We also showed that in the near-horizon limit the brane solution gives rise to a class of $\mathcal{N}=4$ AdS$_2\times S^3\times S^2\times S^1\times I_z\times I_r$ geometries. These backgrounds are defined by the functions $H_{\rm D7}(z)$, $H_{\rm D5}(z,r)$, $H_{\rm NS5}(r)$ solving the equations of motion of the D5-NS5-D7 bound state, given by equation (7.92). We also mentioned that our solutions can be related via double analytic continuation to the $\mathcal{N}=(0,4)$ AdS$_3\times S^2$ solutions constructed in [46]. These other solutions originate from D3-D5-NS5 branes ending on a D5-NS5-D7 system, and under certain assumptions can be interpreted as holographic duals to surface defects within the 5d Sp(N) fixed point theory. One can check that the equations describing the D5-NS5-D7 subsystem of our brane set-up, given

[24]In order to have AdS$_2$ with unitary radius we performed the rescaling $t\to 2^{-1}q_{\rm D1}^{3/2}t$.
[25]See the solutions (5.13) of [46].

by (7.92), are exactly the same ones that allowed to find such defect interpretation in [46]. Therefore, we can take the same profiles for $H_{\rm D7}$, $H_{\rm D5}$ and $H_{\rm NS5}$ in order to find AdS$_6$ arising in the asymptotics[26]. These profiles are given by [159],

$$H_{\rm D5}=1+\frac{q_{\rm D5}}{\left(4q_{\rm NS5}r+\frac{4}{9}q_{\rm D7}z^3\right)^{5/3}}\,,\qquad H_{\rm NS5}=\frac{q_{\rm NS5}}{r}\,,\qquad H_{\rm D7}=q_{\rm D7}z\,,\tag{7.95}$$

where the parameters $q_{\rm D5}$, $q_{\rm D7}$ and $q_{\rm NS5}$ are the charges of the D5-, D7- and NS5-branes. As in [46], the AdS$_6\times S^2\times\Sigma_2$ geometry constructed in [159, 169] comes out after the change of coordinates,

$$r=9^{-1}q_{\rm D7}\,\mu^3\,\cos\alpha^2\,,\quad z=q_{\rm NS5}^{1/3}\,\mu\,\sin\alpha^{2/3}\,,\tag{7.96}$$

with $\mu>0$ and $\alpha\in[0,\frac{\pi}{2}]$. Indeed, rewriting the backgrounds (7.94), considering the warp factors (7.95) written in this parametrisation and taking the $\mu\to 0$ limit, one obtains,

$$\begin{aligned}ds_{10}^2=&s^{-1/3}[\overbrace{4^{-1}q_{\rm D1}\,q_{\rm NS5}^{2/3}\mu^2\left(ds^2_{\rm AdS_2}+4ds^2_{S^3}\right)+\frac{d\mu^2}{\mu^2}}^{\text{locally AdS}_6\text{ geometry}}+\\&+\frac{4}{9}d\alpha^2+9q_{\rm NS5}^2c^{-2}s^{2/3}d\psi^2+9^{-1}c^2ds^2_{S^2}]\,,\end{aligned}\tag{7.97}$$

where the 6d external part of the metric asymptotes locally to an AdS$_6$ geometry with unit radius. From this expression it is manifest that in the $\mu\to 0$ limit, the solutions take the form of a AdS$_6\times S^2\times\Sigma_2$ vacuum, where the Riemann surface Σ_2 is an annulus parametrised by the coordinates (α,ψ). Note however that AdS$_6$ arises only locally since extra, subleading, fluxes are also present in the solution that break the AdS$_6$ isometries. We also remark that, as the internal space in (7.97) is non-compact along the μ direction, the holographic central charge for the dual superconformal quantum mechanics turns out to be infinite. Indeed, using the formula for the AdS$_2$ case (7.43)[27] (see [105, 106, 172]), one finds the following holographic central charge,

$$\begin{aligned}c_{\rm hol}=&\qquad\frac{3}{8\pi^6}\int_{M_8}d^8y\,\sqrt{g_8}\,e^{-2\Phi}\\&\propto\quad q_{\rm D1}^{3/2}\,q_{\rm D5}^3\int d\psi\,d\alpha\,d\mu\cos^3\alpha\,\sin^{1/3}\alpha\,\mu^2\,,\end{aligned}\tag{7.98}$$

where the integration has been performed along the M_8 8d internal manifold of the AdS$_2$ spacetime. In this expression the divergence along the μ direction (which plays the role of AdS$_6$ radial coordinate) is manifest. This is exactly the situation one would expect for a 1d CFT dual to a conformal defect embedded in a higher dimensional CFT (see [34, 46, 66]).

Finally, it is easy to check that the new AdS$_2\times S^3\times S^2\times S^1\times\Sigma_2$ solutions defined by (7.94) are related by T-duality along the ψ direction to the AdS$_2\times S^3\times$ CY$_2\times I$

[26] For a detailed derivation see subsection 5.3 of [46].

[27] We fixed the ten-dimensional Newton's constant as $G_N=8\pi^6$.

solutions to massive IIA supergravity constructed in [43] in the case where $\text{CY}_2 = \mathbb{R}^4$. After the duality, the S^2 and the ψ direction give rise to a second S^3, which together with the r direction build up the $\mathbb{R}^4$. As already mentioned, it was shown in [34, 46] that these Type IIA solutions describe D0-F1-D4′ branes ending on the D4-D8 system. Furthermore, the detailed analysis of the dual field theory performed in [43] allowed to interpret the D0-branes as baryon vertices associated to the D8-branes of the background, and the D4′-branes as baryon vertices associated to the D4-branes[28]. Analogously, the D1-F1-D3 defect branes present in our AdS$_2$ solutions find an interpretation as D1 and D3 baryon vertices for the D7 and D5 background branes, respectively. The T-duality symmetry that relates these constructions guarantees that the 1d quivers constructed in [43], now built out of D1-D3 colour branes and D7-D5 flavour branes, describe 1d QMs that flow in the IR to the SCQMs dual to our solutions.

7.7. $\text{SL}(2,\mathbb{R})$ rotation of the D1-F1-D3-D5-NS5 brane set-up

In this section we focus on the subclass of solutions associated to the brane intersection depicted in Table 7.8 in the absence of D7-branes. Acting with a rotation included in the $\text{SL}(2,\mathbb{R})$ S-duality group of Type IIB supergravity, we obtain a covariant class of solutions depending on the parameter associated to the $\text{SL}(2,\mathbb{R})$ transformation. As usual, since only $\text{SL}(2,\mathbb{Z})$ is a symmetry of Type IIB string theory, continuous transformations determine new inequivalent backgrounds in the supergravity limit.

The exclusion of D7-branes is required so that a local analysis of $\text{SL}(2,\mathbb{R})$ rotations can be performed. This is because the NS7-brane (the S-dual of the D7-brane) is not well understood due to its highly non-perturbative nature. Note that this leaves the supersymmetries unaltered. Globally one is, of course, free to take the general brane set-up depicted in Table 7.8 and perform an S-duality transformation involving the D7-branes. However, we will refrain from doing this, as we are mainly interested in the local, supergravity description.

Taking as seed solution the brane intersection described by (7.90), setting $H_{D7} = 1$ and applying rules introduced in subsection 1.4.2, we arrive at the following family of

[28] The field theory is described by quiver-like constructions involving different nodes, and therefore different gauge groups, for both the D4- and the D8-branes. It is worth pointing out that in these constructions the D4- and D8-branes turn from colour branes, where the 5d Sp(N) gauge theory lives, to flavour branes, once the defect branes are introduced. The reader is referred to [43, 161] for more details on this description.

backgrounds,

$$
\begin{aligned}
ds_{10}^2 =&\Delta^{1/2}\Big[H_{\mathrm{D5}}^{-1/2}\left(-H_{\mathrm{D1}}^{-1/2}H_{\mathrm{D3}}^{-1/2}H_{\mathrm{F1}}^{-1}\,dt^2+H_{\mathrm{D1}}^{1/2}H_{\mathrm{D3}}^{1/2}\big(d\rho^2+\rho^2 ds^2_{S^3}\big)\right)\\
&+H_{\mathrm{D5}}^{1/2}H_{\mathrm{D1}}^{1/2}H_{\mathrm{D3}}^{1/2}H_{\mathrm{F1}}^{-1}dz^2+H_{\mathrm{D5}}^{-1/2}H_{\mathrm{NS5}}H_{\mathrm{D1}}^{-1/2}H_{\mathrm{D3}}^{1/2}d\psi^2\\
&+H_{\mathrm{D5}}^{1/2}H_{\mathrm{NS5}}H_{\mathrm{D1}}^{1/2}H_{\mathrm{D3}}^{-1/2}\big(dr^2+r^2ds^2_{S^2}\big)\Big]\,,\\
e^{\Phi}=&\Delta\, H_{\mathrm{D5}}^{-1/2}H_{\mathrm{NS5}}^{1/2}H_{\mathrm{D1}}^{1/2}H_{\mathrm{F1}}^{-1/2}\,,\quad C_{(0)}=\Delta^{-1}\left(\frac{H_{\mathrm{D5}}}{H_{\mathrm{NS5}}}\frac{H_{\mathrm{F1}}}{H_{\mathrm{D1}}}-1\right)\,sc\,,\\
H_{(3)}=&-c\,\partial_\rho H_{\mathrm{F1}}^{-1}dt\wedge d\rho\wedge dz+c\,\partial_r H_{\mathrm{NS5}}\,r^2\,d\psi\wedge\mathrm{vol}_{S^2}-s\,\partial_\rho H_{\mathrm{D1}}^{-1}dt\wedge d\rho\wedge d\psi\\
&-s\,\partial_r H_{\mathrm{D5}}\,r^2dz\wedge\mathrm{vol}_{S^2}+s\,H_{\mathrm{F1}}H_{\mathrm{D3}}^{-1}H_{\mathrm{NS5}}\,r^2\,\partial_z H_{\mathrm{D5}}\,dr\wedge\mathrm{vol}_{S^2}\,,\\
F_{(3)}=&-c\,\Delta^{-1}\partial_\rho H_{\mathrm{D1}}^{-1}dt\wedge d\rho\wedge d\psi+c\,\Delta^{-1}H_{\mathrm{F1}}H_{\mathrm{D3}}^{-1}H_{\mathrm{NS5}}\,r^2\,\partial_z H_{\mathrm{D5}}\,dr\wedge\mathrm{vol}_{S^2}\\
&-c\,\Delta^{-1}\partial_r H_{\mathrm{D5}}\,r^2dz\wedge\mathrm{vol}_{S^2}+s\,\Delta^{-1}\frac{H_{\mathrm{D5}}}{H_{\mathrm{NS5}}}\frac{H_{\mathrm{F1}}}{H_{\mathrm{D1}}}\,\partial_\rho H_{\mathrm{F1}}^{-1}dt\wedge d\rho\wedge dz\\
&-s\,\Delta^{-1}\frac{H_{\mathrm{D5}}}{H_{\mathrm{NS5}}}\frac{H_{\mathrm{F1}}}{H_{\mathrm{D1}}}\partial_r H_{\mathrm{NS5}}\,r^2\,d\psi\wedge\mathrm{vol}_{S^2}\,,\\
F_{(5)}=&-H_{\mathrm{D5}}H_{\mathrm{NS5}}\,\partial_\rho H_{\mathrm{D3}}^{-1}\,r^2\,dt\wedge d\rho\wedge dr\wedge\mathrm{vol}_{S^2}+\partial_\rho H_{\mathrm{D3}}\,\rho^3\,\mathrm{vol}_{S^3}\wedge dz\wedge d\psi\,,\\
\Delta=&\,c^2+\frac{H_{\mathrm{D5}}}{H_{\mathrm{NS5}}}\frac{H_{\mathrm{F1}}}{H_{\mathrm{D1}}}\,s^2\,,
\end{aligned}
\tag{7.99}
$$

where the family is parametrised by ξ and we have denoted $s=\sin\xi$, $c=\cos\xi$. The equations of motion and Bianchi identities are preserved by the SL(2,$\mathbb{R}$) rotation, therefore H_{D5} and H_{NS5} must still satisfy equation (7.92) for $H_{\mathrm{D7}}=1$. We highlight that, in the absence of D7-branes, we have that

$$
\nabla^2_{\mathbb{R}^4_\rho}H_{\mathrm{D1}}=0\quad\text{and}\quad\nabla^2_{\mathbb{R}^4_\rho}H_{\mathrm{F1}}=0\quad\text{with}\quad H_{\mathrm{D3}}=H_{\mathrm{F1}}\neq H_{\mathrm{D1}}
\tag{7.100}
$$

are satisfied instead of (7.91). We can then choose the particular solutions

$$
H_{\mathrm{D1}}=1+\frac{q_{\mathrm{D1}}}{\rho^2}\,,\quad H_{\mathrm{F1}}=1+\frac{q_{\mathrm{F1}}}{\rho^2}\,,
\tag{7.101}
$$

and send $\rho\to 0$. In this way we get a new class of $\mathcal{N}=4$ $\mathrm{AdS}_2\times S^3\times S^2\times S^1\times I_z\times I_r$

backgrounds to Type IIB supergravity of the form[29]

$$
\begin{aligned}
ds^2_{10} =& 4^{-1}\Delta^{1/2}\bigg[q_{\rm D1}^{1/2}q_{\rm F1}^{1/2}H_{\rm D5}^{-1/2}\left[ds^2_{\rm AdS_2}+4ds^2_{S^3}\right]+q_{\rm D1}^{1/2}q_{\rm F1}^{-1/2}H_{\rm D5}^{1/2}dz^2\\
&+H_{\rm D5}^{-1/2}H_{\rm NS5}\,q_{\rm D1}^{-1/2}q_{\rm F1}^{1/2}d\psi^2+H_{\rm D5}^{1/2}H_{\rm NS5}q_{\rm D1}^{1/2}q_{\rm F1}^{-1/2}\left(dr^2+r^2ds^2_{S^2}\right)\bigg]\,,\\
e^{\Phi} =& \Delta H_{\rm D5}^{-1/2}H_{\rm NS5}^{1/2}\,q_{\rm D1}^{1/2}q_{\rm F1}^{-1/2}\qquad \text{with}\qquad \Delta=c^2+\frac{q_{\rm F1}}{q_{\rm D1}}\frac{H_{\rm D5}}{H_{\rm NS5}}\,s^2\,,\\
H_{(3)} =& -2^{-1}c\,q_{\rm D1}^{1/2}{\rm vol}_{\rm AdS_2}\wedge dz+c\,\partial_rH_{\rm NS5}\,r^2d\psi\wedge{\rm vol}_{S^2}-2^{-1}s\,q_{\rm D1}^{-1/2}q_{\rm F1}{\rm vol}_{\rm AdS_2}\wedge d\psi\\
&-s\,\partial_rH_{\rm D5}\,r^2dz\wedge{\rm vol}_{S^2}+sH_{\rm NS5}\,r^2\,\partial_zH_{\rm D5}\,dr\wedge{\rm vol}_{S^2}\,,\\
F_{(1)} =& sc\,\Delta^{-2}H_{\rm NS5}^{-1}\frac{q_{\rm F1}}{q_{\rm D1}}\left[\partial_zH_{\rm D5}dz+\left(\partial_rH_{\rm D5}-H_{\rm NS5}^{-1}H_{\rm D5}\partial_rH_{\rm NS5}\right)dr\right],\\
F_{(3)} =& -2^{-1}c\,\Delta^{-1}q_{\rm D1}^{-1/2}q_{\rm F1}{\rm vol}_{\rm AdS_2}\wedge d\psi-c\,\Delta^{-1}\,\partial_rH_{\rm D5}\,r^2dz\wedge{\rm vol}_{S^2}+\\
&+c\,\Delta^{-1}H_{\rm NS5}\,r^2\,\partial_zH_{\rm D5}\,dr\wedge{\rm vol}_{S^2}+2^{-1}s\,\Delta^{-1}H_{\rm D5}H_{\rm NS5}^{-1}q_{\rm F1}q_{\rm D1}^{-1/2}{\rm vol}_{\rm AdS_2}\wedge dz\\
&-s\,\Delta^{-1}H_{\rm D5}H_{\rm NS5}^{-1}q_{\rm F1}q_{\rm D1}^{-1}\partial_rH_{\rm NS5}\,r^2d\psi\wedge{\rm vol}_{S^2}\,,\\
F_{(5)} =& -2^{-1}q_{\rm D1}^{1/2}H_{\rm D5}H_{\rm NS5}\,r^2\,{\rm vol}_{\rm AdS_2}\wedge dr\wedge{\rm vol}_{S^2}-2q_{\rm F1}\,{\rm vol}_{S^3}\wedge dz\wedge d\psi\,.
\end{aligned}
\tag{7.102}
$$

Here $H_{\rm D5}$ and $H_{\rm NS5}$ must satisfy the equations

$$
\nabla^2_{\mathbb{R}^3_r}H_{\rm D5}+H_{\rm NS5}\,\partial_z^2H_{\rm D5}=0\quad\text{and}\quad\nabla^2_{\mathbb{R}^3_r}H_{\rm NS5}=0\,. \tag{7.103}
$$

The above class of solutions describes (p',q') strings and D3-branes ending on orthogonal (p,q) 5-branes. It is in this sense more general than the class of solutions constructed in section 7.6. This is reflected by the fact that the D5 and NS5 charges are now distributed along the (z,ψ,ρ) directions while the D1 and F1 charges are mixed along (z,ψ). The interpretation of these solutions should be as holographic duals to D3 baryon vertices introduced in the 5d field theory living in D5-NS5 branes, with F1 (D1) strings in the completely antisymmetric representation of the D5 (NS5) gauge groups stretched between the D3 and the D5 (NS5) branes. It would be interesting to provide a concrete realisation of this set-up, along the lines of [43, 45, 47, 65].

It will be useful for our constructions in section 7.8 to have the explicit form of the solutions (7.102) particularised to $\xi=0,\frac{\pi}{2}$, that is, the two families of solutions in this

[29] As in the previous section we rescaled the coordinates as $t\to 2^{-1}q_{\rm D1}^{1/2}q_{\rm F1}t$ to have AdS$_2$ with unit radius.

class that are S-dual to one another. For $\xi=0$ we have

$$\begin{aligned}
ds_{10}^2 =&4^{-1}q_{\rm D1}^{1/2}q_{\rm F1}^{1/2}H_{\rm D5}^{-1/2}(ds^2_{\rm AdS_2}+4ds^2_{S^3})+q_{\rm D1}^{1/2}q_{\rm F1}^{-1/2}H_{\rm D5}^{1/2}dz^2\\
&+H_{\rm D5}^{-1/2}H_{\rm NS5}\,q_{\rm D1}^{-1/2}q_{\rm F1}^{1/2}d\psi^2+H_{\rm D5}^{1/2}H_{\rm NS5}q_{\rm D1}^{1/2}q_{\rm F1}^{-1/2}\left(dr^2+r^2ds^2_{S^2}\right)\,,\\
e^{\Phi} =&H_{\rm D5}^{-1/2}H_{\rm NS5}^{1/2}\,q_{\rm D1}^{1/2}q_{\rm F1}^{-1/2}\,,\\
H_{(3)} =&-2^{-1}\,q_{\rm D1}^{1/2}{\rm vol}_{\rm AdS_2}\wedge dz+\,\partial_r H_{\rm NS5}\,r^2d\psi\wedge{\rm vol}_{S^2}\,,\\
F_{(3)} =&-2^{-1}q_{\rm D1}^{-1/2}q_{\rm F1}{\rm vol}_{\rm AdS_2}\wedge d\psi-\,\partial_r H_{\rm D5}\,r^2dz\wedge{\rm vol}_{S^2}+\\
&+H_{\rm NS5}\partial_z H_{\rm D5}\,r^2\,dr\wedge{\rm vol}_{S^2}\,,\\
F_{(5)} =&-2^{-1}q_{\rm D1}^{1/2}H_{\rm D5}H_{\rm NS5}\,r^2\,{\rm vol}_{\rm AdS_2}\wedge dr\wedge{\rm vol}_{S^2}-2q_{\rm F1}\,{\rm vol}_{S^3}\wedge dz\wedge d\psi\,.
\end{aligned}\tag{7.104}$$

Note that this class of solutions is a generalisation of the backgrounds (7.94) with $H_{\rm D7}=1$. In this case, $H_{\rm D1}\neq H_{\rm F1}$ and therefore both q_{D1} and q_{F1} quantised charges are present. In turn, for $\xi=\frac{\pi}{2}$ the fields read

$$\begin{aligned}
ds_{10}^2 =&4^{-1}q_{\rm F1}H_{\rm NS5}^{-1/2}(ds^2_{\rm AdS_2}+4ds^2_{S^3})+H_{\rm NS5}^{-1/2}H_{\rm D5}dz^2+\\
&+q_{\rm F1}q_{\rm D1}^{-1}H_{\rm NS5}^{1/2}d\psi^2+H_{\rm NS5}^{1/2}H_{\rm D5}(dr^2+r^2ds^2_{S^2})\,,\\
e^{\Phi} =&H_{\rm NS5}^{-1/2}H_{\rm D5}^{1/2}q_{\rm F1}^{1/2}q_{\rm D1}^{-1/2}\,,\\
H_{(3)} =&-2^{-1}q_{\rm F1}q_{\rm D1}^{-1/2}\,{\rm vol}_{\rm AdS_2}\wedge d\psi-\partial_r H_{\rm D5}r^2dz\wedge{\rm vol}_{S^2}+\\
&+H_{\rm NS5}\partial_z H_{\rm D5}r^2\,dr\wedge{\rm vol}_{S^2}\,,\\
F_{(3)} =&2^{-1}q_{\rm D1}^{1/2}\,{\rm vol}_{\rm AdS_2}\wedge dz-\partial_r H_{\rm D5}r^2\,d\psi\wedge{\rm vol}_{S^2}\,,\\
F_{(5)} =&-2^{-1}q_{\rm D1}^{1/2}H_{\rm D5}H_{\rm NS5}r^2\,{\rm vol}_{\rm AdS_2}\wedge dr\wedge{\rm vol}_{S^2}-2q_{\rm F1}{\rm vol}_{S^3}\wedge dz\wedge d\psi\,.
\end{aligned}\tag{7.105}$$

Finally, we can provide a unified expression for the central charge of the whole family of $\mathrm{SL}(2,\mathbb{R})$ solutions, since this quantity is $\mathrm{SL}(2,\mathbb{R})$ invariant. Substituting the metric and dilaton of the backgrounds (7.102) in (7.98) we indeed find

$$c_{\rm hol}=\frac{3}{8\pi^6}\int_{M_8}d^8y\,\sqrt{g_8}\,e^{-2\Phi}=\frac{3}{8\pi^6}\,q_{\rm D1}^{1/2}\,q_{\rm F1}\,{\rm Vol}_{S^3}{\rm Vol}_{S^2}\int d\psi\,dr\,dz\,r^2H_{\rm D5}\,H_{\rm NS5}\,,\tag{7.106}$$

which is independent on the ξ-parameter.

7.7.1. Web of dualities and M-theory origin

In this subsection we discuss the Type IIA realisation and M-theory origin of the S-dual solutions to Type IIB that we built in the previous subsection. As we already mentioned the defect interpretation within $\mathrm{AdS}_6\times S^2\times\Sigma_2$ is lost. Instead, the solutions allow for an interesting realisation as line defects within the 6d $\mathcal{N}=(1,0)$ CFT dual to $\mathrm{AdS}_7/\mathbb{Z}_k\times S^4$ in M-theory.

The two S-dual solutions with $\xi=0,\frac{\pi}{2}$ in (7.104) and (7.105) are related by T-duality to the $\mathrm{AdS}_2\times S^3\times\mathbb{R}^4/\mathbb{Z}_k$ solutions constructed in [43][30], and to the $\mathrm{AdS}_3/\mathbb{Z}_{k'}\times S^3\times S^2$

[30]Restricted to the massless case, since we are not allowing for D7-branes.

solutions to massless IIA supergravity constructed in [58][31], respectively. As shown in [58] these solutions share a common origin in M-theory, in the form of $\mathrm{AdS}_3 \times S^3 \times S^3/\mathbb{Z}_k$ backgrounds ($\mathrm{AdS}_3/\mathbb{Z}_{k'}$ in our case), also classified in said reference. These solutions to 11d supergravity were shown to asymptote to $\mathrm{AdS}_7/\mathbb{Z}_k \times S^4$ in the UV. Our solutions are thus interpreted as duals to line defects in the 6d $\mathcal{N} = (1,0)$ CFT dual to this background, once uplifted to M-theory. The web of dualities connecting these classes of solutions is depicted in Figure 7.14, that we now explain in detail. Starting with the bottom left

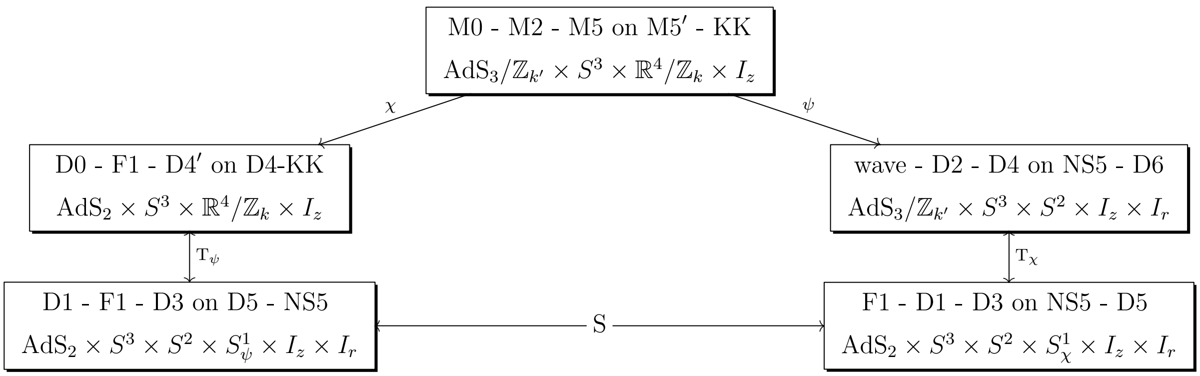


Figure 7.14: Web of dualities that relate the new AdS_2 solutions in Type IIB written in (7.104) and (7.105) to the Type IIA and M-theory solutions constructed in [43] and [58].

solution of Type IIB and performing a T-duality along the S^1_ψ circle, an S^3 is built up with the S^1_ψ and the S^2. This S^3 gives rise to an $\mathbb{R}^4/\mathbb{Z}_k$ space together with the I_r interval. Here the integer k is the number of NS5-branes present in the Type IIB solution, which become KK-monopoles in Type IIA. The result is a Type IIA background contained within the class found in [43], for $\mathrm{CY}_2 = \mathbb{R}^4/\mathbb{Z}_k$. The underlying brane set-up is described by a D4-KK-F1-D4′-D0 intersection studied in [34] and it is depicted in Table 7.9a. We already referred to this T-duality transformation in Section 7.6, for the more general situation in which D7-branes were also present. If we now uplift the Type IIA solution to M-theory, the

branes	t	ρ	φ^1	φ^2	φ^3	z	ψ	r	θ^1	θ^2
D4	×	×	×	×	×	–	–	–	–	–
KK	×	×	×	×	×	×	ISO	–	–	–
F1	×	–	–	–	–	×	–	–	–	–
D4′	×	–	–	–	–	–	×	×	×	×
D0	×	–	–	–	–	–	–	–	–	–

(a) Brane intersection consisting on D0-F1-D4′ branes ending on a D4-KK bound system. This Type IIA brane set-up is T-dual to the the original Type IIB one depicted in Table 7.8 in the absence of D7-branes.

branes	t	χ	ρ	φ^1	φ^2	φ^3	z	r	θ^1	θ^2
NS5	×	×	×	×	×	×	–	–	–	–
D6	×	×	×	×	×	×	×	–	–	–
D2	×	×	–	–	–	–	×	–	–	–
D4	×	×	–	–	–	–	–	×	×	×
W	×	ISO	–	–	–	–	–	–	–	–

(b) Brane intersection consisting on W-D2-D4 branes ending on a NS5-D6 bound system. This Type IIA brane set-up is related by a chain of TST dualities to the the one in Table 7.8 in the absence of D7-branes, as it can be seen in Figure 7.14.

Table 7.9: The $\frac{1}{8}$-BPS brane set-ups in Type IIA appearing in Figure 7.14.

result is an $\mathrm{AdS}_3/\mathbb{Z}_{k'}$ space, built up with the AdS_2 and the M-theory circle (parametrised

[31] With the AdS_3 modded out by $\mathbb{Z}_{k'}$, but this is a trivial extension of the solutions in [58].

by the χ coordinate). Here k' is the number of F1-strings in the Type IIA solution, which become waves, or units of momentum, in M-theory. The M-theory intersection underlying these solutions is depicted in Table 7.10 and it is defined by an intersection of M5′-KK-M2-M5-M0 branes. The corresponding class of $\mathrm{AdS}_3/\mathbb{Z}_{k'} \times S^3 \times \mathbb{R}^4/\mathbb{Z}_k \times I_z$

branes	t	χ	ρ	φ^1	φ^2	φ^3	z	ψ	r	θ^1	θ^2
M5′	×	×	×	×	×	×	–	–	–	–	–
KK	×	×	×	×	×	×	×	ISO	–	–	–
M2	×	×	–	–	–	–	×	–	–	–	–
M5	×	×	–	–	–	–	–	×	×	×	×
M0	×	ISO	–	–	–	–	–	–	–	–	–

Table 7.10: 1/8-BPS brane set-up in M-theory consisting on M2-M5-M0 branes ending on M5′-branes with KK-monopoles. The reduction to Type IIA can be performed over either χ or ψ, giving rise to the Type IIA brane set-ups in Table 7.9 that parametrise the isometric directions and are respectively associated to the momentum waves M0 and the KK-monopoles.

solutions to M-theory was found in [173]. In turn, it belongs to the more general class of $\mathrm{AdS}_3 \times S^3 \times S^3/\mathbb{Z}_k \times \Sigma_2$ solutions presented in [58], in our case orbifolded by $\mathbb{Z}_{k'}$.

Taking now these 11d solutions as our starting point, but reducing instead along the $S^1_\psi/\mathbb{Z}_k$ Hopf fibre of the $S^3/\mathbb{Z}_k$ contained in $\mathbb{R}^4/\mathbb{Z}_k$, we obtain a solution in Type IIA in the class constructed in [58][32], with extra k' waves, or units of momentum. The corresponding brane set-up is presented in Table 7.9b and it is given by an intersection of D6-NS5-D4-D2 branes with momentum waves W. T-dualising along the Hopf fibre of the $\mathrm{AdS}_3/\mathbb{Z}_{k'}$ subspace we finally arrive at the Type IIB solution shown at the bottom right of Figure 7.14, containing k' F1-strings. As expected due to their common M-theory origin, both solutions in Type IIB are related to each other by S-duality.

7.8. Non-Abelian T-duals to the S-dual solutions

In this section we present new AdS_2 solutions to Type IIA supergravity preserving 4 supercharges obtained by performing a non-Abelian T-duality transformation along the S^3 on the two S-dual backgrounds with $\xi = 0$ and $\xi = \pi/2$ given in (7.104) and (7.105). These Type IIA backgrounds depend on two defining functions $H_{\mathrm{D5}} = H_{\mathrm{D5}}(z, r)$ and

[32] And later generalised to the massive case in [66].

$H_{\rm NS5}=H_{\rm NS5}(r)$ satisfying the master equations below [33],

$$\nabla^2_{\mathbb{R}^3_r}H_{\rm D5}+\frac{q_{\rm F1}}{q_{\rm D3}}H_{\rm NS5}\,\partial_z^2H_{\rm D5}=0\quad\text{and}\quad\nabla^2_{\mathbb{R}^3_r}H_{\rm NS5}=0\,. \tag{7.107}$$

Under non-Abelian T-duality the S^3 of the original background is transformed into an open subset of $\mathbb{R}^3$, parametrised by the radial coordinate R and the 2-sphere $\tilde{S}^2$. For $\xi=0$ the new class of non-Abelian T-dual solutions is given by,

$$\begin{aligned}
ds^2_{10}&=4^{-1}q_{\rm D1}^{1/2}q_{\rm D3}^{1/2}H_{\rm D5}^{-1/2}ds^2_{\rm AdS_2}+q_{\rm D1}^{1/2}q_{\rm D3}^{1/2}q_{\rm F1}^{-1}H_{\rm D5}^{1/2}dz^2+q_{\rm D1}^{-1/2}q_{\rm D3}^{1/2}H_{\rm D5}^{-1/2}H_{\rm NS5}\,d\psi^2+\\
&\quad+q_{\rm D1}^{1/2}q_{\rm D3}^{-1/2}H_{\rm D5}^{1/2}H_{\rm NS5}\left(dr^2+r^2ds^2_{S^2}\right)+q_{\rm D1}^{-1/2}q_{\rm D3}^{-1/2}H_{\rm D5}^{1/2}4(dR^2+HR^2ds^2_{\tilde{S}^2})\,,\\
e^{\Phi}&=8q_{\rm D1}^{-1/4}q_{\rm F1}^{-1/2}q_{\rm D3}^{-3/4}H_{\rm D5}^{1/4}H_{\rm NS5}^{1/2}\,H^{1/2}\,,\\
H_{(3)}&=-2^{-1}\,q_{\rm D1}^{1/2}q_{\rm D3}^{1/2}q_{\rm F1}^{-1/2}\mathrm{vol}_{\rm AdS_2}\wedge dz+\,\partial_rH_{\rm NS5}\,r^2d\psi\wedge\mathrm{vol}_{S^2}\\
&\quad+\partial_zHRdz\wedge\mathrm{vol}_{\tilde{S}^2}+\partial_rHRdr\wedge\mathrm{vol}_{\tilde{S}^2}+\partial_R((H-1)R)\;dR\wedge\mathrm{vol}_{\tilde{S}^2}\,,\\
F_{(2)}&=-4^{-1}q_{\rm D3}\,dz\wedge d\psi\,,\\
F_{(4)}&=d[2^{-1}q_{\rm D1}^{-1/2}q_{\rm F1}^{1/2}q_{\rm D3}^{1/2}(3/2+(H-1)^{-1})R^2\mathrm{vol}_{\rm AdS_2}\wedge d\psi]\\
&\quad+4q_{\rm D1}^{-1}HR^3H_{\rm D5}\,dz\wedge d\psi\wedge\mathrm{vol}_{\tilde{S}^2}-4^{-1}q_{\rm D1}H_{\rm D5}H_{\rm NS5}\,r^2\,dz\wedge dr\wedge\mathrm{vol}_{S^2}+\\
&\quad+r^2R(q_{\rm F1}q_{\rm D3}^{-1}H_{\rm NS5}\,\partial_zH_{\rm D5}dr-\partial_rH_{\rm D5}\,dz)\;\wedge\mathrm{vol}_{S^2}\wedge dR=,
\end{aligned}\tag{7.108}$$

where we have defined the function below,

$$H=\frac{q_{\rm D1}q_{\rm D3}}{q_{\rm D1}q_{\rm D3}+16R^2H_{\rm D5}}\,. \tag{7.109}$$

[33] In this section we restore the integration constant $q_{\rm D3}$ associated to D3 defect branes in the S-dual Type IIB backgrounds (7.104) and (7.105). We recall that this parameter was fixed as $q_{\rm D3}=q_{\rm F1}$ at the level of the brane solution $H_{\rm D3}=1+\frac{q_{\rm D3}}{\rho^2}$, $H_{\rm F1}=1+\frac{q_{\rm F1}}{\rho^2}$ in (7.102) by the conditions $\nabla^2_{\mathbb{R}^4_\rho}H_{\rm D3}=\nabla^2_{\mathbb{R}^4_\rho}H_{\rm F1}=0$ and $H_{\rm D3}=H_{\rm F1}$ coming from the equations of motion for the defect branes (written in (7.100)). The freedom to keep $q_{\rm D3}$ unconstrained at the near-horizon is provided by the fact that the condition $H_{\rm D3}=H_{\rm F1}$ is a particular realisation of the slightly more general condition $H_{\rm D3}H'_{\rm F1}=H'_{\rm D3}H_{\rm F1}$, implied by the equations of motion. Outside of the near-horizon these two conditions are equivalent and imply that $q_{\rm D3}=q_{\rm F1}$, but in the $\rho\to0$ limit the absence of the "1" factor in the harmonic functions $H_{\rm D3}=\frac{q_{\rm D3}}{\rho^2}$, $H_{\rm F1}=\frac{q_{\rm F1}}{\rho^2}$ allows one to avoid any constraint on $q_{\rm D3}$ in terms of the other integration constants. The AdS$_2$ factor with unitary radius in the metrics of the S-dual solutions in Type IIB is realised by the rescaling of the time direction $t\to2^{-1}q_{\rm D1}^{1/2}q_{\rm D3}^{1/2}q_{\rm F1}^{1/2}\,t$.

In turn, for $\xi = \frac{\pi}{2}$ we find the new class,

$$
\begin{aligned}
ds^2_{10} &= 4^{-1} q_{\mathrm{F1}}^{1/2} q_{\mathrm{D3}}^{1/2} H_{\mathrm{NS5}}^{-1/2} ds^2_{\mathrm{AdS}_2} + q_{\mathrm{F1}}^{-1/2} q_{\mathrm{D3}}^{1/2} H_{\mathrm{D5}} H_{\mathrm{NS5}}^{-1/2} dz^2 + H_{\mathrm{NS5}}^{1/2} q_{\mathrm{F1}}^{1/2} q_{\mathrm{D3}}^{1/2} q_{\mathrm{D1}}^{-1} d\psi^2 + \\
&\quad + q_{\mathrm{F1}}^{1/2} q_{\mathrm{D3}}^{-1/2} H_{\mathrm{D5}} H_{\mathrm{NS5}}^{1/2} \left(dr^2 + r^2 ds^2_{S^2}\right) + 4 q_{\mathrm{F1}}^{-1/2} q_{\mathrm{D3}}^{-1/2} H_{\mathrm{NS5}}^{1/2} (dR^2 + \tilde{H} R^2 ds^2_{\tilde{S}^2}) \,, \\
e^{\Phi} &= 8 H_{\mathrm{D5}}^{1/2} H_{\mathrm{NS5}}^{1/4} q_{\mathrm{D1}}^{-1/2} q_{\mathrm{F1}}^{-1/4} q_{\mathrm{D3}}^{-3/4} \tilde{H}^{1/2} \,, \\
H_{(3)} &= -2^{-1} q_{\mathrm{F1}}^{1/2} q_{\mathrm{D3}}^{1/2} q_{\mathrm{D1}}^{-1/2} \mathrm{vol}_{\mathrm{AdS}_2} \wedge d\psi + \\
&\quad + (q_{\mathrm{F1}} q_{\mathrm{D3}}^{-1} H_{\mathrm{NS5}} \, \partial_z H_{\mathrm{D5}} dr - \partial_r H_{\mathrm{D5}} \, dz) \, r^2 d\psi \wedge \mathrm{vol}_{S^2} + d((\tilde{H} - 1) R \mathrm{vol}_{\tilde{S}^2}) \,, \\
F_{(2)} &= -4^{-1} q_{\mathrm{D3}} \, dz \wedge d\psi \,, \\
F_{(4)} &= -d[2^{-1} q_{\mathrm{D1}}^{1/2} q_{\mathrm{F1}}^{-1/2} q_{\mathrm{D3}}^{1/2} (3/2 + (\tilde{H} - 1)^{-1}) R^2 \mathrm{vol}_{\mathrm{AdS}_2} \wedge dz] + \\
&\quad + r^2 (4^{-1} q_{\mathrm{F1}} H_{\mathrm{D5}} H_{\mathrm{NS5}} \, dr + R \partial_r H_{\mathrm{D5}} \, dR) \wedge d\psi \wedge \mathrm{vol}_{S^2} \\
&\quad - 4^{-1} q_{\mathrm{D3}} R^{-1} (\tilde{H} - 1) \, dz \wedge d\psi \wedge \mathrm{vol}_{\tilde{S}^2} \,,
\end{aligned}
\tag{7.110}
$$

where we have denoted

$$
\tilde{H} = \frac{q_{\mathrm{F1}} q_{\mathrm{D3}}}{q_{\mathrm{F1}} q_{\mathrm{D3}} + 16 R^2 H_{\mathrm{NS5}}} \,. \tag{7.111}
$$

The fluxes of these solutions are compatible with the brane configurations shown in Table 7.11. We point out that, as usual for non-Abelian T-dual solutions, a clear prescription to construct the full brane solutions describing the set-ups of Table 7.11 and reproducing (7.108) and (7.110) in the near-horizon limit is not available. Nevertheless we can consider

branes	t	ρ	R	χ^1	χ^2	z	ψ	r	θ^1	θ^2
D4	×	×	−	×	×	−	×	−	−	−
D2	×	×	−	−	−	−	×	−	−	−
NS5	×	×	×	×	×	×	−	−	−	−
D2′	×	−	×	−	−	−	×	−	−	−
D4′	×	−	×	×	×	−	×	−	−	−
F1	×	−	−	−	−	×	−	−	−	−
D4″	×	−	×	−	−	−	−	×	×	×
D6	×	−	×	×	×	−	−	×	×	×
NS5′	×	−	−	−	−	×	×	×	×	×

branes	t	ρ	R	χ^1	χ^2	z	ψ	r	θ^1	θ^2
D4	×	×	−	×	×	×	−	−	−	−
D2	×	×	−	−	−	×	−	−	−	−
NS5	×	×	×	×	×	−	×	−	−	−
D2′	×	−	×	−	−	×	−	−	−	−
D4′	×	−	×	×	×	×	−	−	−	−
F1	×	−	−	−	−	−	×	−	−	−
D4″	×	−	×	−	−	−	−	×	×	×
D6	×	−	×	×	×	−	−	×	×	×
NS5′	×	−	−	−	−	×	×	×	×	×

Table 7.11: Brane set-ups compatible with the fluxes of the non-Abelian T-dual solutions (7.108) and (7.110). The coordinates (R, χ^1, χ^2) parametrise the open subset of $\mathbb{R}^3$ generated by the action of non-Abelian T-duality on the S^3 factor of the Type IIB backgrounds.

their M-theory uplifts. In subsection 7.7.1 we discussed the M-theory interpretation of the Abelian T-duals of the Type IIB backgrounds with $\xi = 0$ and $\xi = \frac{\pi}{2}$, observing that the two corresponding 11d solutions arise from the same intersection in M-theory (with different smearing of brane charges). Even if for the non-Abelian T-dual backgrounds we do not have full control over the brane solutions behind the AdS_2 backgrounds (7.108) and (7.110), it is possible to show that their M-theory uplifts are related to each other, provided that one makes some assumptions on the spacetime dependence of the function

H_{D5}, which implies a particular choice of the charge distribution of branes underlying the non-Abelian T-dual solutions.

The backgrounds (7.108) and (7.110) can be uplifted to 11d supergravity by applying (1.226). For the 11d backgrounds to be related, one needs to choose the same gauge potential for the $F_{(2)}$ flux of the original Type IIA solutions, namely

$$C_{(1)} = \frac{q_{\mathrm{D3}}}{8}\,(\psi dz - z d\psi)\ , \tag{7.112}$$

which is invariant under the following relabelling of the coordinates, $(z,\psi)\to(\psi,-z)$. We observe that the parameter q_{D3}, whose inclusion in the non-Abelian T-dual backgrounds was discussed in footnote 33, gains a natural interpretation in M-theory as KK-monopole charge.

It was shown in [93] that the Abelian T-dual of a certain background can be obtained from the corresponding non-Abelian T-dual one by sending the radial direction of the dual space $\mathbb{R}^3$ to infinity and further compactifying it to the interval $[0,\pi]$. Taking this limit in the solution (7.108), we recover the Abelian T-dual of the $\xi=0$ solution (7.104), where now $R\in[0,\pi]$. From this Abelian T-dual, we can take the uplift to 11d along the χ direction, rotate the coordinates as $(\chi,R)\to(R,-\chi)$ and go back to Type IIA. This recovers the Abelian T-dual of the $\xi=\frac{\pi}{2}$ solution. Such a procedure confirms the reliability of the non-Abelian T-dual backgrounds, since the corresponding Abelian T-duals are shown to be related to the S-dual solutions in Type IIB with $\xi=0$ and $\xi=\frac{\pi}{2}$. Furthermore, the two circular coordinates (χ,R) parametrise the 2-torus in M-theory that provides the geometrisation of the S-duality transformation in Type IIB.

As it was expected, the 11d uplifts of the solutions (7.108) and (7.110) are not related anymore by a simple rotation of the coordinates as for their Abelian limits. This is reflecting an "exotic" charge distribution as underlying the intersections depicted in Table 7.11, which modifies the standard chain of dualities connecting Type IIB string theory to M-theory. Such an "exotic" charge distribution could be related to the presence of dyonic membranes, which, as shown in [59], define an additional warping between the AdS factor and the internal space, as in (7.108) and (7.110). A dyonic membrane is basically an exotic bound state where a regular brane carries charge of another kind of brane dissolved in its worldvolume. For instance, in the aforementioned reference the M5 dyonic membrane, an M5-brane with M2 charge dissolved in its worldvolume, was explored.

Conclusions

In this thesis, we have built new AdS_2 and AdS_3 solutions to Type II supergravities and explored the holographic correspondence through them.

As explained in chapter 5, this was attained by following a series of steps. The first one is to obtain one of these low-dimensional AdS backgrounds. This was achieved either by taking the near-horizon limit of brane configurations, via S- or T-duality of known backgrounds or by restricting a more general class of solutions with a certain amount of supersymmetry (computed using G-structure tools). After finding one such solution, the next step consists on computing the quantised charges in order to obtain the brane distribution and then consider a compatible Hanany-Witten brane set-up. Next one considers the massless modes of open strings ending on the different branes, which gives rise to the possible multiplets of the dual quiver theory. These theories are conjectured to be UV deformations of the CFTs dual to the corresponding AdS supergravity solutions. In order to support this hypothesis, one can then compare the central charge computed from the supergravity solution to that of the quiver field theory living in the underlying brane set-up. These two quantities coincide in the IR for our solutions, heavily implying that the aforementioned hypothesis should be true.

Chapter 6 was dedicated to the $\mathrm{AdS}_3/\mathrm{CFT}_2$ scenario. We started in section 6.1 by presenting a new class of $\mathrm{AdS}_3\times S^3\times M_4$ solutions to massive IIA supergravity and studying its supersymmetry. We showed that in the most general case it arises in the near-horizon limit of a D2-D4-D6-NS5-D8 brane intersection. Next we explored two relevant subclasses of solutions.

The first one consists on the case where $M_4 = S^2\times\Sigma_2$, where Σ_2 is a Riemann surface. In section 6.2 we showed that such geometries in the UV flow asymptotically locally to the $\mathrm{AdS}_7\times S^2\times I$ backgrounds built in [126]. We then embedded the 2d $\mathcal{N}=(0,4)$ quivers associated to the AdS_3 solutions into the 6d quivers that describe the 6d $\mathcal{N}=(1,0)$ CFTs dual to the AdS_7 spaces. Our analysis proved the exact agreement between the field theory and holographic central charges, even if both quantities are divergent due to the existence of the non-compact direction inherent to the defect. The defect interpretation is based on the fact that the presence of the non-compact direction allows to build up the AdS_7 geometry asymptotically and therefore to complete the non-compact AdS_3 solutions in the UV.

The second subclass of AdS_3 solutions takes place when $M_4=\mathbb{T}^3\times I$. The study of these backgrounds was particularised for the massless and massive cases. The former was explored in section 6.3, where we showed that supersymmetry is enhanced to $\mathcal{N} = (4, 4)$ and that this subclass arises in the near-horizon limit of a D2-D4-NS5 brane intersection and is holographically dual to 2d CFTs with 8 supercharges. Although these set-ups were studied long ago in [138,139], their dual field theories had not been explored and, therefore, our results have contributed in filling this gap. We remark that the global description we found for these AdS_3 solutions requires the presence of ONS5 orientifold fixed planes. Although they are well-defined objects in string theory, they must be fully localised and not smeared, as they are along the $\mathbb{T}^3$ in our case. We argued that this smearing may be just an artefact of the supergravity approximation which disappears in string theory. Nevertheless, if one considers a more conservative approach, the existence of solutions with smeared ONS5 planes suggest that there may be similar solutions where the ONS5 planes are localised, as it is often the case when O-planes are present. Such constructions are much harder than the ones we used and lie outside the scope of this thesis, but our results provide some hope of them being possible. Besides, we connected this class of AdS_3 solutions of massless Type IIA with M-theory, thus relating our solutions to those constructed in [38], characterised by an $\mathrm{AdS}_3\times\mathrm{S}^2 \times \mathbb{T}^4 \times I$ geometry. This realisation provides a new interpretation for the dual $\mathcal{N} = (4, 4)$ quiver QFTs we built and also for the $\mathcal{N} = (0, 4)$ ones in [38] as deformations of a 2d $\mathcal{N} = (4, 4)$ QFT whose amount of preserved supersymmetry depends on how it is deformed in the UV. We have completed our analysis with a study in Type IIB string theory, thus connecting our $\mathrm{AdS}_3/\mathrm{CFT}_2$ pair with the one studied in [38] by S-duality. The realisation in Type IIB shows that mirror symmetry in 2d interchanges the scalars in the hypermultiplets and twisted hypermultiplets, instead of the scalars in the vector multiplets and hypermultiplets (and therefore the Coulomb and Higgs branches) as in 3d [108, 151]. That mirror symmetry can still be realised in this way in theories without a Coulomb branch is a remarkable output of our analysis. These AdS_3 solutions in Type IIB provide concrete examples within the broad classification of $\mathrm{AdS}_3\times\mathrm{S}^2\times\mathrm{M}_5$ vacua with M_5 supporting an identity-structure derived in [51].

Finally, in section 6.4 we studied the massive case of this class of $\mathrm{AdS}_3 \times S^3 \times \mathbb{T}^3 \times I$ solutions. We obtained solutions with non-compact parts glued together with localised D8-branes, bounded between D8/O8s. These solutions were globally embedded in Type I′ string theory. This permitted us to propose a dual CFT, which was supported by the exact agreement of the central charges of both theories. In this case the condition for anomaly cancellation of the 2d quivers implies an additional constraint on the dual supergravity background. We had to implement this condition by hand, but it would be interesting to reproduce it with a gravity computation.

In chapter 7, we studied new AdS_2 solutions and, when possible, presented the underlying brane intersection and constructed the dual quiver quantum mechanics. This provides new evidence for the $\mathrm{AdS}_2/\mathrm{SCQM}$ correspondence.

In section 7.1 we built a vast class of $\mathrm{AdS}_2 \times S^2 \times S^2 \times \mathbb{R}^2 \times S^1 \times I$ solutions to Type IIA supergravity preserving four supercharges. They arise in the near-horizon limit of D2-F1-D4′-NS5′ branes ending on a bound state of D4-NS5 branes. A particular solution within this class is determined by a choice of the D4-NS5 charge distribution. A particular semi-localised profile for said branes gives rise to two interesting regimes. The first one consists on approaching the D2-F1-D4′-NS5′ defect branes, which are resolved into a fully backreacted $\mathrm{AdS}_2 \times S^2 \times S^2 \times \mathbb{R}^2 \times S^1 \times I$ geometry. The second regime becomes manifest only after a non-linear change of coordinates of the previous solution, and allows one to move away from the defect branes. In this regime an AdS_5 geometry arises asymptotically, which corresponds to the near-horizon geometry of the D4-NS5 branes. This particular AdS_5 vacuum is the T-dual of the $\mathrm{AdS}_5 \times S^5/\mathbb{Z}_n$ solution to Type IIB supergravity, holographically dual to 4d $\mathcal{N} = 4$ SYM modded by $\mathbb{Z}_n$. This fact has allowed us to interpret this class of solutions as dual to line defect CFTs within 4d $\mathcal{N} = 4$ SYM modded by $\mathbb{Z}_n$.

We then turned to the Type IIB analysis in section 7.2 by considering solutions linked via T-duality along a circle direction ψ to the ones studied in the previous section. In this case we obtained a new family of $\mathrm{AdS}_2 \times S^2 \times S^2 \times \mathbb{R}^2 \times S^1 \times I$ solutions with the same supersymmetries as before, now emerging in the near-horizon limit of D1-F1-D5-NS5 branes ending on a D3-KK system. We computed the analogue to the second regime described above, where an $\mathrm{AdS}_5 \times S^5/\mathbb{Z}_n$ solution of Type IIB supergravity emerges asymptotically locally. The solutions thus admit as well a line defect interpretation, this time in terms of D1-F1-D5-NS5 defect branes.

In section 7.3 we went back to the F1-D2-D4′-NS5′-D4-NS5 brane intersection and generalised it to include D6-branes. The choice of the semi-localised profile for the D4-NS5 branes is no longer available so the regime where the solution asymptotes locally to the AdS_5 vacuum is lost here. We then focused on a detailed analysis of the dual quiver quantum mechanics, taking a simplified ansatz which consisted on taking y to be a circle direction so the D4- and D6-branes are smeared along it. Remarkably, these are the same quiver quantum mechanics that flow in the IR to the 1d CFTs studied in [43], dual to the $\mathcal{N} = 4$ $\mathrm{AdS}_2 \times S^3 \times \mathrm{CY}_2 \times I$ solutions to massive Type IIA studied therein. The underlying reason behind this agreement is that, when the branes are smeared on y, our brane system is related by two T-dualities to the brane system discussed in [43], consisting on a D0-F1-D4-D4′-D8 intersection. The 1d dual CFT associated to this class of solutions was interpreted in terms of baryon vertices within 5d fixed point theories living in D4-D8 intersections, and an asymptotically locally AdS_6 geometry was shown to arise for certain solutions. Our findings show that the same quiver quantum mechanics describe in the UV line defect CFTs associated to baryon vertices within 4d $\mathcal{N} = 2$ SCFTs. However, in our case we are still lacking a defect completion within an AdS_5 vacuum in Type IIA. Nevertheless, the smearing in y allows to make connection with the class of $\mathrm{AdS}_3 \times S^2 \times S^2 \times S^1 \times \Sigma_2$ solutions to Type IIB supergravity studied in [46], for which an interpretation as surface defects within $\mathrm{AdS}_6 \times S^2 \times \Sigma_2$ Type IIB vacua was found.

We constructed in section 7.4 a new class of AdS_2 solutions to Type IIA supergravity with $\mathcal{N} = 4$ supersymmetry realised as $\mathrm{AdS}_2 \times S^2 \times S^2$ foliations over 4 intervals. These backgrounds arise after performing a non-Abelian T-duality transformation (with respect to a freely acting SU(2) group) on the class of solutions to Type IIB supergravity of section 7.2.

We then focused on their defect CFT interpretation in section 7.5. We saw that for a particular brane profile the resulting background asymptotes locally to a Gaiotto-

Maldacena geometry, which suggests that it should be dual to a line defect CFT within the 4d $\mathcal{N}=2$ SCFT dual to this geometry. By computing the quantised charges, we realised that the supergravity solution was generated by D2-F1-D4′-NS5′ branes ending on a D4-D6-NS5 bound system. From this brane set-up we constructed a 1d quiver quantum mechanics that, we propose, flows in the IR to the SCQM dual to the AdS$_2$ solution. We stress that the 1d quiver field theory that we have constructed is an elaborated quantum mechanics described by a D2-brane box model of the type constructed in [149]. It is, in deed, the first holographic realisation of a general brane box. We showed that this quiver can be interpreted as a result of embedding the defect branes of the solution in the 4d $\mathcal{N}=2$ background theory. Following [43] we gave an interpretation to the massive F1-strings present in the solution in terms of baryon vertices within the 4d $\mathcal{N}=2$ SCFT, analogously to what was done in section 7.3. This is consistent with an interpretation of the AdS$_2$ solution as describing backreacted baryon vertices within the 4d $\mathcal{N}=2$ SCFT, living in a D4-NS5-D6 subsystem of the complete brane set-up. In this interpretation the SCQM arises in the low energy limit of a system of D4-NS5-D6 branes in which one-dimensional defects are introduced. These defects consist on D4′-brane baryon vertices, connected to the D4-branes with F1-strings, and D2-brane baryon vertices connected to the D6 with F1-strings. In the IR the gauge symmetry on the D4-branes, which played the role of colour branes in the 4d SCFT, becomes global, turning them from colour to flavour branes. In turn, the D2-branes, stretched between the two field theory directions present in the brane set-up, become the new colour branes of the backreacted geometry. Extra NS5′-branes present in the brane set-up make this possible.

We next considered in section 7.6 a class of $\mathcal{N}=4$ AdS$_2 \times S^3 \times S^2 \times S^1$ backgrounds fibred over two intervals in Type IIB supergravity. Such solutions arise in the near-horizon limit of a brane solution describing D1-F1-D3 branes ending on the D5-NS5-D7 bound state, which is related by T-duality along the y direction to the F1-D2-D4′-NS5′-D4-NS5-D6 intersection we studied in section 7.3. By choosing semi-localised profiles for the background D5-NS5-D7 branes and taking the near-horizon limit, we arrived at an asymptotically locally AdS$_6 \times S^2 \times \Sigma_2$ geometry, which is the Abelian T-dual of the Brandhuber-Oz solution of massive Type IIA supergravity. This behaviour allowed us to propose an interpretation of the AdS$_2$ solution as holographically dual to an $\mathcal{N}=4$ superconformal quantum mechanics realising a defect within the $\mathcal{N}=1$ Sp(N) 5d SCFT dual to the AdS$_6$ geometry.

In section 7.7, we focused on the particular subclass of $\mathcal{N}=4$ AdS$_2 \times S^3 \times S^2 \times S^1$ solutions fibred over two intervals featured by the absence of D7-branes. Even if this requirement implies that the defect interpretation in AdS$_6$ is lost, this subclass is interesting since we could act locally with an SL$(2,\mathbb{R})$ transformation to generate a vast class of inequivalent backgrounds parametrised by a continuous parameter $\xi \in [0, \frac{\pi}{2}]$. We then considered the two S-dual backgrounds with $\xi = 0$ and $\xi = \frac{\pi}{2}$, and studied their Type IIA realisation by acting with Abelian T-duality along the S^1 present in both backgrounds. In this way we constructed the entire chain of dualities providing the M-theory origin of our S-dual pair of solutions. This allowed us to show that they belong to the general class of $\mathcal{N}=(0,4)$ AdS$_3$ solutions to M-theory classified in [173]. Remarkably, we showed that the T-dual of the $\xi = \frac{\pi}{2}$ solutions is related to the $\mathrm{AdS}_3 \times S^3 \times S^3$ backgrounds studied in [58], which were shown to asymptote locally to the $\mathrm{AdS}_7/\mathbb{Z}_k \times S^4$ vacuum geometry of M-theory. Thus, in the absence of D7-branes we lost the line defect interpretation within AdS_6 in Type IIB, but we recovered a surface defect interpretation within the $\mathcal{N} = (1, 0)$ 6d SCFT dual to the $\mathrm{AdS}_7/\mathbb{Z}_k$ solution in M-theory.

We concluded in section 7.8 by deriving the non-Abelian T-duals in Type IIA of the S-dual pairs with $\xi = 0$ and $\xi = \frac{\pi}{2}$ and discussing their embeddings in M-theory.

All in all, we have further the study of the holographic correspondence for low-dimensional AdS backgrounds. In doing this, we have also opened new paths which can be followed in order to obtain new interesting results. For instance, new AdS_2 and AdS_3 solutions to Type II supergravities have been constructed, but we still lack a complete classification of these solutions. In particular, non-Abelian T-duality deserves a special mention. It has being used to construct new AdS_2 solutions, also proving very useful for holography. Furthermore, the success of its applicability in our work suggests that it may be possible to use this tool to obtain a general classification of AdS_2 solutions that asymptote to Gaiotto-Maldacena geometries in the UV. In more general terms, non-Abelian T-duality still lacks a full understanding so it deserves further study.

It would also be interesting to construct a more general and systematic classification of $\mathcal{N} = 4$ $\mathrm{AdS}_2 \times S^3$ solutions to Type IIB supergravity, containing those in sections 7.6 and 7.7. In particular including an additional warping between the AdS_2 and the S^3 factors. One could try to search for these solutions in lower dimensional gauged supergravities, as initiated in [34]. These more general backgrounds would be described in terms of a brane intersection involving dyonic membranes, as it has been highlighted in M-theory for $\mathrm{AdS}_3 \times S^3$ backgrounds [59]. Another interesting research direction is the construction of the quiver defining the superconformal quantum mechanics dual to the AdS_2 solution studied in subsection 7.6.1, following the ideas of [58, 66, 67]. Such a field theory would explicitly describe a conformal line defect within the 5d SCFT dual to the $\mathrm{AdS}_6 \times S^2 \times \Sigma_2$ background emerging in the asymptotics.

We have also shown that mirror symmetry can be realised in the absence of a Coulomb branch. In particular, we saw an example of a CFT_2 where S-duality interchanges twisted and untwisted hypermultiplets. This is quite surprising and deserves more research.

Besides, the first example of a holographic dual of a general brane box system was obtained for a particular AdS_2 solution. However, one expects that this interpretation of the dual SCQM as line defect within a 4d $\mathcal{N} = 2$ SCFT should be possible for a class of solutions which locally asymptotes to more general Gaiotto-Maldacena solutions.

With respect to the $\mathrm{AdS}_1/\mathrm{SCQM}$ correspondence, we have explored the concept of central charge of a SCQM. In particular, we have followed the hypothesis that the same formula that is valid for 2d CFTs should be valid for SCQMs. The agreement of the central charges obtained in this way with that of the dual supergravity solutions provides support for this conjecture. Nevertheless, more investigation is necessary in order to arrive at a final conclusion.

Conclusiones

En esta tesis hemos construido nuevas soluciones AdS_2 y AdS_3 a las supergravedades de Tipo II y explorado la correspondencia holográfica a través de ellas.

Como se explica en el capítulo 5, esto se logró a través de una serie de pasos. El primero es obtener uno de estos fondos AdS de dimensión baja. Esto se consiguió tomando el límite de horizonte cercano, mediante dualidad-S o -T de fondos conocidos o restringiendo una clase más general de soluciones con una cierta cantidad de supersimetría (calculada usando herramientas de estructura-G). Tras encontrar una tal solución, el siguiente paso consiste en calcular las cargas cuantizadas para obtener la distribución de branas y luego considerar una configuración de branas de Hanany-Witten compatible. Después uno considera los modos sin masa de las cuerdas abiertas que acaban en las diferentes branas, lo cual da lugar a los posibles multipletes de la teoría de *quiver* dual. Se conjetura que estas teorías son deformaciones en el UV de las CFTs duales a la correspondiente solución AdS de supergravedad. Para apoyar esta hipótesis, uno puede comparar la carga central obtenida a partir de la solución de supergravedad con la de la teoría de campos de *quiver* que vive en la configuración de branas subyacente. Estas dos cantidades coinciden en el IR para nuestras soluciones, implicando que la hipótesis antes mencionada debería ser cierta.

El capítulo 7 se dedicó al escenario $\text{AdS}_3/\text{CFT}_2$. Comenzamos en la sección 6.1 presentando una nueva clase de soluciones $\text{AdS}_3 \times S^3 \times M_4$ de supergravedad masiva de Tipo IIA y estudiando su supersimetría. Mostramos que en el caso más general aparece en el límite de horizonte cercano de una intersección de branas D2-D4-D6-NS5-D8. Tras esto exploramos dos clases de soluciones relevantes.

La primera consiste en el caso donde $M_4 = S^2 \times \Sigma_2$ con Σ_2 una superficie de Riemann. En la sección 6.2 mostramos que tales geometrías en el UV fluyen asintótica y localmente a los fondos $\text{AdS}_7 \times S^2 \times I$ construidos en [126]. Entonces embebimos los *quivers* 2d con $\mathcal{N} = (0, 4)$ asociados a las soluciones AdS_3 en los *quivers* 6d que describen las CFTs 6d con $\mathcal{N} = (1, 0)$ duales a los espacios AdS_7. Nuestro análisis prueba la coincidencia exacta entre la cargas central de la teoría de campos y la holográfica, incluso siendo ambas cantidades divergentes debido a la naturaleza no compacta del defecto. La interpretación de defectos se basa en el hecho de que la presencia de la dirección no compacta permite construir la geometría asintóticamente AdS_7 y, por tanto, completar las soluciones AdS_3 no compactas en el UV.

La segunda subclase de soluciones AdS_3 tiene lugar cuando $M_4 = \mathbb{T}^3 \times I$. El estudio de estos fondos se particularizó para los casos sin y con masa. El primero fue explorado en la sección 6.3, donde mostramos que la supersimetría aumenta a $\mathcal{N} = (4, 4)$ y que esta subclase aparece en el límite de horizonte cercano de una intersección de branas D2-D4-NS5 y es holográficamente dual a CFTs 2d con 8 supercargas. A pesar de que estas configuraciones fueron estudiadas hace mucho tiempo en [138, 139], sus teorías de campos duales no habían sido exploradas y, por tanto, nuestros resultados han contribuido a completar esta laguna. Destacamos que la descripción global que hallamos para estas soluciones AdS_3 requiere de la presencia de planos fijos *orientifold* ONS5. Aunque son objetos bien definidos en teoría de cuerdas, deben estar completamente localizados y no deslocalizadas, como lo están a lo largo del $\mathbb{T}^3$ en nuestro caso. Argumentamos que la deslocalización podría ser simplemente un artefacto de la aproximación de supergravedad que desaparece en teoría de cuerdas. Sin embargo, si uno considera un planteamiento más conservador, la existencia de soluciones con planos ONS5 deslocalizados sugiere que debe haber soluciones similares donde los planos ONS5 están localizados, como suele ser el caso cuando aparecen O-planos. Tales construcciones son mucho más complicadas que las que empleamos y están fuera del alcance de esta tesis, pero nuestras soluciones proveen esperanza de que son posibles. Por otro lados, conectamos esta clase de soluciones AdS_3 de Tipo IIA sin masa con teoría M, de este modo relacionando nuestras soluciones con aquellas construidas en [38], caracterizadas por una geometría $\mathrm{AdS}_3 \times S^2 \times \mathbb{T}^4 \times I$. Este conocimiento provee una nueva interpretación para las QFTs de *quiver* duales con $\mathcal{N} = (4, 4)$ que construimos y también para aquellas con $\mathcal{N} = (0, 4)$ en [38] somo deformaciones de una QFT 2d con $\mathcal{N} = (4, 4)$ cuya cantidad de supersimetría preservada depende de cómo haya sido deformada en el UV. Hemos completado nuestro análisis con un estudio en teoría de cuerdas Tipo IIB, conectando de esta manera nuestro par $\mathrm{AdS}_3/\mathrm{CFT}_2$ con el estudiado en [38] a través de dualidad-S. Este desarrollo en Tipo IIB muestra que la simetría especular en 2d intercambia los escalares de los multipletes vectoriales y los hipermultipletes (y por tanto las ramas de Coulomb y Higgs) como en 3d [108, 151]. Que la simetría especular pueda aún llevarse a cabo de esta manera en teorías que carecen de una rama de Coulomb es un resultado destacable de nuestro análisis. Estas soluciones AdS_3 en Tipo IIB proveen ejemplos concretos dentro de la amplia clasificación de vacíos $\mathrm{AdS}_3 \times S^2 \times M_5$ con M_5 compatible con una estructura de identidad derivada en in [51].

Finalmente, en la sección 6.4 estudiamos el caso masivo de la clase de soluciones $\mathrm{AdS}_3 \times S^2 \times \mathbb{T}^4 \times I$. Obtuvimos soluciones con partes no compactas pegadas entre sí con D8-branas localizadas, delimitadas entre D8/O8s. Estas soluciones pueden ser embebidas globalmente en teoría de cuerdas Tipo I′. Esto nos permitió proponer una CFT dual, la cual fue apoyada por la coincidencia exacta de las cargas centrales de ambas teorías. En este caso la condición para la cancelación de anomalías de los *quivers* 2d implica una ligadura adicional en el fondo de supergravedad dual. Tuvimos que implementar esta condición a mano, pero sería interesante reproducirlo con un cálculo de supergravedad.

En el capítulo 7, estudiamos nuevas soluciones AdS_2 y, cuando fue posible, presentamos las intersecciones de branas subyacentes y construimos la mecánica cuántica de *quiver* dual. Esto provee nueva evidencia para la correspondencia AdS_2/SCQM.

En la sección 7.1 construimos una vasta clase de soluciones $\mathrm{AdS}_2 \times S^2 \times S^2 \times \mathbb{R}^2 \times S^1 \times I$

de supergravedad Tipo IIA que preservan cuatro supercargas. Estas aparecen en el límite de horizonte cercano de branas D2-F1-D4′-NS5′ acabando en un estado ligado de branas D4-NS5. Una solución particular dentro de esta clase queda determinada por la elección de la distribución de carga D4-NS5. Cierto perfil semilocalizado para estas branas da lugar a dos regímenes interesantes. El primero consiste en acercarse a las branas de defecto D2-F1-D4′-NS5′, las cuales se resuelven en una geometría $\mathrm{AdS}_2 \times S^2 \times S^2 \times \mathbb{R}^2 \times S^1 \times I$ completamente retrorreaccionada. El segundo régimen se manifiesta solo tras un cambio de coordenadas no lineal de la solución anterior, y permite a uno alejarse de las branas de defecto. En este régimen, una geometría AdS_5 emerge asintóticamente, lo cual corresponde a la geometría de horizonte cercano de las branas D4-NS5. Este vacío AdS_5 en particular es el dual-T de la solución $\mathrm{AdS}_5 \times S^5/\mathbb{Z}_n$ de supergravedad Tipo IIB holográficamente dual a SYM 4d con $\mathcal{N} = 4$ modulado por $\mathbb{Z}_n$. Este hecho nos ha permitido interpretar esta clase de soluciones como duales a CFTs de defectos de línea dentro de SYM 4d con $\mathbb{N} = 4$ modulado por $\mathbb{Z}_n$.

Luego nos dedicamos al análisis en Tipo IIB en la sección 7.2 considerando soluciones relacionadas vía dualidad-T a lo largo de una dirección circular ψ a las estudiadas en la sección previa. En este caso obtuvimos una nueva clase de soluciones $\mathrm{AdS}_2 \times S^2 \times S^2 \times \mathbb{R}^2 \times S^1 \times I$ con las mismas supersimetrías que antes, ahora emergiendo del límite de horizonte cercano de branas D1-F1-D5-NS5 acabando en un sistema D3-KK. Calculamos el análogo al segundo régimen descrito arriba, donde una solución $\mathrm{AdS}_5 \times S^5/\mathbb{Z}_n$ de supergravedad de Tipo II emerge asintótica y localmente. Las soluciones entonces admiten también una interpretación como defectos de línea, esta vez en términos de las branas de defecto D1-F1-D5-NS5.

En la sección 7.3 volvemos a la intersección F1-D2-D4′-NS5′-D4-NS5 y la generalizamos para incluir D6-branas. La elección del perfil semilocalizado para las D4-NS5 branas ya no es válida, por lo que el régimen donde la solución se aproxima asintótica locamente al vacío AdS_5 se pierde aquí. Nos enfocamos en un análisis detallado de la mecánica cuántica de *quiver* dual, tomando un ansatz simplificado que consistió en suponer que y es una dirección circular de modo que las D4- y D6-branas están deslocalizadas a lo largo de esta. Es destacable que estas son las mismas mecánicas cuánticas de *quiver* que fluyen en el IR a las CFTs estudiadas en [43], duales a las soluciones $\mathrm{AdS}_2 \times S^3 \times CY_2 \times I$ de Tipo IIA masiva con $\mathcal{N} = 4$ allí estudiadas. La razón subyacente tras este acuerdo es que, cuando las branas están deslocalizadas en y, nuestro sistema de branas está relacionado mediante dos dualidades-T al sistema de branas discutido en [43], que consiste en una intersección D0-F1-D4-D4′-D8. La CFT 1d dual asociada a esta clase de soluciones fue interpretada en términos de vértices bariónicos dentro de teorías de punto fijo 5d que viven en intersecciones D4-D8, y se demostró que una geometría asintótica y localmente AdS_5 emerge en ciertas soluciones. Nuestros descubrimientos demuestran que las mismas mecánicas cuánticas de *quiver* describen en el UV CFTs de defecto de línea asociadas a vértices bariónicos dentro de SCFTs 4d con $\mathcal{N} = 2$. No obstante, en nuestro caso carecemos de una compleción de defecto dentro de un vacío AdS_5 en Tipo IIA. Sin embargo, la deslocalización en y permite conectar con la clase de soluciones $\mathrm{AdS}_3 \times S^2 \times S^2 \times S^1 \times \Sigma_2$ de supergravedad de Tipo IIB estudiadas en [46], para las cuales se halló la interpretación como defectos de superficie dentro de vacíos $\text{AdS}_6 \times S^2 \times \Sigma_2$ de Tipo IIB.

Construimos en la sección 7.4 una nueva clase de soluciones AdS_2 de supergravedad de Tipo IIA con supersimetría $\mathcal{N} = 4$ realizada como foliaciones de $\text{AdS}_2 \times S^2 \times S^2$ sobre 4 intervalos. Estos fondos aparecen tras realizar una transformación de dualidad-T no abeliana (con respecto a un grupo SU(2) que actúa libremente) sobre la clase de soluciones de supergravedad Tipo IIB de la sección 7.2.

Después nos enfocamos en la interpretación como CFT de defecto en la sección 7.5. Vimos que para un perfil de branas particular el fondo resultante se aproxima asintótica y localmente a una geometría de Gaiotto-Maldacena, lo cual sugiere que debería ser dual a una CFT de defecto de línea dentro de la SCFT 4d con $\mathcal{N} = 2$ dual a esta geometría. Calculando las cargas cuantizadas, nos dimos cuenta de que la solución de supergravedad fue generada por branas D2-F1-D4′-NS5′ acabando en un sistema ligado D4-D6-NS5. Para esta configuración de branas construimos una mecánica cuántica de *quiver* 1d que, proponemos, fluye en el IR a la SCQM dual a la solución AdS_2. Recalcamos que la teoría de *quiver* 1d que hemos construido es una mecánica cuántica bastante compleja descrita por un modelo de caja de D2-branas del tipo construido en [149]. El nuestro es, de hecho, la primera descripción holográfica de una caja de branas general. Mostramos que este *quiver* puede interpretarse como el resultado de embeber las branas de defecto de la solución en la teoría de fondo 4d con $\mathcal{N} = 2$. Siguiendo [43] dimos una interpretación a las F1-cuerdas masivas presentes en la solución en términos de vértices bariónicos dentro de la SCFT 4d con $\mathcal{N} = 2$, análogamente a lo hecho en la sección 7.3. Esto es consistente con la interpretación de la solución AdS_2 como vértices bariónicos retrorreaccionados dentro de la SCFT 4d con $\mathcal{N} = 2$, la cual vive en el subsistema D4-NS5-D6 de nuestra configuración de branas completa. En esta interpretación la SCQM emerge en el límite de baja energía de un sistema de D4-NS5-D6 branas en el que se introducen defectos unidimensionales. Estos defectos consisten en vértices bariónicos de D4-branas, conectadas a las D4′-branas a través de F1-cuerdas, y vértices bariónicos de D2-branas, conectas a las D6-branas con F1-cuerdas. En el IR la simetría *gauge* en las D4-branas, las cuales jugaron el papel de branas de color en la SCFT 4d, se vuelve global, cambiándolas de branas de color a branas de sabor. A su vez, las D2-branas, extendidas entre las dos direcciones de teoría de campos presentes en la configuración de branas, se convierten en las nuevas branas de color de la geometría retrorreaccionada. Las NS5′-branas adicionales presentes en la configuración de branas hacen que esto sea posible.

A continuación consideramos en la sección 7.6 una clase de fondos $\text{AdS}_2 \times S^3 \times S^2 \times S^1$ fibrados sobre dos intervalos en supergravedad Tipo IIB. Tales soluciones aparecen en el límite de horizonte cercano de una solución de branas que describe D1-F1-D3 branas acabando en el estado ligado D5-NS5-D7, que está relacionada mediante dualidad-T a lo largo de la dirección y a la intersección F1-D2-D4′-NS5′-D4-NS5-D6 que estudiamos en la sección 7.3. Eligiendo perfiles semilocalizados para las D5-NS5-D7 branas de fondo y tomando el límite de horizonte cercano, llegamos a una geometría asintótica y localmente $\text{AdS}_6 \times S^2 \times \Sigma_2$, que es la dual-T abeliana de la solución de Brandhuber-Oz de supergravedad masiva Tipo IIA. Este comportamiento nos permitió proponer una interpretación para la solución AdS_2 como dual holográfica a una mecánica cuántica superconforme con

$\mathcal{N}=4$ que describe un defecto dentro de la SCFT Sp(N) 5d con $\mathcal{N}=1$ dual a la geometría AdS_6.

En la sección 7.7 nos enfocamos en la subclase particular de soluciones $\mathrm{AdS}_2\times S^3\times S^2\times S^1$ fibradas sobre dos intervalos caracterizadas por la ausencia de D7-branas. incluso si este requerimiento implica que la interpretación de defecto en AdS_6 se pierde, esta subclase es interesante dado que podemos actuar localmente en ella con una transformación $\mathrm{SL}(2,\mathbb{R})$ para generar una vasta clase de fondos no equivalentes parametrizados por un parámetro continuo $\xi\in[0,\frac{\pi}{2}]$. Entonces consideramos los fondos duales-S con $\xi=0$ y $\xi=\frac{\pi}{2}$, y estudiamos su realización en Tipo II tras actuar en ellos con dualidad-T abeliana a lo largo de la S^1 presente en ambos fondos. De esta manera construimos una cadena completa de dualidades y proveemos el origen de teoría M de nuestra pareja de soluciones duales-S. Esto nos permitió mostrar que pertenecen a la misma clase general de soluciones AdS_3 con $\mathcal{N}=(0,4)$ de teoría M clasificadas en [173]. Mostramos que la dual-T a la solución con $\xi=\frac{\pi}{2}$ está relacionada con los fondos $\mathrm{AdS}_3\times S^3\times S^3$ estudiados en [58], los cuales se aproximan asintótica y localmente al vacío $\mathrm{AdS}_7/\mathbb{Z}_k\times S^4$ de teoría M. De este modo, en ausencia de D7-branas perdemos la interpretación como defecto de línea dentro de AdS_6 en Tipo IIB, pero recuperamos una interpretación como defecto de superficie dentro de la SCFT 6d con $\mathcal{N}=(1,0)$ dual al la solución $\mathrm{AdS}_7/\mathbb{Z}_k$ en teoría M.

Concluimos en la sección 7.8 derivando los duales-T no abelianos en Tipo IIA del par dual-S con $\xi=0$ y $\xi=\frac{\pi}{2}$ y discutiendo su embebimiento en teoría M.

Para resumir, hemos avanzado el estudio de la correspondencia holográfica para fondos AdS de dimensión baja. Al hacer esto, también hemos abierto nuevos caminos que pueden ser seguidos para obtener nuevos resultados interesantes. Por ejemplo, hemos construido nuevas soluciones AdS_2 y AdS_3 de supergravedad Tipo II, pero aún carecemos de una clasificación completa de dichas soluciones. En particular, la dualidad-T no abeliana merece una mención especial. Se ha usado para construir nuevas soluciones AdS_2, resultando muy útil para la holografía. Aún más, el éxito de su aplicabilidad en nuestro trabajo sugiere que podría se posible usar esta herramienta para obtener una clasificación general de soluciones AdS_2 que se aproximan asintótica y localmente a geometría de Gaiotto-Maldacena en el UV. En términos generales, la dualidad-T no abeliana carece de una comprensión completa y, por tanto, merece seguir siendo estudiada.

También sería interesante construir una clasificación más sistemática y general de las soluciones $\mathrm{AdS}_2\times S^3$ de supergravedad Tipo IIB, la cual contenga aquellas de las secciones 7.6 y 7.7. En particular que incluya una distorsión adicional entre los factores AdS_2 y S^3. Uno podría intentar buscar estas soluciones en supergravedades gaugeadas de dimensión baja, como se inició en [59]. Otra línea de investigación interesante consiste en construir los *quivers* que definen la mecánica cuántica superconforme dual a la solución AdS_2 estudiada en la subsección 7.6.1, siguiendo las ideas de [58, 66, 67]. Tal teoría de campos describiría explícitamente un defecto de línea conforme dentro de la SCFT 5d dual al fondo $\mathrm{AdS}_6\times S^2\times\Sigma_2$ que aparece en las asíntotas.

Además, el primer ejemplo de dual holográfico a un sistema de cajas de branas general se obtuvo para una solución AdS_2 particular. Sin embargo, uno espera que esta interpretación de la SCQM dual como defecto de línea dentro de una SCFT 4d con $\mathcal{N}=2$

debería ser posible para una clase de soluciones que se aproxime asintótica y localmente a una clase más general de soluciones de Gaiotto-Maldacena.

Con respecto a la correspondencia AdS_1/SCQM, hemos explorado el concepto de carga central de una SCQM. En particular, hemos seguido la hipótesis de que la fórmula válida para las CFTS 2d debería ser también válida para las SCQM. La concordancia de las cargas centrales obtenidas de esta manera con aquellas de las soluciones de supergravedad dual provee apoyo para esta conjetura. Sin embargo, se requiere de más investigación para llegar a una conclusión final.

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